題目: 無線充電IC設計

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電子設計自動化

經歷: 國家晶片中心晶片製作審查委員,
太陽光電發電系統設置審查委員,
IC設計公司研發顧問,電機技師,
永續能源與資源管理師, 永續發展碳管理師

Latest Publications

- Peng-Chang Huang, Yeong-Chau Kuo*, Yi-Chen Liu, and Tai-Haur Kuo, “An Analog Optimum Torque Control IC for a 200W Wind Energy Harvesting System,” accepted by and to be published in IEEE Trans. Circuits Syst. II, Exp. Briefs.
- Y. C. Kuo*, Y. M. Huang, and L. J. Liu, "Integrated Circuit and System Design for Renewable Energy Inverters," International Journal of Electrical Power & Energy Systems, Vol. 64, pp. 50-57, Jan. 2015. (SCI/SSCI, impact factor: 3.289, SCI/SSCI rank factor: 51/262, ENGINEERING, ELECTRICAL & ELECTRONIC, times cited: 6)
- Y. C. Kuo*, Y. J. Luo, and L. J. Liu, "Synthesizable Integrated Circuit and System Design for Solar Chargers," IEEE Transactions on Power Electronics, Vol. 28, Iss. 9, pp. 4260-4266, Sep. 2013. (SCI, impact factor: 7.151, SCI/SSCI rank factor: 13/262, ENGINEERING, ELECTRICAL & ELECTRONIC, times cited: 4)
- Y. C. Kuo*, W. H. Tung, and L. J. Liu, "Smart Integrated Circuit and System Design for Renewable Energy Harvesters," IEEE Journal of Photovoltaics, Vol. 3, Iss. 1, pp. 401-406, Jan. 2013. (SCI, impact factor: 3.712, SCI/SSCI rank factor: 25/92, ENERGY & FUELS, times cited: 5)
- Y. C. Kuo*, Y. J. Luo, and L. J. Liu, "Synthesis Design of Digital Solar Energy Harvesting Integrated Circuits and Systems," IET Optoelectronics, Vol. 6, Iss. 6, pp. 282-289, Dec. 2012. (SCI, impact factor: 1.165, SCI/SSCI rank factor: 176/262, ENGINEERING, ELECTRICAL & ELECTRONIC)
- Y. C. Kuo*, C. H. Liu, and L. J. Liu, "Synthesisable Solar-Harvesting Integrated Circuit and System," IET Renewable Power Generation, Vol. 6, Iss. 6, pp. 408-413, Nov. 2012. (SCI, impact factor: 2.635, SCI/SSCI rank factor: 76/262, ENGINEERING, ELECTRICAL & ELECTRONIC)

Latest Patents and Honor

Patent

- JM Liu, YC Kuo, and TH Kuo, Analog Variable-Frequency Controller and Switching Converter Therewith, Invention patent, USA, Patent. No.: US 7923975 B2, 2011.
- Y. C. Kuo*, W. C. Liu, and T. H. Kuo, “Analog Controller for Inverter,” Invention patent, USA, US 8,842,456 B2, Sep. 23, 2014.
- 劉家銘、郭永超、郭泰豪,類比式可變頻率控制器, 發明專利, 臺灣,發明第I396373號.
- 郭永超、李貞慶、郭泰豪、黃鵬彰,類比式最大功率追蹤控制方法, 發明專利, 臺灣,發明第I381263號.
- 郭永超*、郭泰豪, 換流器之類比控制器, 發明專利, 臺灣, 發明第I444807號, 2014/7/11.

Honor

- 2018 無人機及智慧製造自造者競賽優等(無人機之動態充電積體電路與系統合成設計)
- 2018年金融科技創新競賽佳作(內建人工智慧金融模型之無線能源傳輸積體電路系統設計)
- 2015年龍騰微笑創業競賽晉級總決賽(智慧型無線充電器)
- 2014年全國大學校院智慧電子系統(IE)設計競賽優等獎(太陽能無線充電積體電路與系統設計)
- 2011年旺宏金矽獎設計組優勝獎(能量採集積體電路系統之自動合成設計)
- 2010年旺宏金矽獎設計組評審團銅獎與指導教授獎(以類比積體電路實現太陽能最大發電功率追蹤器)
- 2010年立錡盃電源IC設計暨系統應用競賽佳作獎(智慧型電網與太陽能充電IC之自動合成設計)
- 2009年Discovery Channel採訪報導太陽能充電器之研究成果
- 2009年立錡盃電源IC設計暨系統應用競賽佳作獎(太陽能與風能最大效率之充電器設計)
- 2009年奇景盃IC佈局設計競賽佳作獎

Latest Project and Result

Project

- 107年無人機之動態充電積體電路系統合成設計(MOST 107-2221-E-992-099)
- 106年微換流器之無線充電積體電路系統合成設計(MOST106-2221-E-327-026)
- 104-105年微換流器之積體電路系統合成設計(MOST 104-2221-E-327 -037 -MY2)
- 103年技術移轉授權-智慧型無線充電器(錠暐自動化科技有限公司)
- 103年技術移轉授權-智慧型植物栽培箱(綠光能科技有限公司)
- 103年綠能積體電路系統之合成設計 (主持人, NSC 103-2623-E-327 -001 –ET)
- 102年高低溫度測試模組(主持人, 金屬中心)
- 101年太陽能和風力最佳化轉換效率模擬與理論驗證 (主持人, 金屬中心)
- 100年高效能儲電模組性能測試 (主持人, 金屬中心)
- 100年整合型計畫-可攜式機電裝置之節能關鍵技術研發 (3/3) (共同主持人, NSC 1002218E006001)
- 99年整合型計畫-具最大功率追蹤與控制之併聯型混合太陽能與風力發電電源轉換器晶片設計與系統研製(3/3) (共同主持人, NSC992220E006 006)
- 99年整合型計畫-可攜式機電裝置之節能關鍵技術研發 (2/3) (共同主持人, NSC 992218E006003)
- 98年整合型計畫-具最大功率追蹤與控制之併聯型混合太陽能與風力發電電源轉換器晶片設計 與系統研製(2/3) (共同主持人, NSC 982220E006014)
- 98年整合型計畫-可攜式機電裝置之節能關鍵技術研發 (1/3)(共同主持人, NSC982218E006242)

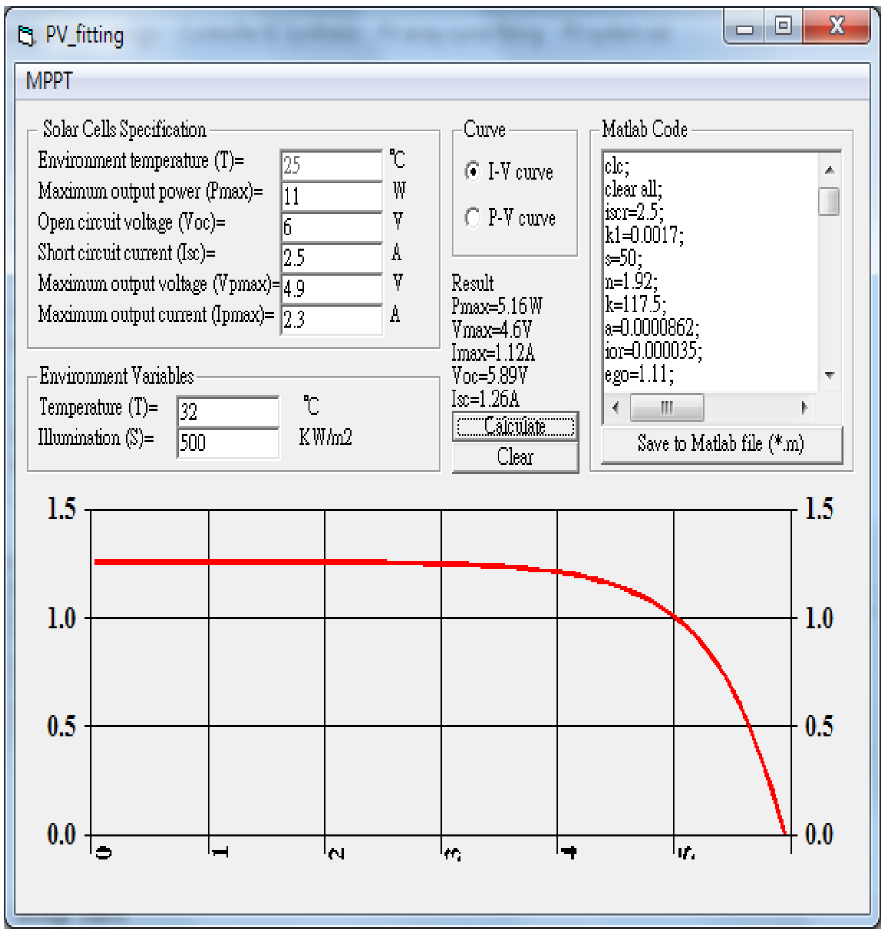
Research result (可供技轉或合作)

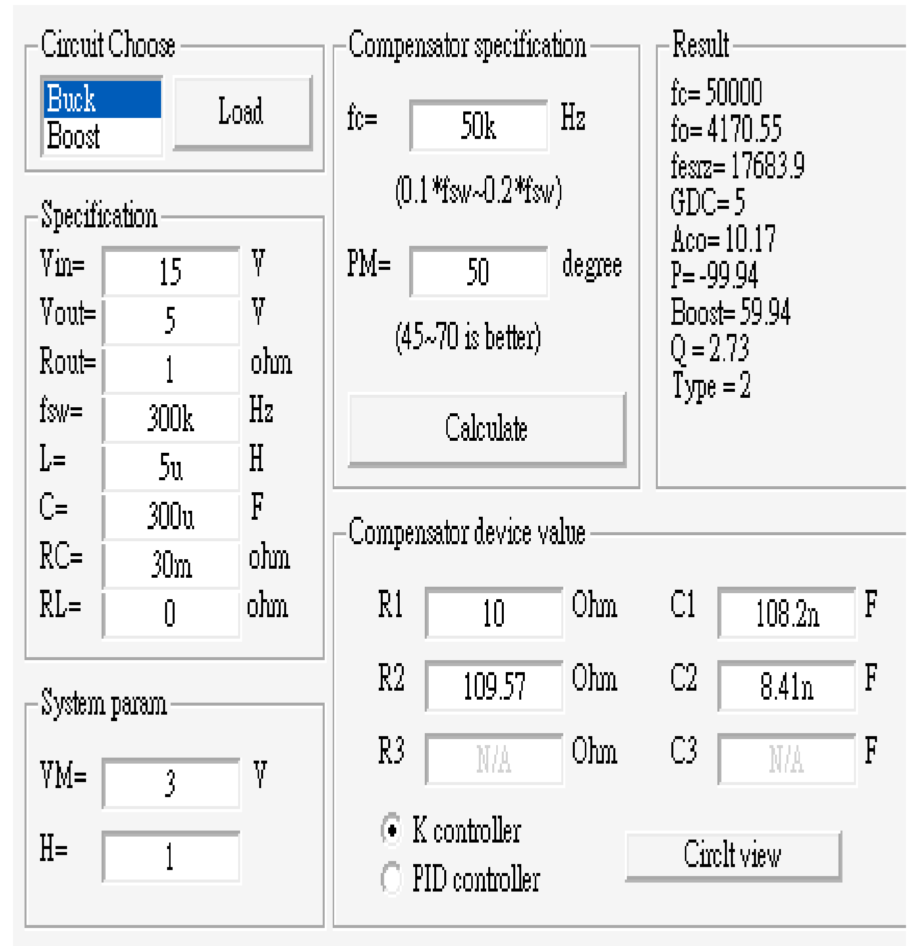
1. 太陽能與風能充電IC與系統設計
2. 電池充電IC與系統設計
3. 綠能IC與系統自動化設計軟體
4. 綠能IC與系統監控軟體

BIPV智慧型無線充電器

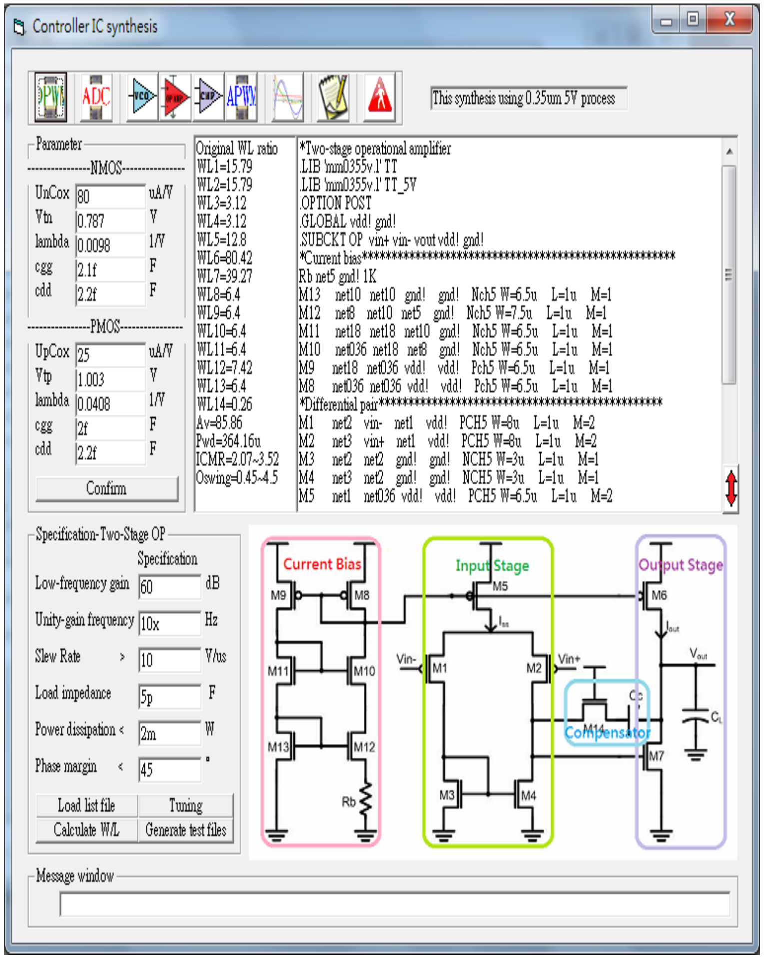
- 建材一體型太陽能板(Building integrated photovoltaic, BIPV) ：可替代建築物的外表包覆材料、代替屋頂、牆面、窗戶、可遮陽，降低建築物外表溫度，兼具建材及發電之功能，降低整體建築成本。
- 全天候的電源供應、結合綠建築的無線充電：由於室內光源發電量低，需透過兼具低功耗和最佳化設計的積體化高效能光能獵能器才能大幅降低功率損耗以增加輸出電能。另外，發展兼具高系統效率和大傳輸距離之無線充電系統則可與建材一體型太陽能板結合，增加電能取得便利性。也能透過能量互補概念連結並解決光源輸出不穩問題，達到不論白天夜晚皆能有效供給電源甚至達到充電效果的目標。
- 結合物聯網的產業應用：在BIPV無線充電模組內，發展一個新型量測系統，此系統主要包含偵測器與無線通訊介面，偵測器能偵測BIPV、無線充電系統與電池的運作資訊，並藉由無線通訊介面形成物聯網，該平台內有本計畫發展的自我診斷軟體，可診斷系統操作情況並作出適當決策，該模組的使用者也能隨時得到模組的運作情形與關鍵參數。

System Synthesis Software



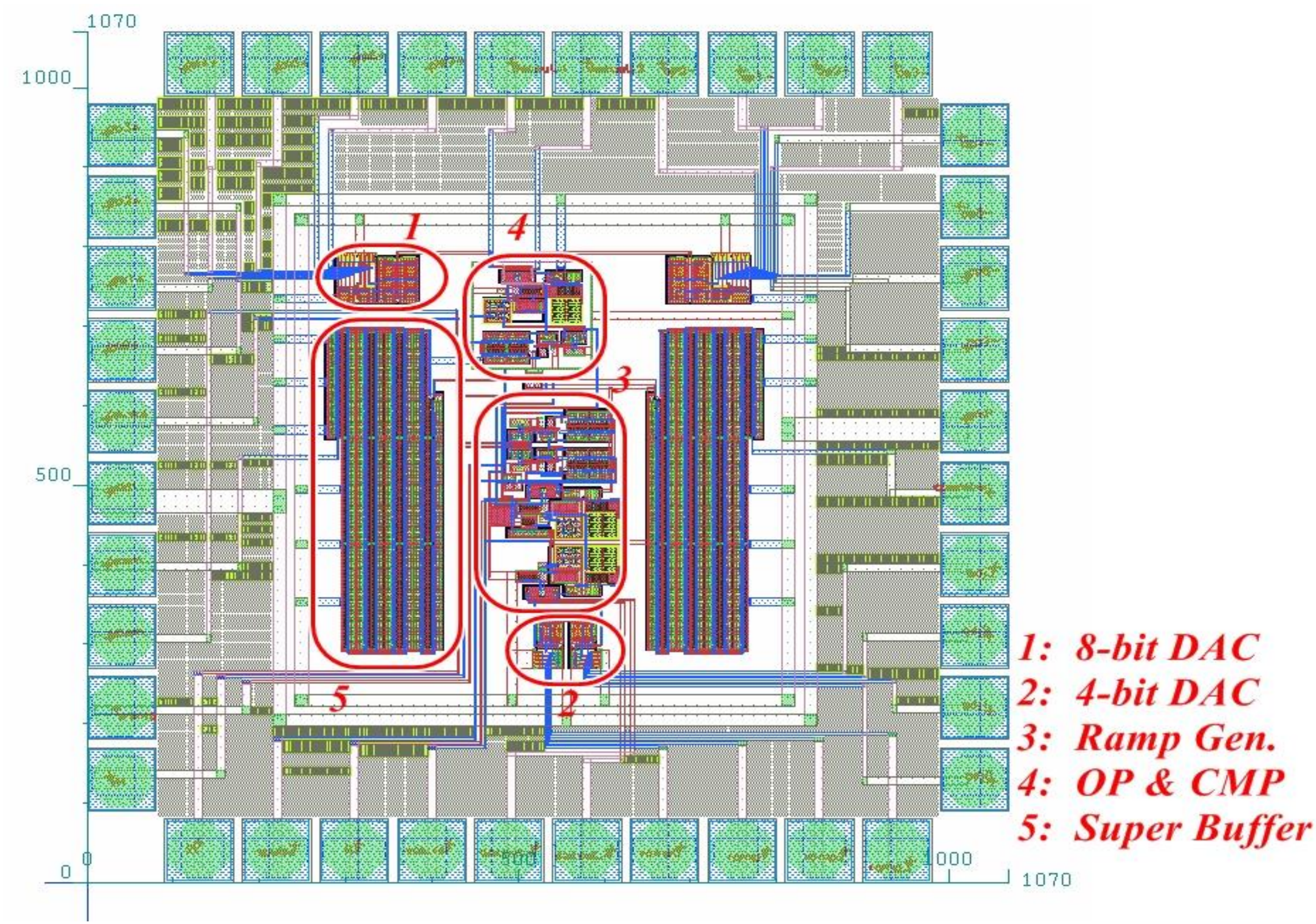


IC Synthesis Software

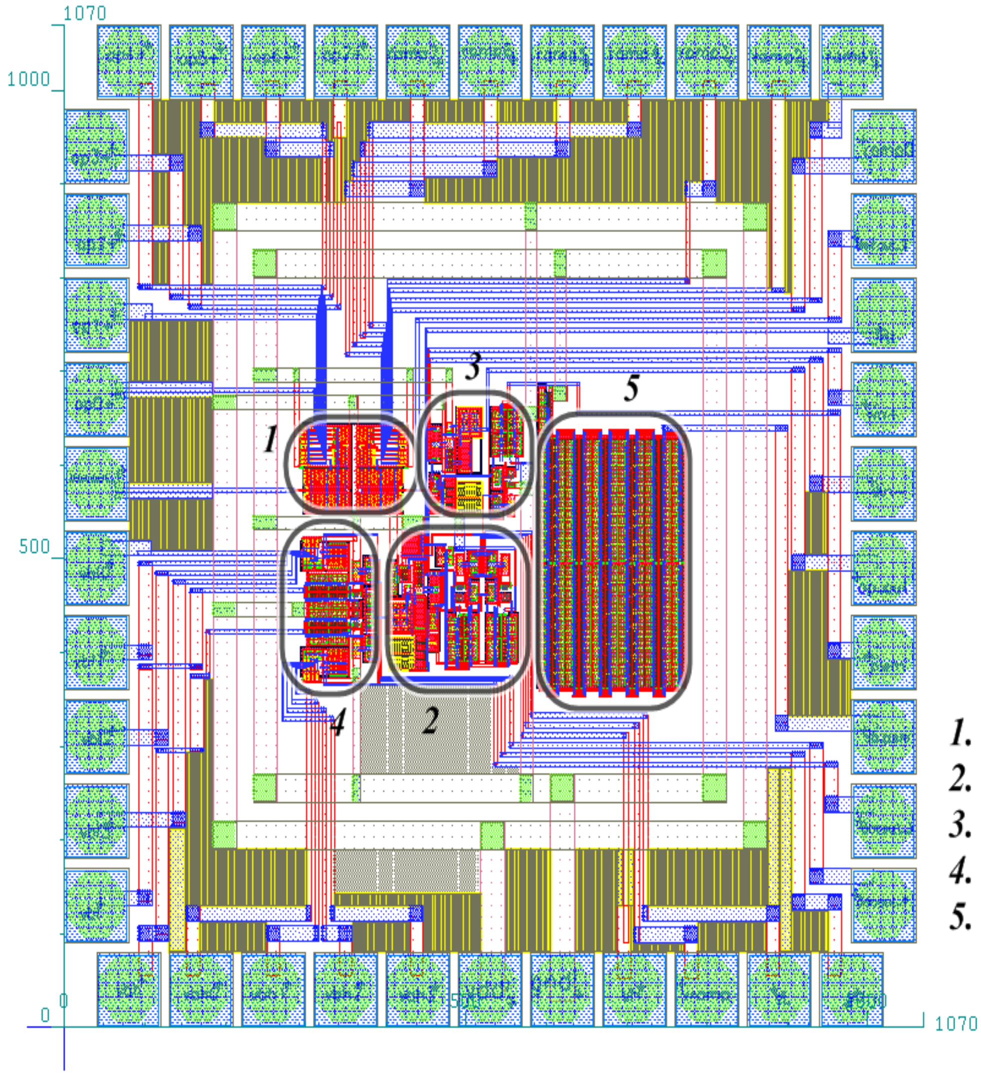


Two-Stage OP		
Term	Target	Simulation
Av	>60dB	75.17dB
Phase Margin	>45°	54.75 °
ICMR	0V-5V	0.06V-4.45V
CMRR	>60 dB	73.52dB
PSRR	>60 dB	76.88dB
Slew Rate	10 V/us	28.75 V/us
Unit-gain frequency	10 MHz	36.92 MHz
Output swing	>3V	0.075V-5V
Offset	>1V	53.31mV
Pwr dissipation	<2mW	0.9mW
Hysteresis Comparator		
Vtrp	110mV	
Ramp Generator		
Frequency	1kHz~60MHz	
Programmable APWM		
Power Dissipation	24.24 mW	
Oscillator Frequency	16 MHz	
Chip size	0.7 mm x 0.7mm	

Controller Architecture (1/2)

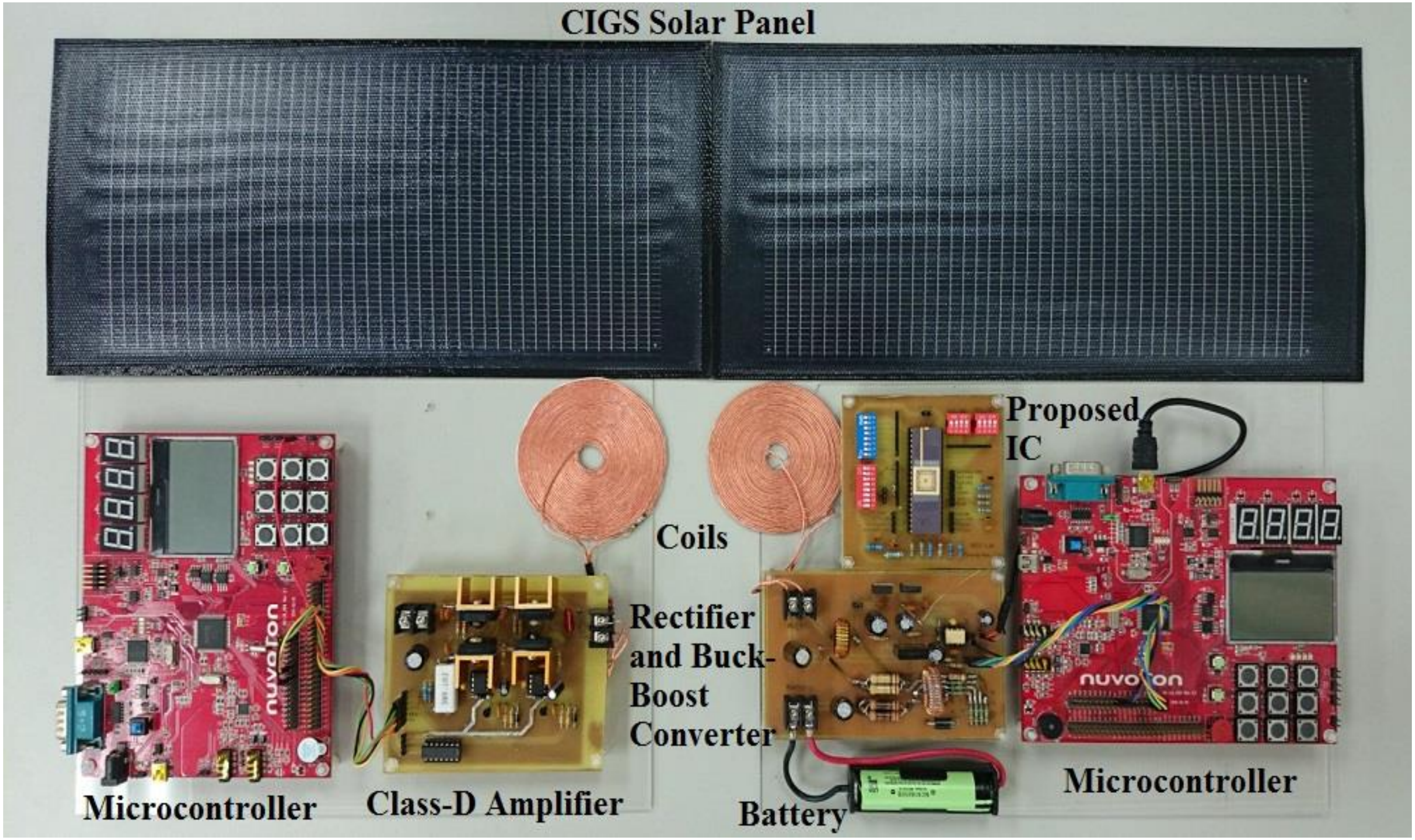


Controller Architecture (2/2)

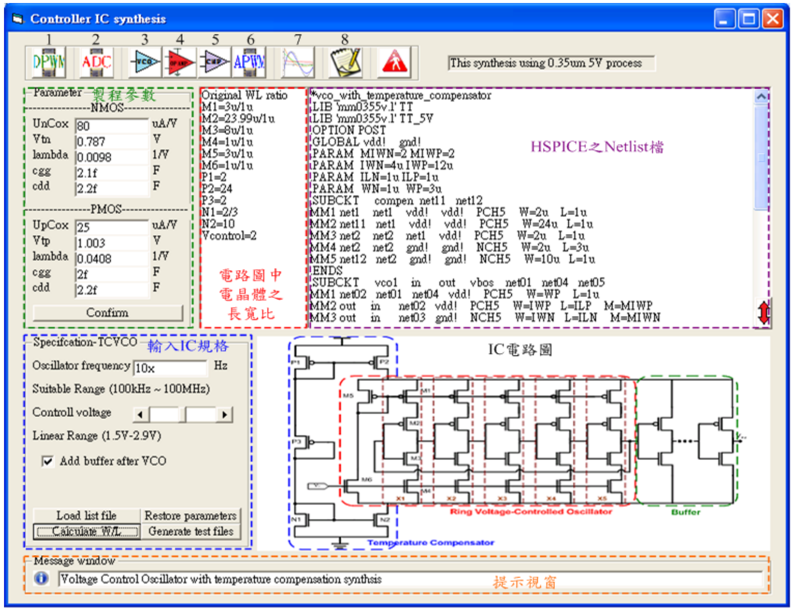


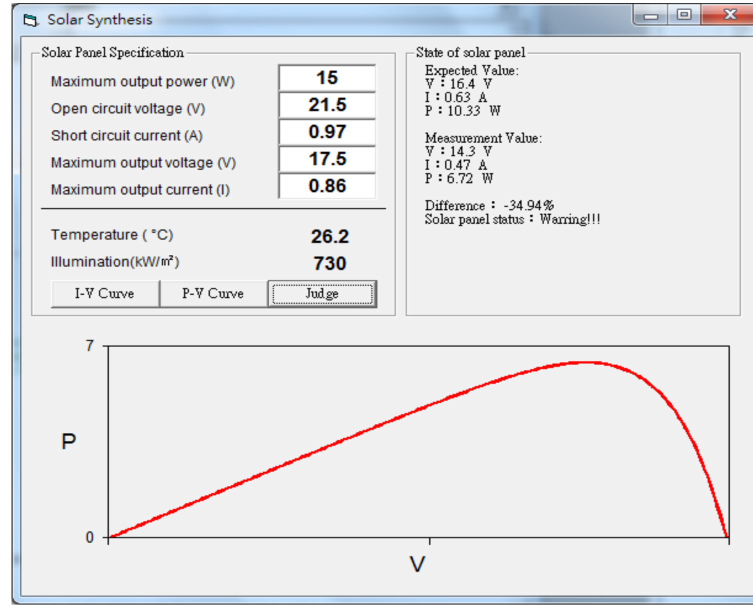
1. 8-bit DAC
2. Ramp Gen.
3. OP & CMP
4. 4-bit DAC & CMP
5. Super Buffer

Measurement



Synthesis and Diagnosis Tool





系統特點：

1.自行研發軟體合成無線充電積體電路系統，減少上市時間

2.本無線充電積體電路系統之電源為再生能源，不用市電

3.研發之軟體，可診斷積體電路系統操作現況，方便維護

4.可適用手機、家電、電動車、BIPV等產品

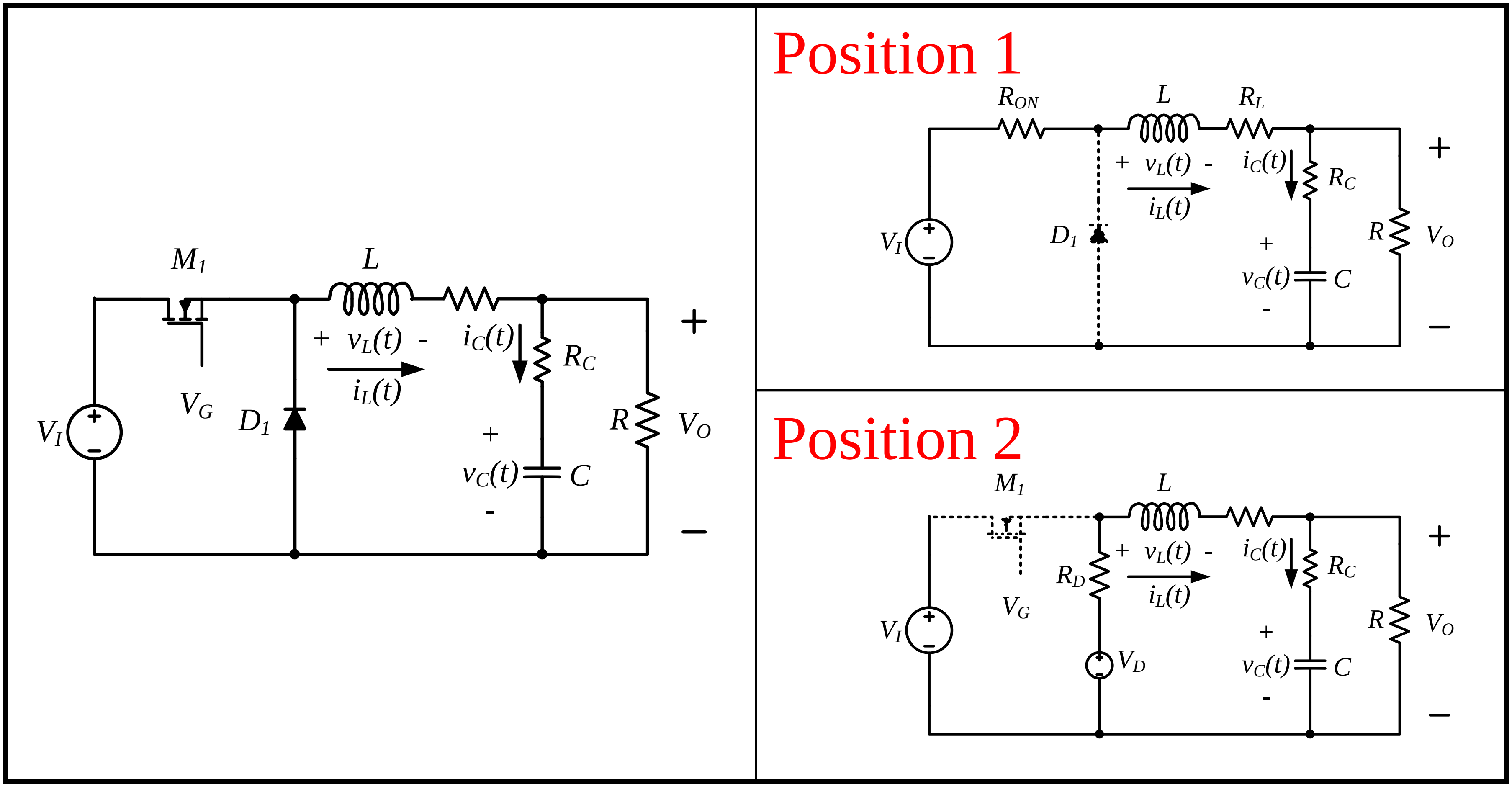
適用領域：能源與環境－其他－能源採集(Energy harvesting)

潛在客戶：手機業者/積體電路業者/軟體業者

Outline

- 1-1. Steady-state analysis
  1) Buck converter
  2) Boost converter
  3) Buck-Boost converter
- 1-2. Transient-state analysis
  1) Buck converter
  2) Boost converter
  3) Buck-Boost converter

Buck Model (1/7)

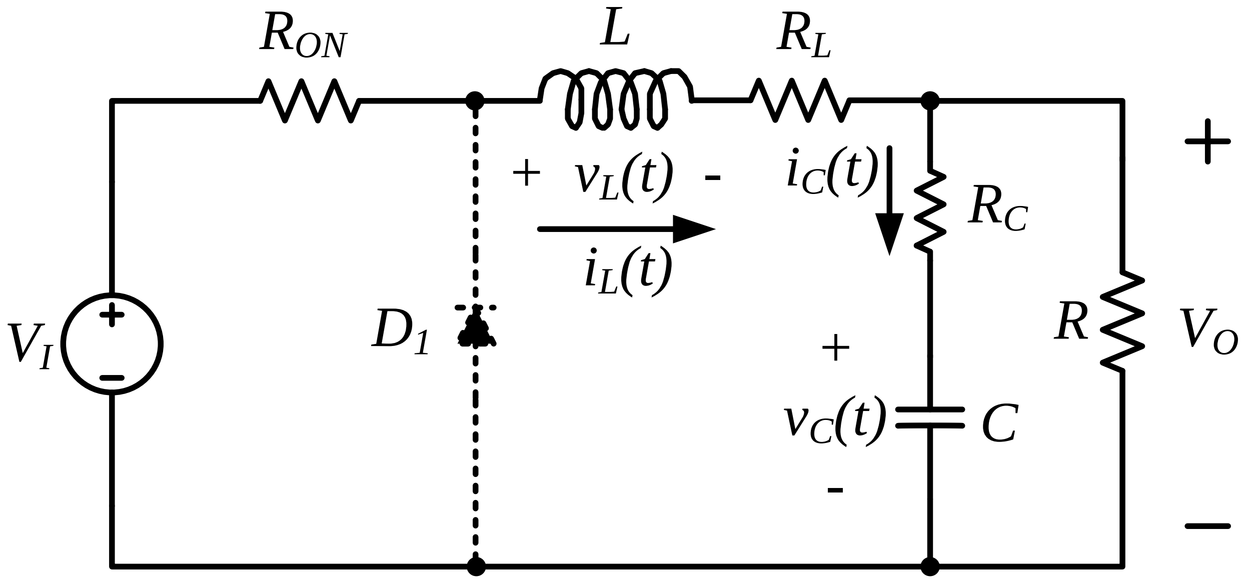


Buck Model (2/7)

• Position 1

$$v_L(t) = V_I - i_L(t)\left(R_L + R_{ON} + R \parallel R_C\right) - v_C(t)\left(\frac{R}{R+R_C}\right)$$

$$i_C(t) = i_L(t)\left(\frac{R}{R+R_C}\right) - v_C(t)\left(\frac{1}{R+R_C}\right)$$

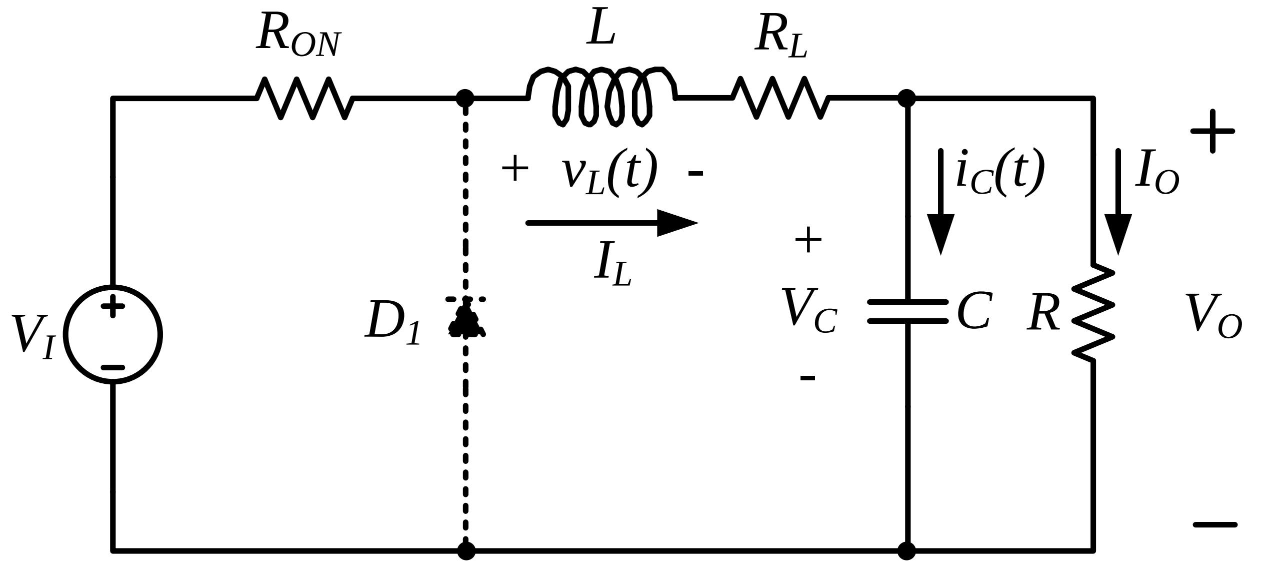


Buck Model (3/7)

- Small-ripple approximation
- Neglect R_C , V_C=V_O

$$v_L(t) = V_I - I_L(R_L + R_{ON}) - V_C$$

$$i_C(t) = I_L - \frac{V_C}{R}$$

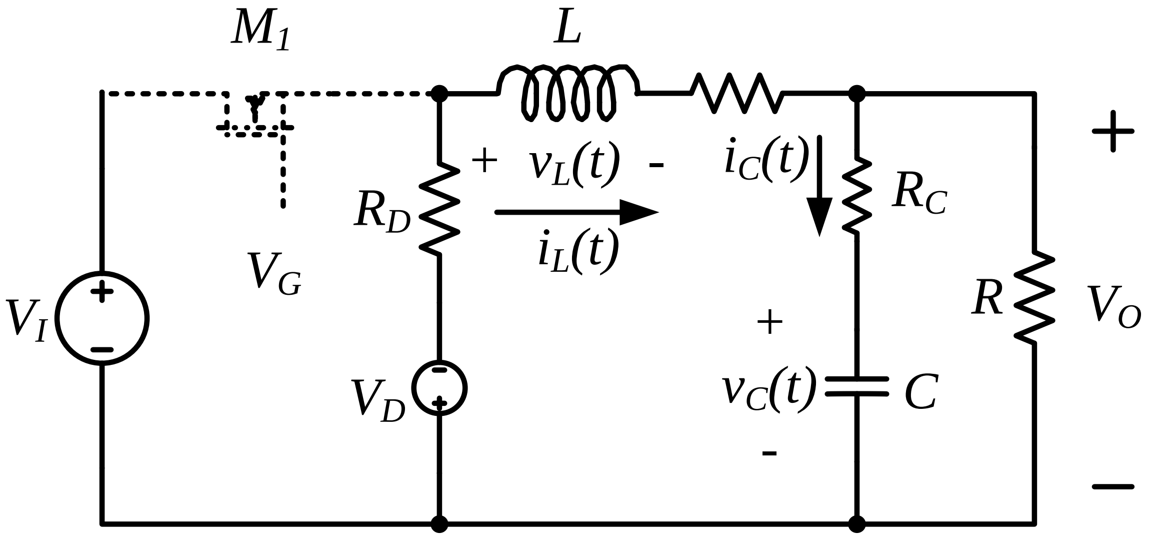


Buck Model (4/7)

•Position 2

$$v_L(t) = -V_D - i_L(t)\left(R_L + R_D + R \parallel R_C\right) + v_C(t)\left(\frac{R}{R+R_C}\right)$$

$$i_C(t) = i_L(t)\left(\frac{R}{R+R_C}\right) - v_C(t)\left(\frac{1}{R+R_C}\right)$$

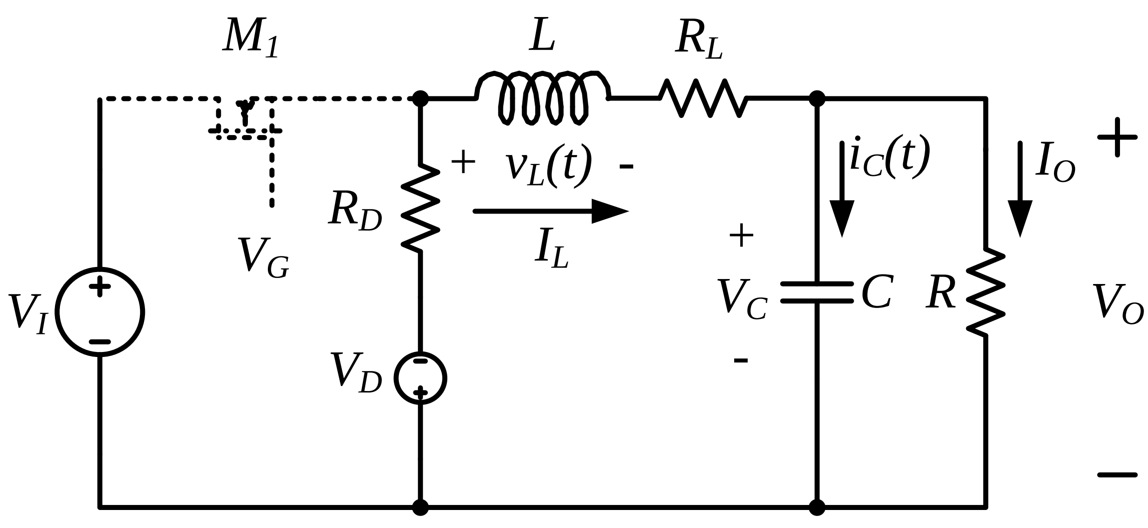


Buck Model (5/7)

- •Small-ripple approximation
- •Neglect R_C , $V_C = V_O$

$$v_L(t) = -V_D - I_L(R_L + R_D) + V_C$$

$$i_C(t) = I_L - \frac{V_C}{R}$$

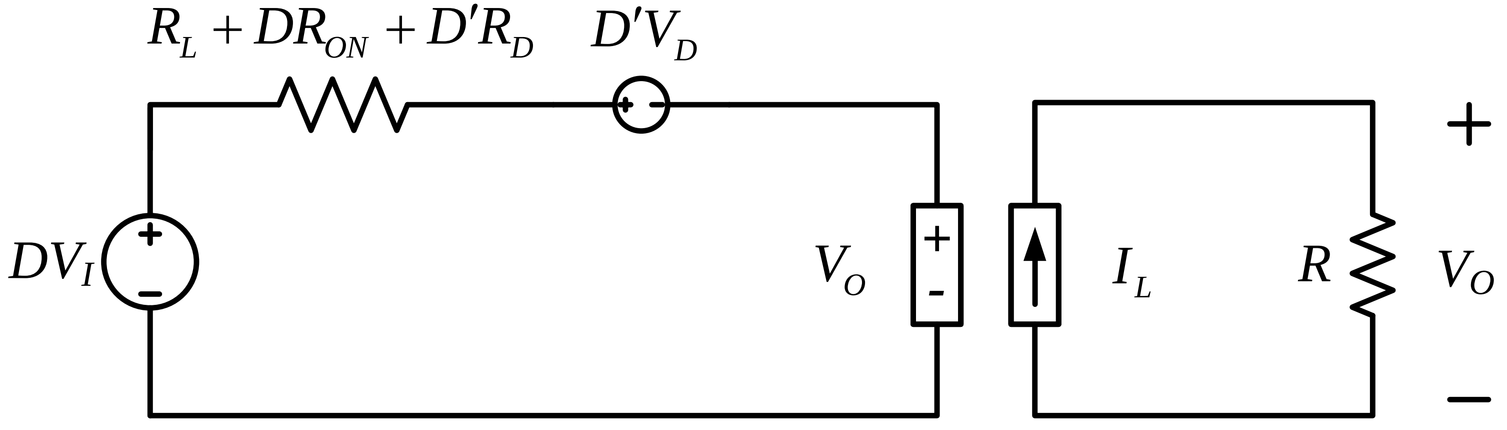


Buck Model (6/7)

- Inductor volt-second balance
- Capacitor Charge Balance

$$\langle v_L(t)\rangle = DV_S - I_L\left(R_L + DR_{ON} + D'R_D\right) - V_O - D'V_D = 0$$

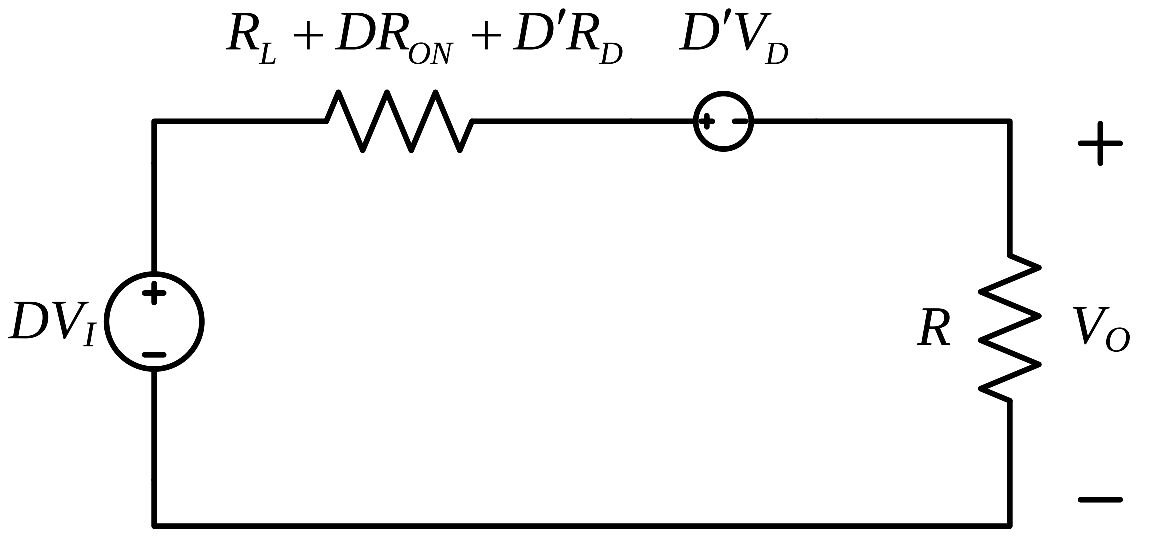
$$\langle i_C(t)\rangle = I_L - \frac{V_O}{R} = 0$$



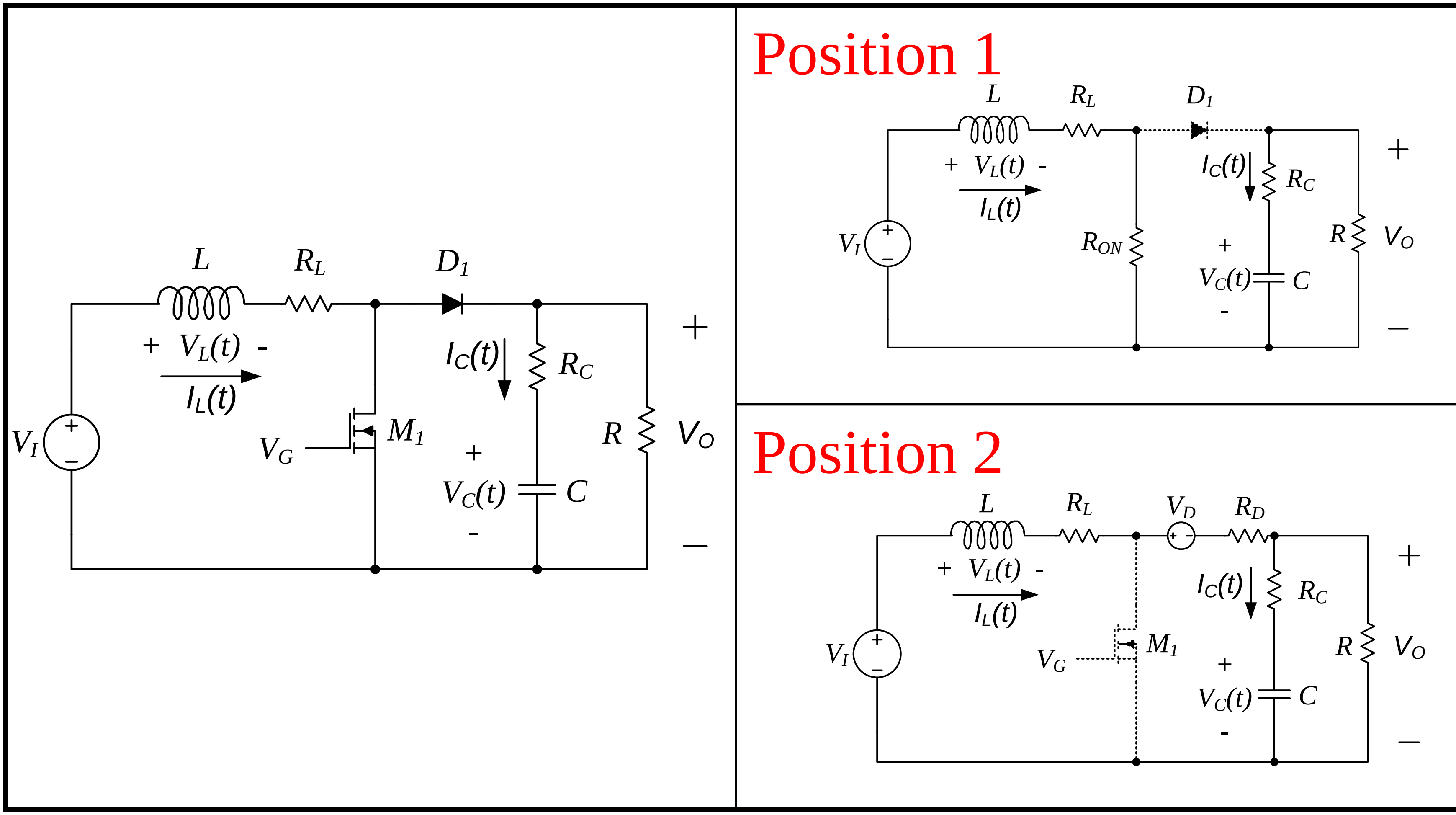
Buck Model (7/7)

• Efficiency

$$\eta = \frac{V_O I_O}{DV_I I_I} = \frac{V_O}{DV_I} = \frac{\left(1 - \frac{D'V_D}{DV_S}\right) R}{R_L + DR_{ON} + D' R_D + R}$$



Boost Model (1/7)

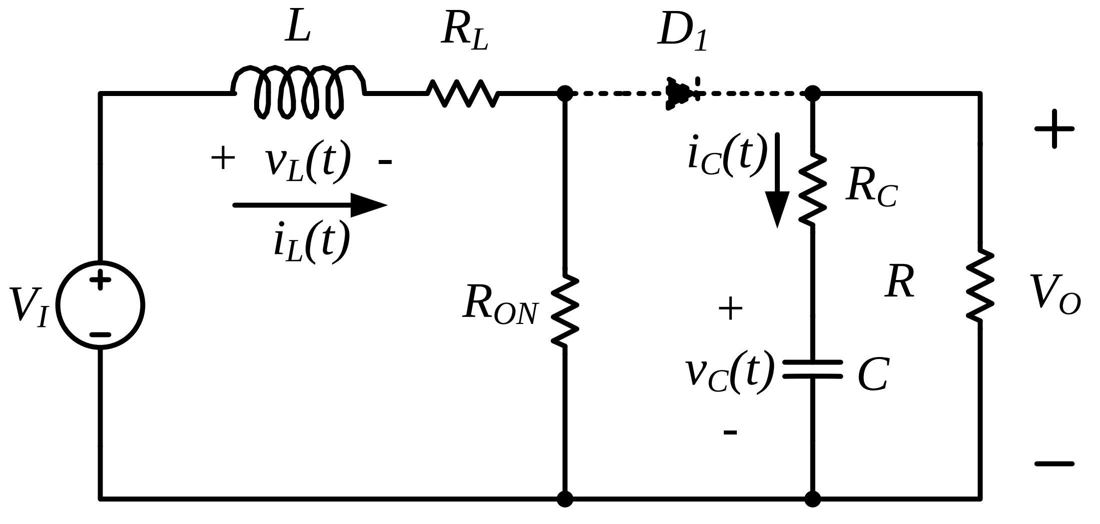


Boost Model (2/7)

- Position 1

$$v_L(t) = V_I - i_L(t)\left(R_L + R_{ON}\right)$$

$$i_C(t) = -v_C(t)\left(\frac{1}{R + R_C}\right)$$

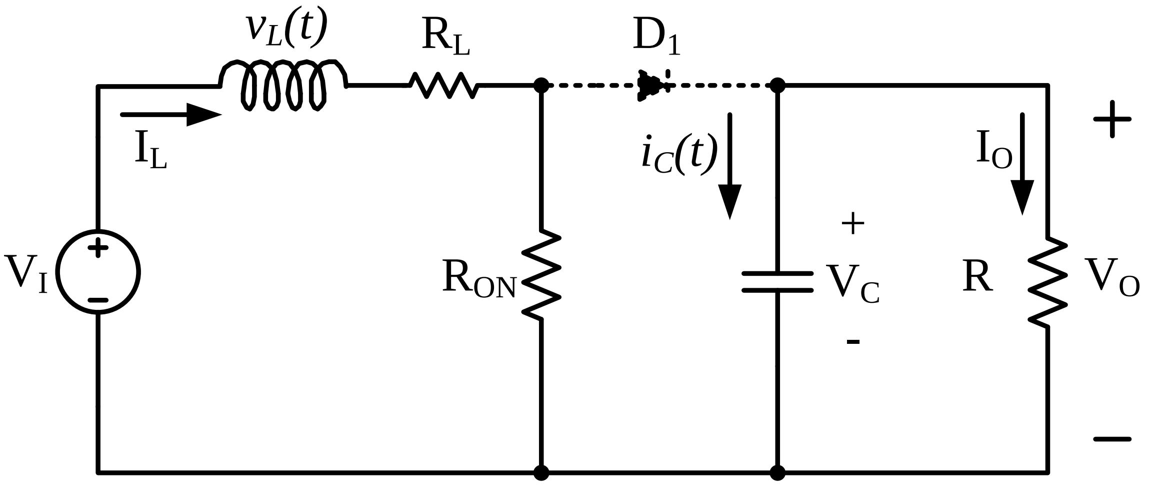


Boost Model (3/7)

- Small-ripple approximation
- Neglect R_C , $V_C = V_O$

$$v_L(t) = V_I - I_L(R_L + R_{ON})$$

$$i_C(t) = -\frac{V_C}{R}$$

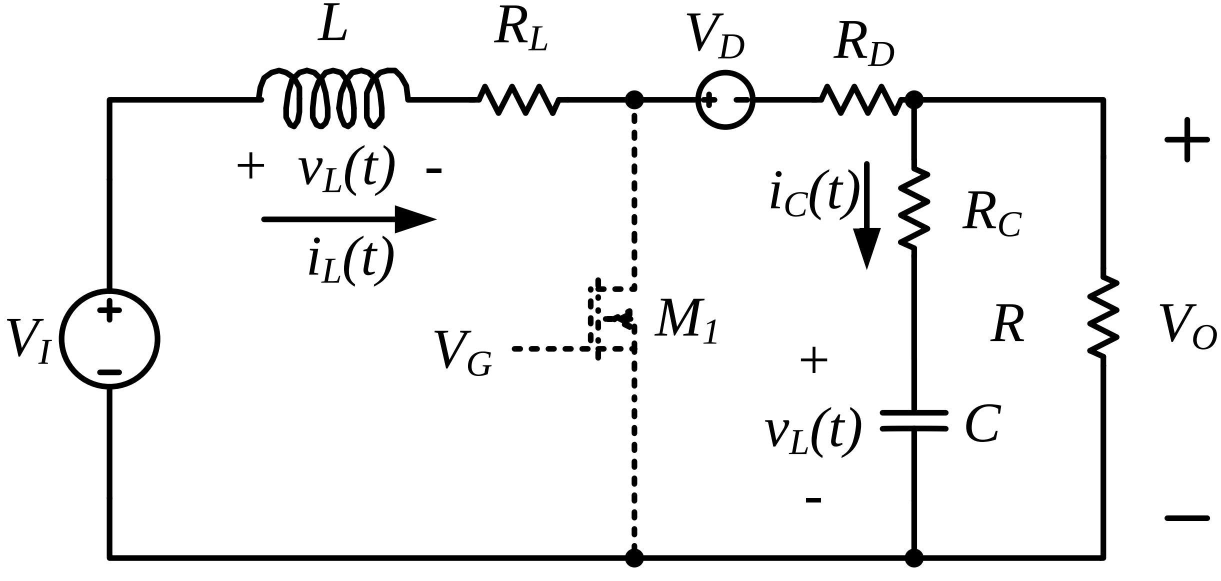


Boost Model (4/7)

•Position 2

$$v_L(t) = V_I - V_D - i_L(t)\left(R_L + R_D + R \parallel R_C\right) - v_C(t)\left(\frac{R}{R+R_C}\right)$$

$$i_C(t) = i_L(t)\left(\frac{R}{R+R_C}\right) - v_C(t)\left(\frac{1}{R+R_C}\right)$$

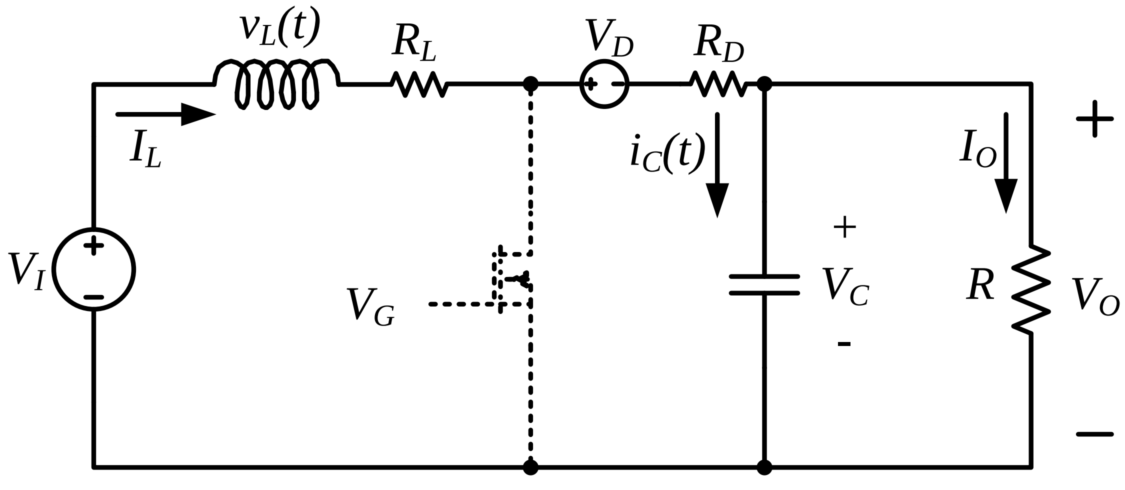


Boost Model (5/7)

- Small-ripple approximation
- Neglect R_C , V_C=V_O

$$v_L\left(t\right)=V_I-I_L\left(R_L+R_D\right)-V_D-V_C$$

$$i_C\left(t\right)=I_L-\frac{V_C}{R}$$

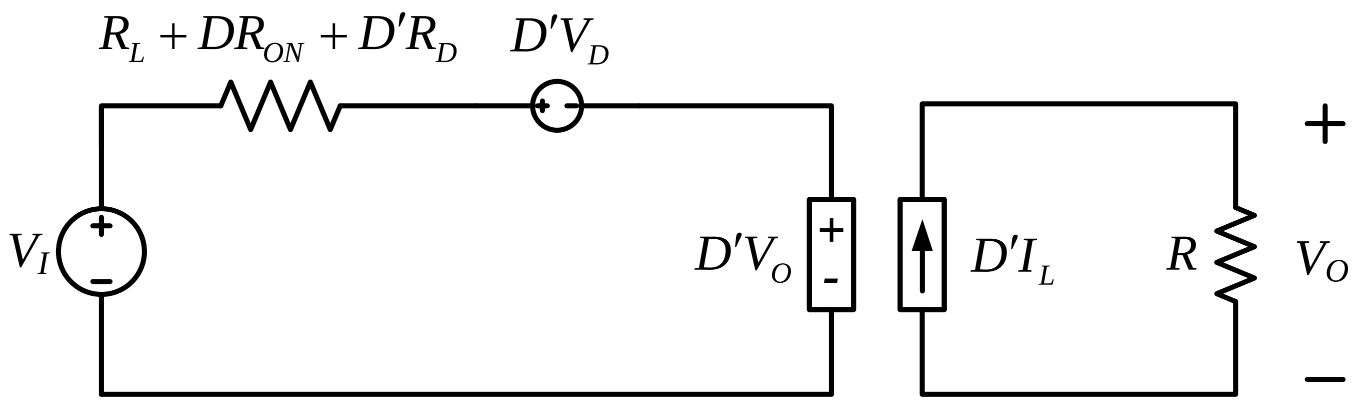


Boost Model (6/7)

- Inductor volt-second balance
- Capacitor charge balance

$$\langle v_L(t)\rangle = V_I - I_L\left(R_L + DR_{ON} + D'R_D\right) - D'\left(V_O + V_D\right) = 0$$

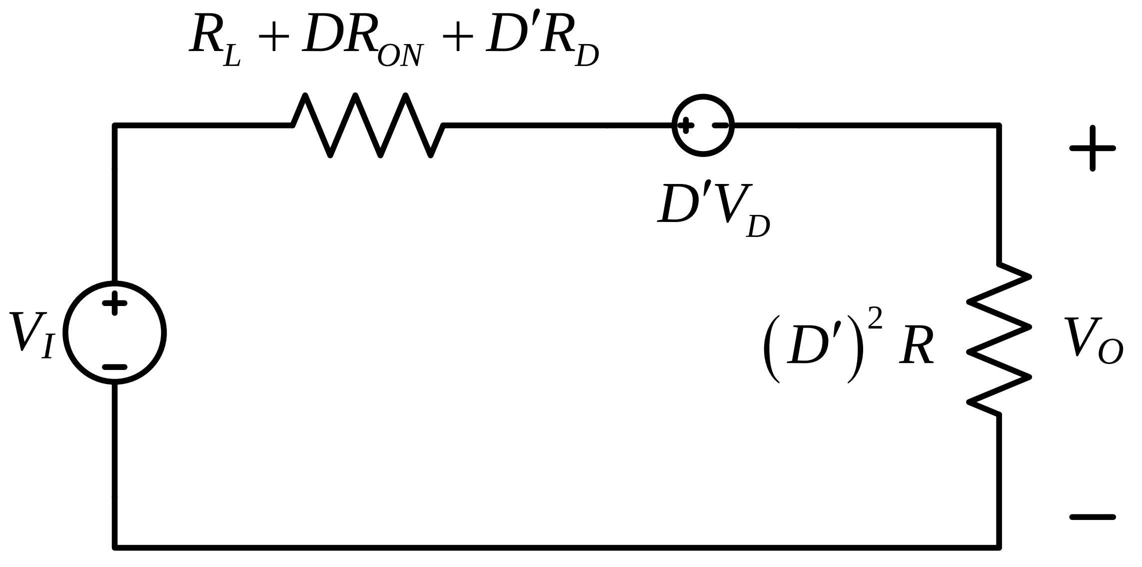
$$\langle i_C(t)\rangle = D'I_L - \frac{V_O}{R} = 0$$



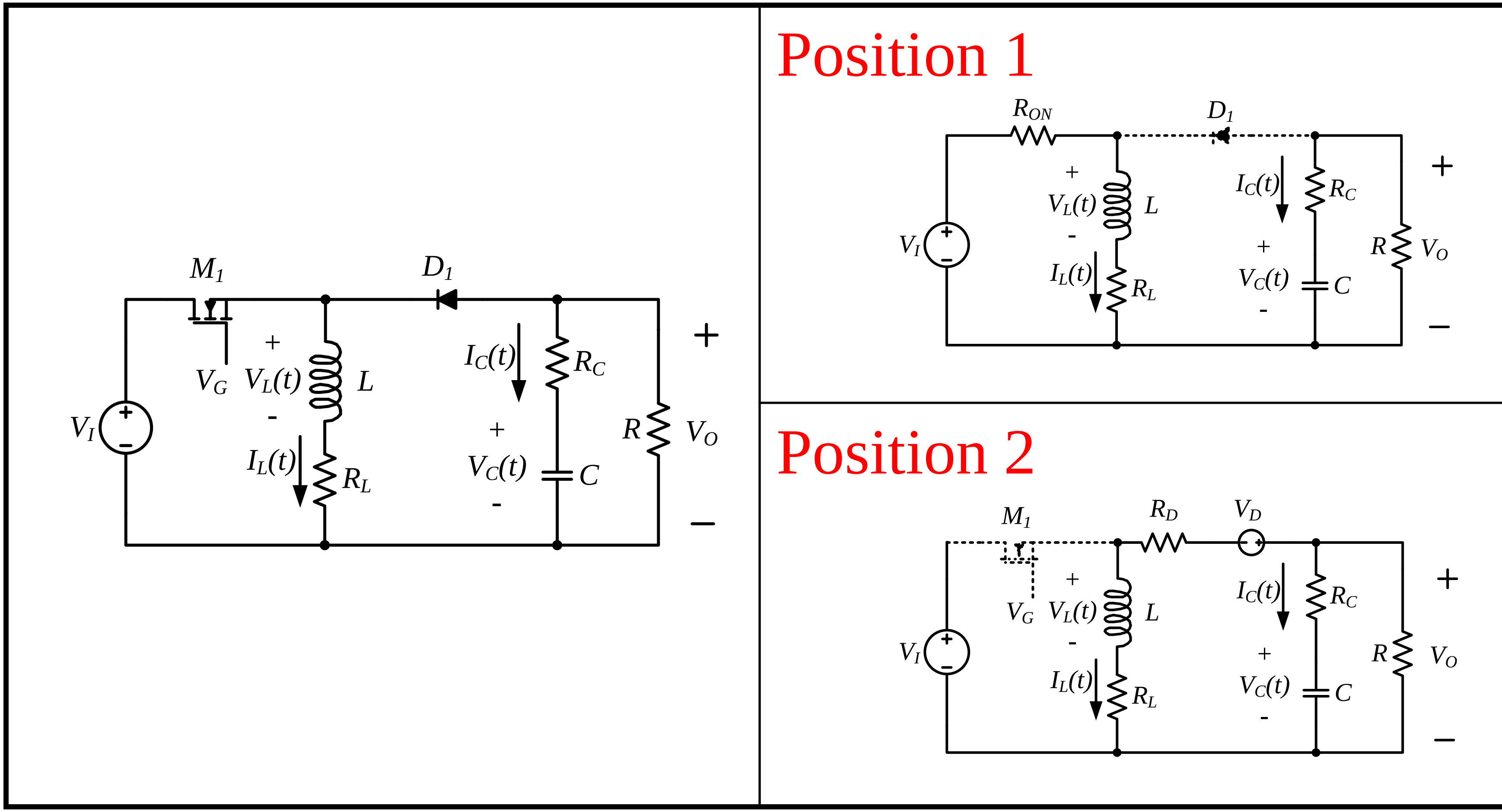
Boost Model (7/7)

- Efficiency

$$\eta = \frac{V_O I_O}{V_I I_I} = \frac{V_O}{V_I} = \frac{\left(1 - D'\frac{V_D}{V_I}\right)(D')^2 R}{R_L + DR_{ON} + D'R_D + (D')^2 R} = \frac{1 - D'\frac{V_D}{V_I}}{1 + \frac{(R_L + DR_{ON} + D'R_D)}{(D')^2 R}}$$



Buck-Boost Model (1/7)

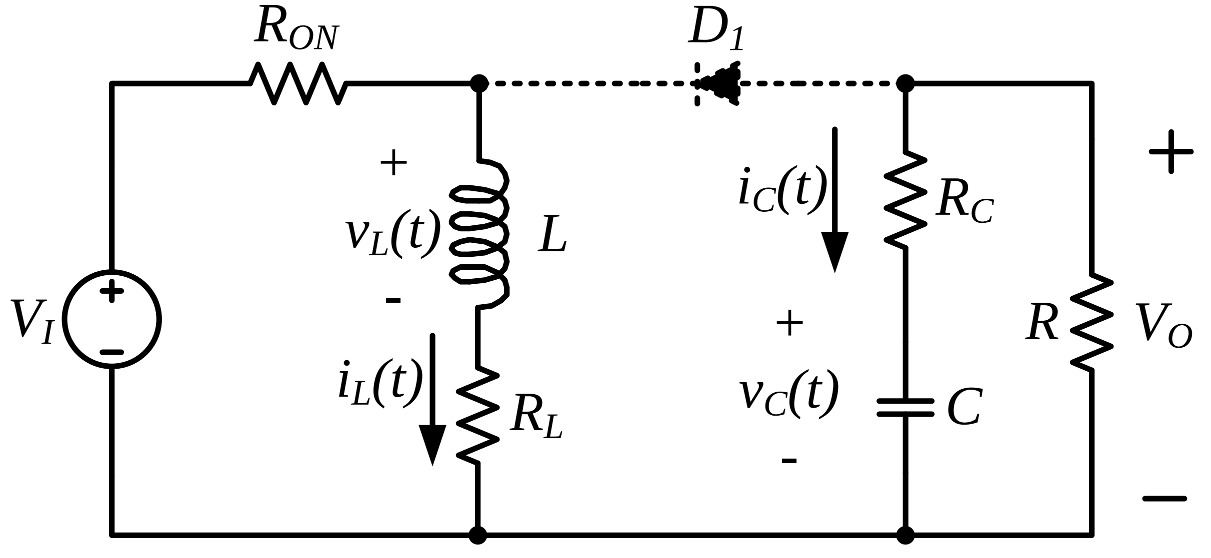


Buck-Boost Model (2/7)

- Position 1

$$v_L(t) = V_I - i_L(t)\left(R_L + R_{ON}\right)$$

$$i_C(t) = -v_C(t)\left(\frac{1}{R + R_C}\right)$$

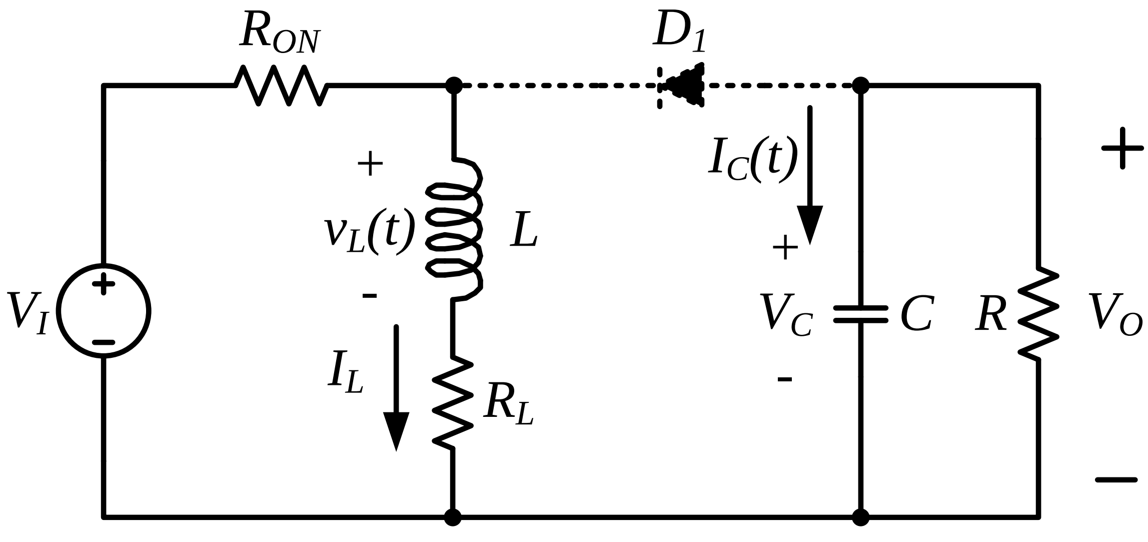


Buck-Boost Model (3/7)

- Small-ripple approximation
- Neglect R_C , $V_C = V_O$

$$v_L(t) = V_I - I_L(R_L + R_{ON})$$

$$i_C(t) = -\frac{V_C}{R}$$

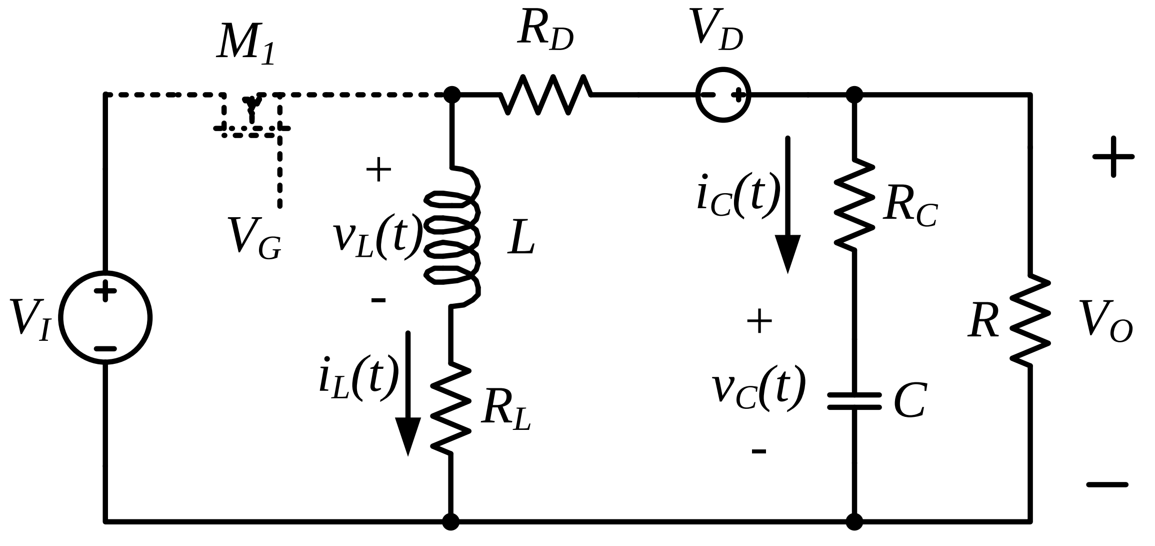


Buck-Boost Model (4/7)

•Position 2

$$v_L(t) = v_C(t)\left(\frac{R}{R+R_C}\right) - i_L(t)\left(R_L + R_D + R \parallel R_C\right) - V_D$$

$$i_C(t) = -i_L(t)\left(\frac{R}{R+R_C}\right) - v_C(t)\left(\frac{1}{R+R_C}\right)$$

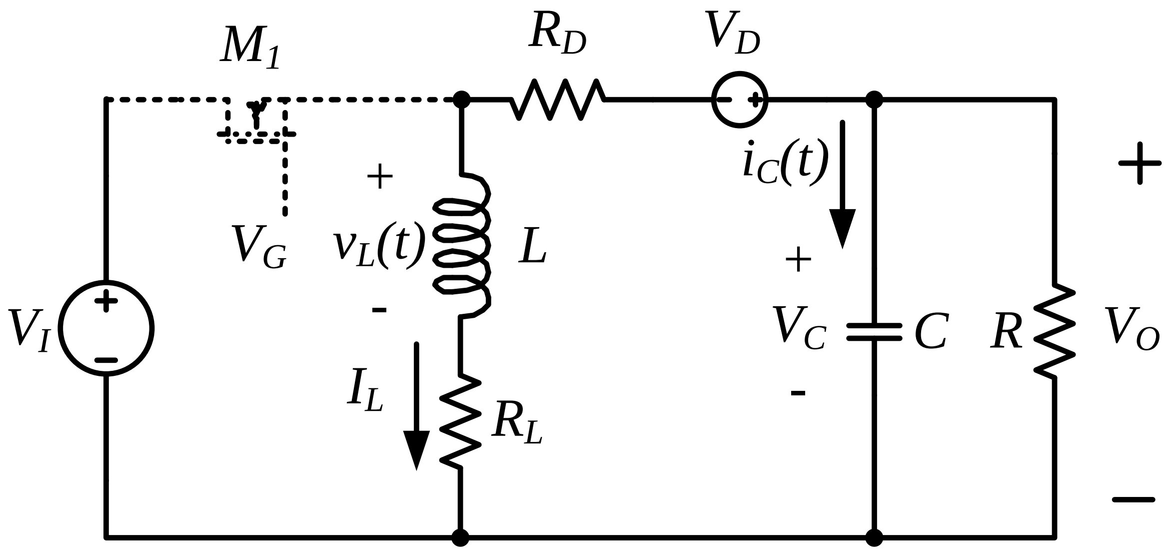


Buck-Boost Model (5/7)

- Small-ripple approximation
- Neglect R_C , V_C=V_O

$$v_L(t) = V_C - I_L(R_L + R_D) - V_D$$

$$i_C(t) = -I_L - \frac{V_C}{R}$$

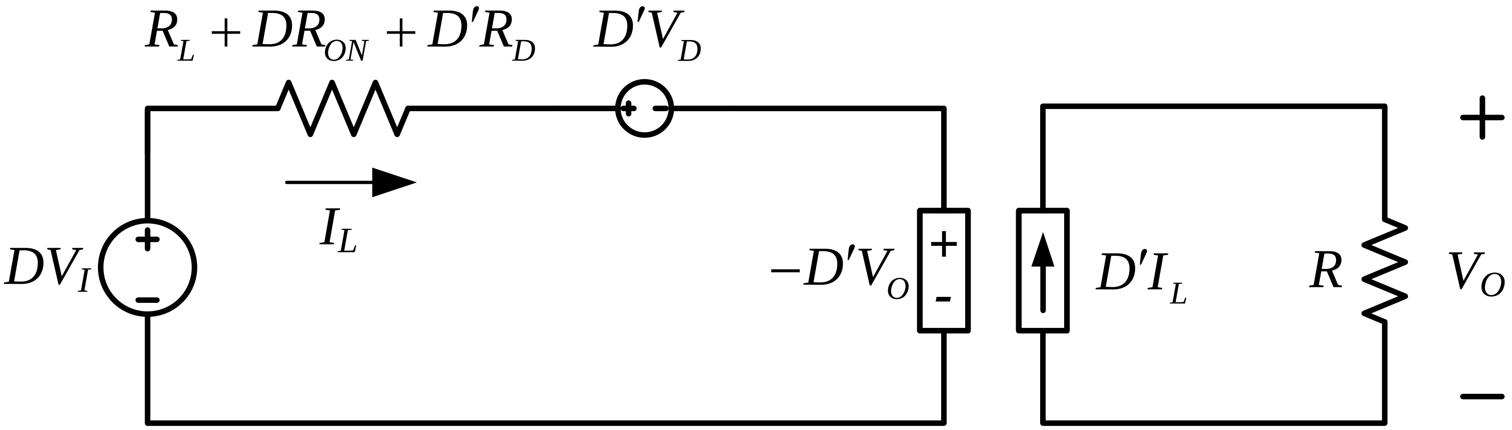


Buck-Boost Model (6/7)

•Inductor volt-second balance
•Capacitor charge balance

$$\left\langle v_L\left(t\right)\right\rangle = DV_I - I_L\left(R_L + DR_{ON} + D'R_D\right) - D'V_D + D'V_O = 0$$

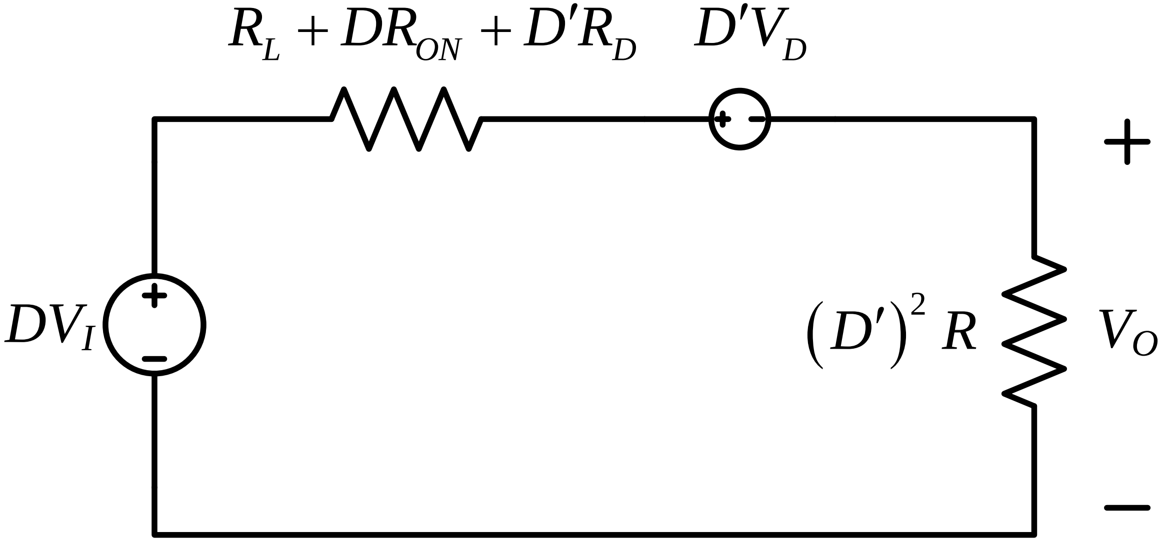
$$\left\langle i_C\left(t\right)\right\rangle = D'I_L - \frac{V_O}{R} = 0$$



Buck-Boost Model (7/7)

- Efficiency

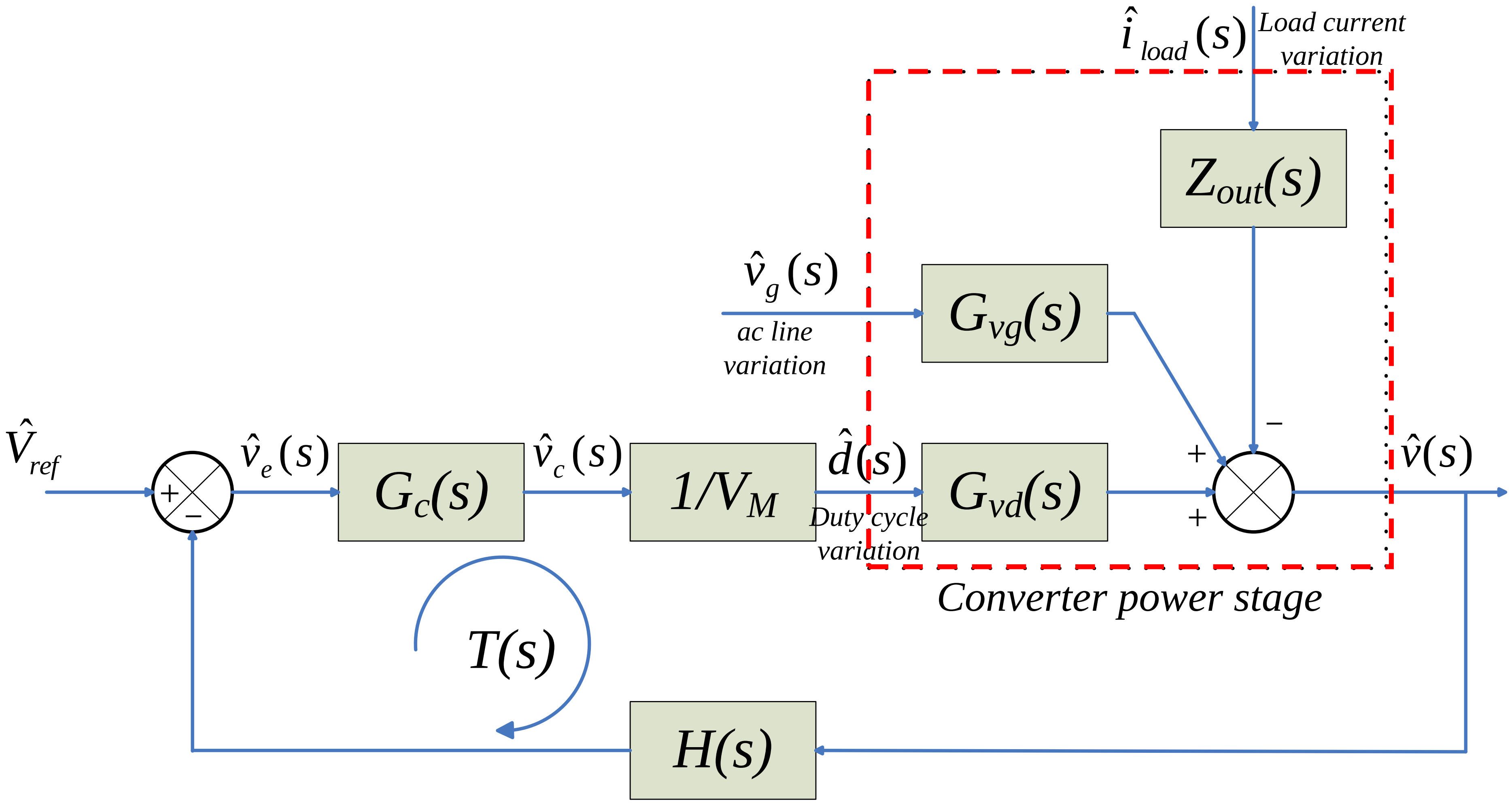
$$\eta = \frac{V_O I_O}{D V_I I_I} = \frac{V_O}{D V_I} = \left(1 - \frac{D' V_D}{D V_I}\right)\left(\frac{(D')^2 R}{R_L + D R_{ON} + D' R_D + (D')^2 R}\right)$$



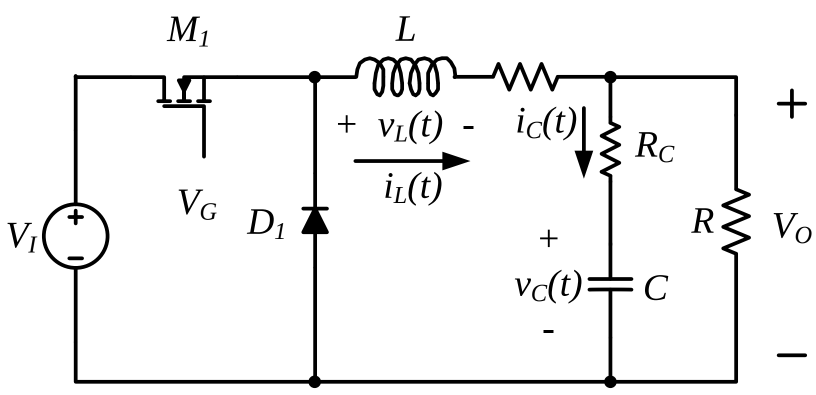
Outline

- 1-1. Steady-state analysis
  1) Buck converter
  2) Boost converter
  3) Buck-Boost converter
- 1-2. Transient-state analysis
  1) Buck converter
  2) Boost converter
  3) Buck-Boost converter

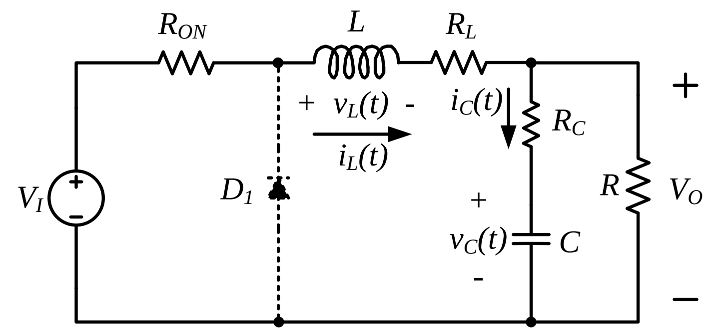
Converter's Transient-state Block



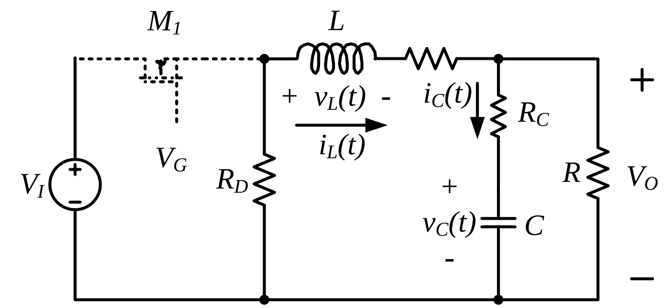
Buck (1/10)



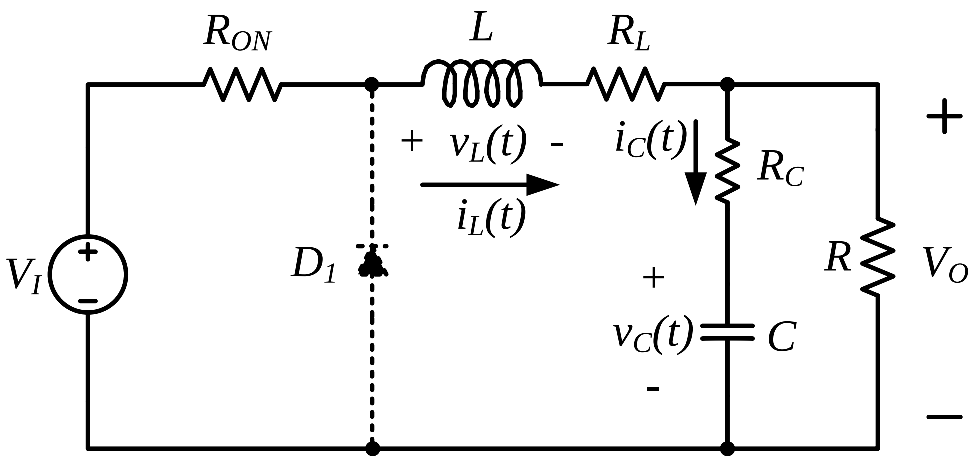
Position 1



Position 2



Buck (2/10)



Position 1

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_{ON}+R_L+R \parallel R_C}{L} & -\frac{R}{(R+R_C)L} \\ \frac{R}{(R+R_C)C} & -\frac{1}{(R+R_C)C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_I$$

$$[v_O] = \begin{bmatrix} R \parallel R_C & \frac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$[i_i] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

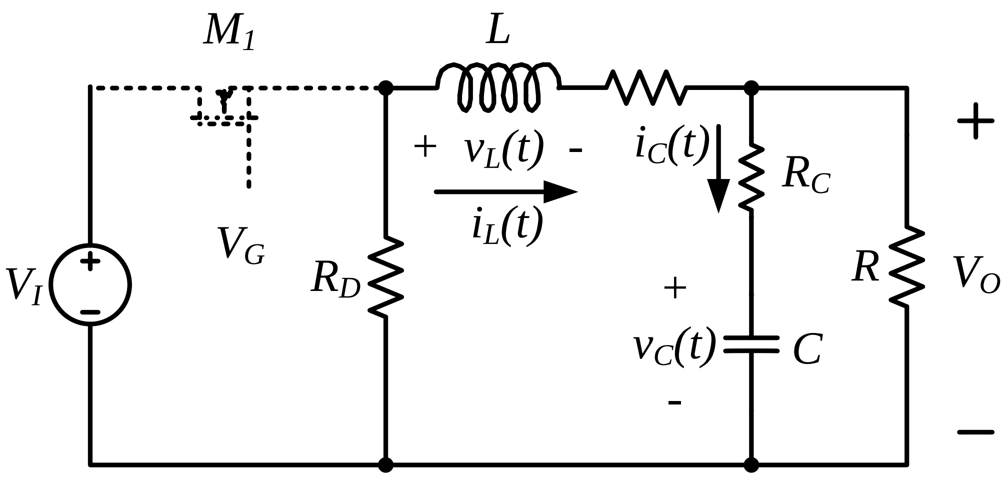
$$A_1 = \begin{bmatrix} -\frac{R_{ON}+R_L+R \parallel R_C}{L} & -\frac{R}{(R+R_C)L} \\ \frac{R}{(R+R_C)C} & -\frac{1}{(R+R_C)C} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} R \parallel R_C & \frac{R}{R+R_C} \end{bmatrix}$$

$$C_1' = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Buck (3/10)



Position 2

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_D + R_L + R \parallel R_C}{L} & -\frac{R}{(R+R_C)L} \\ \frac{R}{(R+R_C)C} & -\frac{1}{C(R+R_C)} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_I$$

$$[v_O] = \begin{bmatrix} R \parallel R_C & \frac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$[i_i] = [0 \quad 0] \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{R_D + R_L + R \parallel R_C}{L} & -\frac{R}{(R+R_C)L} \\ \frac{R}{(R+R_C)C} & -\frac{1}{C(R+R_C)} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} R \parallel R_C & \frac{R}{R+R_C} \end{bmatrix}$$

$$C_1' = [0 \quad 0]$$

Buck (4/10)

$$A = DA_1 + D'A_2 = \begin{bmatrix} -\dfrac{(DR_{ON} + D'R_D + R_L + R \parallel R_C)}{L} & -\dfrac{R}{(R+R_C)L} \\ \dfrac{R}{(R+R_C)C} & -\dfrac{1}{(R+R_C)C} \end{bmatrix}$$

$$B = DB_1 + D'B_2 = \begin{bmatrix} \dfrac{D}{L} \\ 0 \end{bmatrix}$$

$$C = DC_1 + D'C_2 = \begin{bmatrix} R \parallel R_C & \dfrac{R}{R+R_C} \end{bmatrix}$$

$$C' = DC_1' + D'C_2' = \begin{bmatrix} D & 0 \end{bmatrix}$$

$$A_1 - A_2 = \begin{bmatrix} \dfrac{-R_{ON} + R_D}{L} & 0 \\ 0 & 0 \end{bmatrix}$$

$$B_1 - B_2 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}$$

$$C_1 - C_2 = 0$$

$$F = \begin{bmatrix} \dfrac{-R_{ON} + R_D}{L} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \begin{bmatrix} V_I \end{bmatrix}$$

Buck (5/10)

$$(SI-A)^{-1}=\frac{adj(SI-A)}{\left|SI-A\right|}$$

$$(SI-A)^{-1}=\begin{bmatrix} S+\dfrac{DR_{ON}+D'R_D+R_L+R\parallel R_C}{L} & \dfrac{R}{(R+R_C)L} \\ -\dfrac{R}{(R+R_C)C} & S+\dfrac{1}{(R+R_C)C} \end{bmatrix}^{-1}$$

$$adj(SI-A)=\begin{bmatrix} S+\dfrac{1}{(R+R_C)C} & -\dfrac{R}{(R+R_C)L} \\ \dfrac{R}{(R+R_C)C} & S+\dfrac{DR_{ON}+D'R_D+R_L+R\parallel R_C}{L} \end{bmatrix}$$

$$\left|SI-A\right|=S^2+S\left(\frac{1}{L}\right)\left(DR_{ON}+D'R_D+R_L+R\parallel R_C+\frac{L}{\left(R+R_C\right)C}\right)+\left(\frac{DR_{ON}+D'R_D+R_L+R}{LC\left(R+R_C\right)}\right)$$

Buck (6/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = C(SI-A)^{-1}B$$

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \begin{bmatrix} R \parallel R_C & \dfrac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} S+\dfrac{DR_{ON}+D'R_D+R_L+R \parallel R_C}{L} & \dfrac{R}{(R+R_C)L} \\ -\dfrac{R}{(R+R_C)C} & S+\dfrac{1}{(R+R_C)C} \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{D}{L} \\ 0 \end{bmatrix}$$

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{DR}{\Delta}\left(\frac{SR_C C+1}{\left(R+R_C\right)LC}\right)$$

Buck (7/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0} = C(SI-A)^{-1}F$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0} = \begin{bmatrix} R\parallel R_C & \dfrac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} S+\dfrac{DR_{ON}+D'R_D+R_L+R\parallel R_C}{L} & \dfrac{R}{(R+R_C)L} \\ -\dfrac{R}{(R+R_C)C} & S+\dfrac{1}{(R+R_C)C} \end{bmatrix}^{-1} \left(\begin{bmatrix} \dfrac{-R_{ON}+R_D}{L} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \begin{bmatrix} V_I \end{bmatrix} \right)$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0} = C(SI-A)^{-1} \begin{bmatrix} \dfrac{\left(-R_{ON}+R_D\right)I_L+V_I}{L} \\ 0 \end{bmatrix}$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0} = \frac{1}{\Delta}\left(\frac{R\left(\left(-R_{ON}+R_D\right)I_L+V_I\right)\left(SR_CC+1\right)}{\left(R+R_C\right)LC} \right)$$

Buck (8/10)

$$\left.\frac{\hat{i}_i(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = C'(SI-A)^{-1}B$$

$$\left.\frac{\hat{i}_i(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \begin{bmatrix} D & 0 \end{bmatrix} \begin{bmatrix} S+\dfrac{DR_{ON}+D'R_D+R_L+R\parallel R_C}{L} & \dfrac{R}{(R+R_C)L} \\ -\dfrac{R}{(R+R_C)C} & S+\dfrac{1}{(R+R_C)C} \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{D}{L} \\ 0 \end{bmatrix}$$

$$\left.\frac{\hat{i}_i(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{D}{\Delta}\left(\frac{S\left(R+R_C\right)C+1}{\left(R+R_C\right)LC}\right)$$

$$\Delta = S^2 + S\left(\frac{1}{L}\right)\left(DR_{ON}+D'R_D+R_L+R\parallel R_C+\frac{L}{\left(R+R_C\right)C}\right)+\left(\frac{DR_{ON}+D'R_D+R_L+R}{LC\left(R+R_C\right)}\right)$$

$$Z'_{in} = D^2 Z_{in}$$

$$Z_{out}\left(S\right) = \frac{Z_1 Z_2}{Z'_{in}} = \left(1-\frac{R\left(SR_C C+1\right)}{\Delta\left(R+R_C\right)LC}\right)\left(\frac{R\left(SR_C C+1\right)}{S\left(R+R_C\right)C+1}\right)$$

Buck (9/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{DR}{\Delta}\left(\frac{SR_C C+1}{(R+R_C)LC}\right)$$

$$\Delta = S^2 + S\left(\frac{1}{L}\right)\left(DR_{ON} + D'R_D + R_L + R\,\|\,R_C + \frac{L}{(R+R_C)C}\right) + \left(\frac{DR_{ON} + D'R_D + R_L + R}{(R+R_C)LC}\right)$$

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{C_3 S^{-1} + C_4 S^{-2}}{1-\left(-C_1 S^{-1} - C_2 S^{-2}\right)}$$

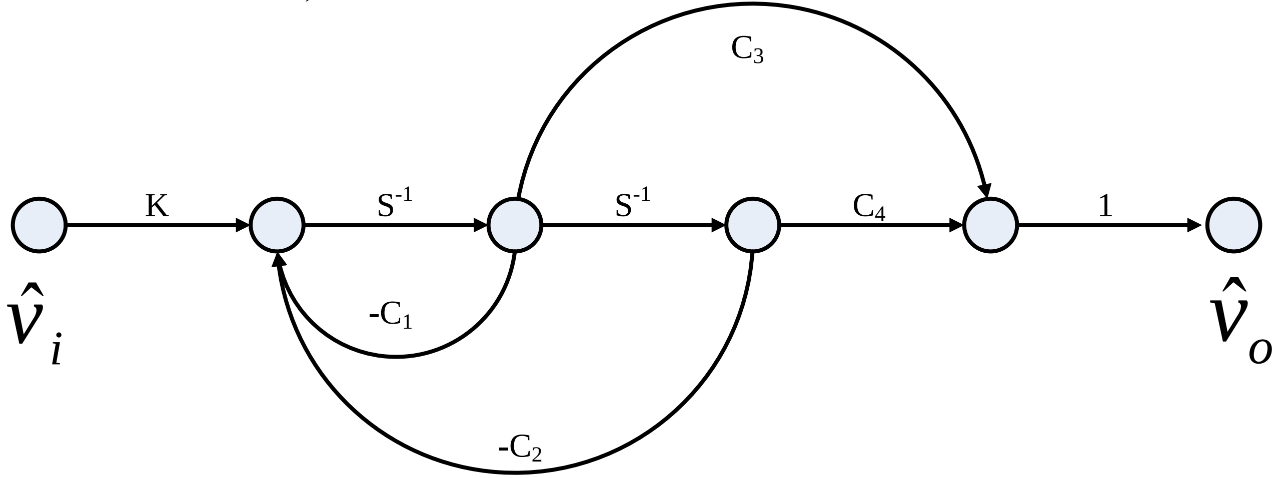
$$C_1 = \left(\frac{1}{L}\right)\left(DR_{ON} + D'R_D + R_L + R\,\|\,R_C + \frac{L}{(R+R_C)C}\right)$$

$$C_2 = \frac{DR_{ON} + D'R_D + R_L + R}{(R+R_C)LC}$$

$$C_3 = R_C C$$

$$C_4 = 1$$

$$K = \frac{DR}{(R+R_C)LC}$$



Buck (10/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0} = \frac{1}{\Delta}\left(\frac{R\left(\left(-R_{ON}+R_D\right)I_L+V_I\right)\left(SR_CC+1\right)}{\left(R+R_C\right)LC}\right)$$

$$\Delta = S^2 + S\left(\frac{1}{L}\right)\left(DR_{ON}+D'R_D+R_L+R\parallel R_C+\frac{L}{(R+R_C)C}\right)+\left(\frac{DR_{ON}+D'R_D+R_L+R}{(R+R_C)LC}\right)$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{V_I(s)=0} = \frac{K(C_3S^{-1}+C_4S^{-2})}{1-(-C_1S^{-1}-C_2S^{-2})}$$

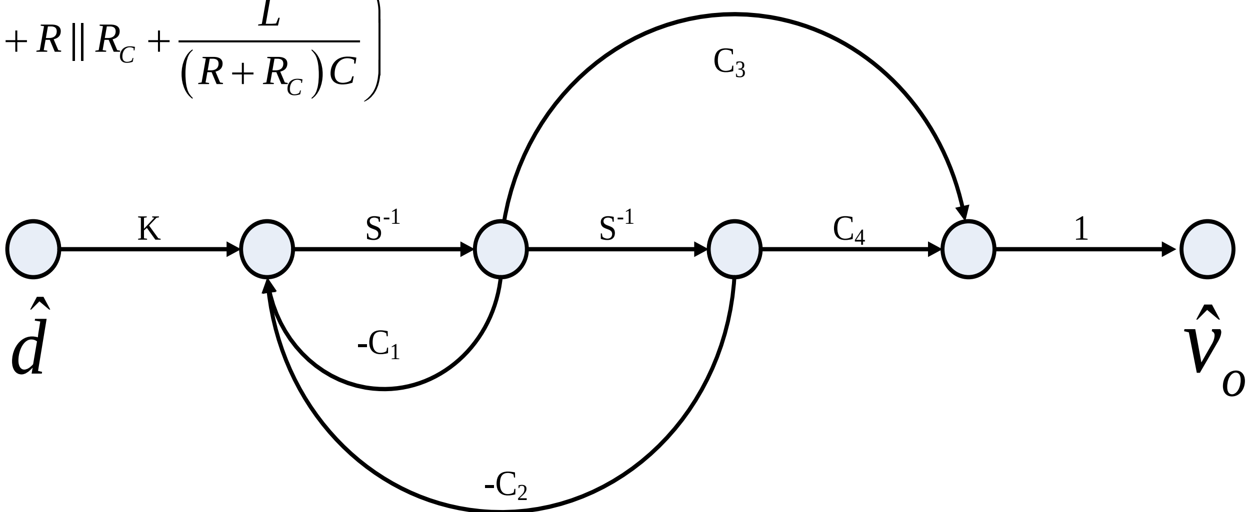
$$C_1 = \left(\frac{1}{L}\right)\left(DR_{ON}+D'R_D+R_L+R\parallel R_C+\frac{L}{\left(R+R_C\right)C}\right)$$

$$C_2 = \frac{DR_{ON}+D'R_D+R_L+R}{\left(R+R_C\right)LC}$$

$$C_3 = R_CC$$

$$C_4 = 1$$

$$K = R\left(\left(-R_{ON}+R_D\right)I_L+V_I\right)$$



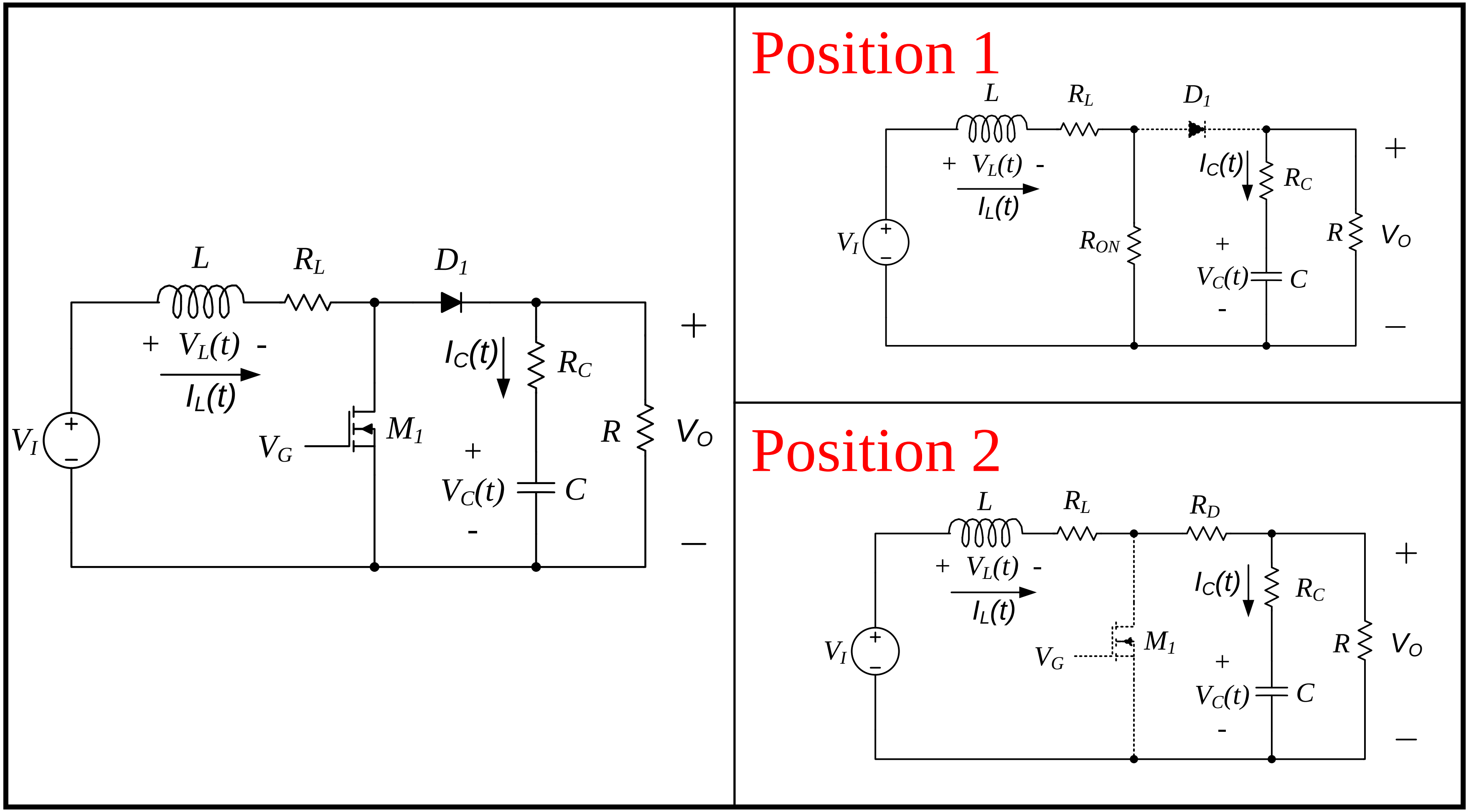
Type	Buck
$G_{vg}(S)$	$\frac{DR}{\Delta}\left(\frac{SR_C C+1}{(R+R_C)LC}\right)$
$G_{vd}(S)$	$\frac{1}{\Delta}\left(\frac{R\left(\left(-R_{ON}+R_D\right)I_L+V_I\right)\left(SR_C C+1\right)}{(R+R_C)LC}\right)$
$Zout(s)$	$\left(1-\frac{R\left(SR_C C+1\right)}{\Delta(R+R_C)LC}\right)\left(\frac{R\left(SR_C C+1\right)}{S(R+R_C)C+1}\right)$
ω_Z	$-\frac{1}{R_C C}$

$$\Delta = S^2 + S\left(\frac{1}{L}\right)\left(DR_{ON} + D'R_D + R_L + R \parallel R_C + \frac{L}{(R+R_C)C}\right) + \left(\frac{DR_{ON} + D'R_D + R_L + R}{LC(R+R_C)}\right)$$

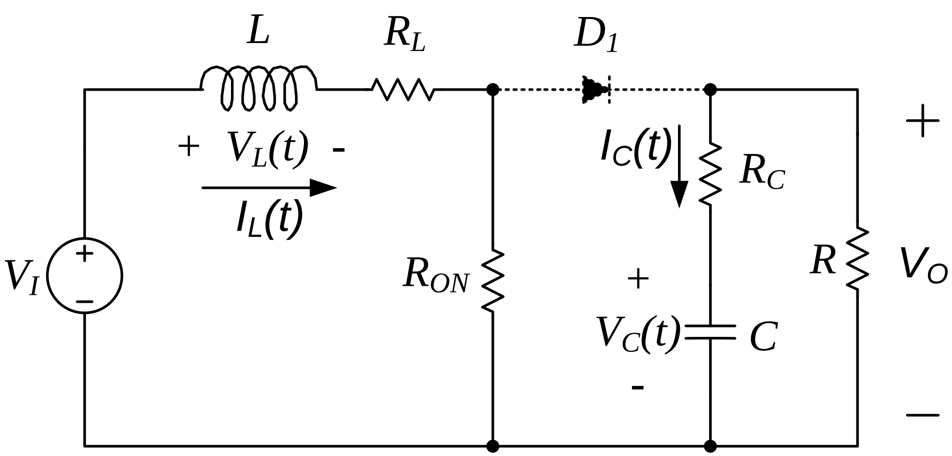
Outline

- 1-1. Steady-state analysis
  1) Buck converter
  2) Boost converter
  3) Buck-Boost converter
- 1-2. Transient-state analysis
  1) Buck converter
  2) Boost converter
  3) Buck-Boost converter

Boost (1/10)


L
RL
D1
+ VL(t) -
IL(t)
VI
VG
M1
IC(t)
RC
VC(t)
C
R
VO
Position 1
RON
Position 2
RD

Boost (2/10)



Position 1

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_{ON}+R_L}{L} & 0 \\ 0 & -\frac{1}{(R+R_C)C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_I$$

$$[v_O] = \begin{bmatrix} 0 & \frac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$[i_i] = [1 \quad 0] \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

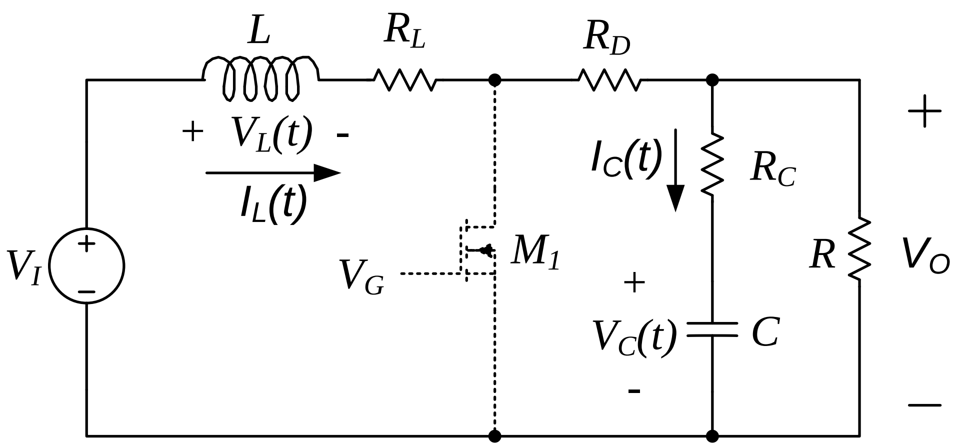
$$A_1 = \begin{bmatrix} -\frac{R_{ON}+R_L}{L} & 0 \\ 0 & -\frac{1}{(R+R_C)C} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & \frac{R}{R+R_C} \end{bmatrix}$$

$$C_1' = [1 \quad 0]$$

Boost (3/10)



Position 2

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_D + R_L + R \| R_C}{L} & -\frac{R}{(R+R_C)L} \\ \frac{R}{(R+R_C)C} & -\frac{1}{C(R+R_C)} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_I$$

$$[v_O] = \begin{bmatrix} R \| R_C & \frac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$[i_i] = [1 \quad 0] \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{R_D + R_L + R \| R_C}{L} & -\frac{R}{(R+R_C)L} \\ \frac{R}{(R+R_C)C} & -\frac{1}{C(R+R_C)} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} R \| R_C & \frac{R}{R+R_C} \end{bmatrix}$$

$$C_2' = [1 \quad 0]$$

Boost (4/10)

$$A = DA_1 + D'A_2 = \begin{bmatrix} -\dfrac{(R_L + DR_{ON} + D'(R_D + R \parallel R_C))}{L} & -\dfrac{D'R}{(R+R_C)L} \\ \dfrac{D'R}{(R+R_C)C} & -\dfrac{1}{(R+R_C)C} \end{bmatrix}$$

$$B = DB_1 + D'B_2 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}$$

$$C = DC_1 + D'C_2 = \begin{bmatrix} D'(R \parallel R_C) & \dfrac{R}{R+R_C} \end{bmatrix}$$

$$C' = DC_1' + D'C_2' = \begin{bmatrix} D' & \dfrac{1}{R_C} \end{bmatrix}$$

$$A_1 - A_2 = \begin{bmatrix} \dfrac{-R_{ON} + R_D + R \parallel R_C}{L} & \dfrac{R}{(R+R_C)L} \\ -\dfrac{R}{(R+R_C)C} & 0 \end{bmatrix}$$

$$B_1 - B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C_1 - C_2 = 0$$

$$F = \begin{bmatrix} \dfrac{-R_{ON} + R_D + R \parallel R_C}{L} & \dfrac{R}{(R+R_C)L} \\ -\dfrac{R}{(R+R_C)C} & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} V_I \end{bmatrix}$$

Boost (5/10)

$$(SI-A)^{-1}=\frac{adj(SI-A)}{\left|SI-A\right|}$$

$$(SI-A)^{-1}=\begin{bmatrix} S+\dfrac{R_L+DR_{ON}+D'(R_D+R\,\|\,R_C)}{L} & \dfrac{D'R}{L(R+R_C)} \\ \dfrac{-D'R}{C(R+R_C)} & S+\dfrac{1}{C(R+R_C)} \end{bmatrix}^{-1}$$

$$adj(SI-A)=\begin{bmatrix} S+\dfrac{1}{C(R+R_C)} & \dfrac{-D'R}{L(R+R_C)} \\ \dfrac{D'R}{C(R+R_C)} & S+\dfrac{R_L+DR_{ON}+D'(R_D+R\,\|\,R_C)}{L} \end{bmatrix}$$

$$\left|SI-A\right|=S^2+S\left(\frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+R\,\|\,R_C\right)\right)}{\left(R+R_C\right)LC}\right)+\left(\frac{R_L+DR_{ON}+D'\left(R_D+R\,\|\,R_C\right)+\dfrac{\left(D'R\right)^2}{R+R_C}}{\left(R+R_C\right)LC}\right)$$

$$\Delta=\left|SI-A\right|$$

Boost (6/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = C(SI-A)^{-1}B$$

$$\Delta = \left|SI-A\right| = S^2 + S\left(\frac{L + C\left(R+R_C\right)\left(R_L + DR_{ON} + D'\left(R_D + R \parallel R_C\right)\right)}{LC\left(R+R_C\right)}\right) + \left(\frac{R_L + DR_{ON} + D'\left(R_D + R \parallel R_C\right) + \dfrac{\left(D'R\right)^2}{R+R_C}}{LC\left(R+R_C\right)}\right)$$

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{1}{\Delta}\begin{bmatrix} D'\left(R \parallel R_C\right) & \dfrac{R}{R+R_C}\end{bmatrix}\begin{bmatrix} \dfrac{SC\left(R+R_C\right)+1}{C\left(R+R_C\right)} & -\dfrac{D'R}{L\left(R+R_C\right)} \\ \dfrac{D'R}{C\left(R+R_C\right)} & \dfrac{SL + R_L + DR_{ON} + D'\left(R_D + R \parallel R_C\right)}{L}\end{bmatrix}\begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}$$

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{1}{\Delta}\left(\frac{D'R\left(SR_C C + 1\right)}{LC\left(R+R_C\right)}\right)$$

Boost (7/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{V}_I(s)=0} = C(SI-A)^{-1}F$$

$$\Delta = |SI-A|$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{V}_I(s)=0} = \frac{1}{\Delta}\begin{bmatrix} D'(R\parallel R_C) & \dfrac{R}{R+R_C}\end{bmatrix}\begin{bmatrix} \dfrac{SC(R+R_C)+1}{C(R+R_C)} & -\dfrac{D'R}{L(R+R_C)} \\ \dfrac{D'R}{C(R+R_C)} & \dfrac{SL+R_L+DR_{ON}+D'(R_D+R\parallel R_C)}{L}\end{bmatrix}\begin{bmatrix} \dfrac{-R_{ON}+R_D+R\parallel R_C}{L} & \dfrac{R}{(R+R_C)L} \\ -\dfrac{R}{(R+R_C)C} & 0\end{bmatrix}\begin{bmatrix} I_L \\ V_C\end{bmatrix}$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{V}_I(s)=0} = \frac{1}{\Delta}\begin{bmatrix} \dfrac{D'R\left(SR_CC+1\right)}{C(R+R_C)} & \dfrac{R(SL+R_L+DR_{ON}+D'R_D+DD'(R\parallel R_C))}{L(R+R_C)}\end{bmatrix}\begin{bmatrix} \dfrac{-R_{ON}+R_D+R\parallel R_C}{L} & \dfrac{R}{(R+R_C)L} \\ -\dfrac{R}{(R+R_C)C} & 0\end{bmatrix}\begin{bmatrix} I_L \\ V_C\end{bmatrix}$$

$$\alpha_1 = \left(-R_{ON}+R_D+R\parallel R_C\right)I_L + \frac{V_C}{R+R_C}$$

$$\alpha_2 = \left(R_L+DR_{ON}+D'R_D+DD'(R\parallel R_C)\right)I_L$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{V}_I(s)=0} = \frac{\alpha_1\left(\mathrm{S}\left(R_CC-\dfrac{R_LI_LL}{D'\alpha_1}\right)+\dfrac{\alpha_1-\alpha_2}{\alpha_1}\right)}{\Delta\left(R+R_C\right)LC}$$

Boost (8/10)

$$\left.\frac{\hat{i}_i(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = C'(SI-A)^{-1}B$$

$$\left.\frac{\hat{i}_i(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{1}{\Delta}\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\dfrac{SC(R+R_C)+1}{C(R+R_C)} & \dfrac{-D'R}{L(R+R_C)} \\ \dfrac{D'R}{C(R+R_C)} & \dfrac{SL+R_L+DR_{ON}+D'(R_D+R\parallel R_C)}{L}\end{bmatrix}\begin{bmatrix}\dfrac{1}{L} \\ 0\end{bmatrix}$$

$$\left.\frac{\hat{i}_i(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{1}{\Delta}\left(\frac{S\left(R+R_C\right)C+1}{\left(R+R_C\right)LC}\right)$$

$$Z'_{out}(S) = \left(\frac{1}{D'}\right)^2 Z_{out}(S) = \left(\frac{1}{D'}\right)^2\left(\frac{\Delta\left(R+R_C\right)LC}{S\left(R+R_C\right)C+1}\right)$$

$$Z_{out}(S) = \left(1-\frac{\left(D'\right)^2 R\left(SR_CC+1\right)}{\Delta LC\left(R+R_C\right)}\right)\left(\frac{R\left(SR_CC+1\right)}{S\left(R+R_C\right)C+1}\right)$$

Boost (9/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{1}{\Delta}\left(\frac{D'R\left(SR_C C+1\right)}{LC\left(R+R_C\right)}\right)$$

$$\Delta = S^2 + S\left(\frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+R\parallel R_C\right)\right)}{\left(R+R_C\right)LC}\right)+\left(\frac{R_L+DR_{ON}+D'\left(R_D+R\parallel R_C\right)+\dfrac{\left(D'R\right)^2}{R+R_C}}{\left(R+R_C\right)LC}\right)$$

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{K(C_3S^{-1}+C_4S^{-2})}{1-(-C_1S^{-1}-C_2S^{-2})}$$

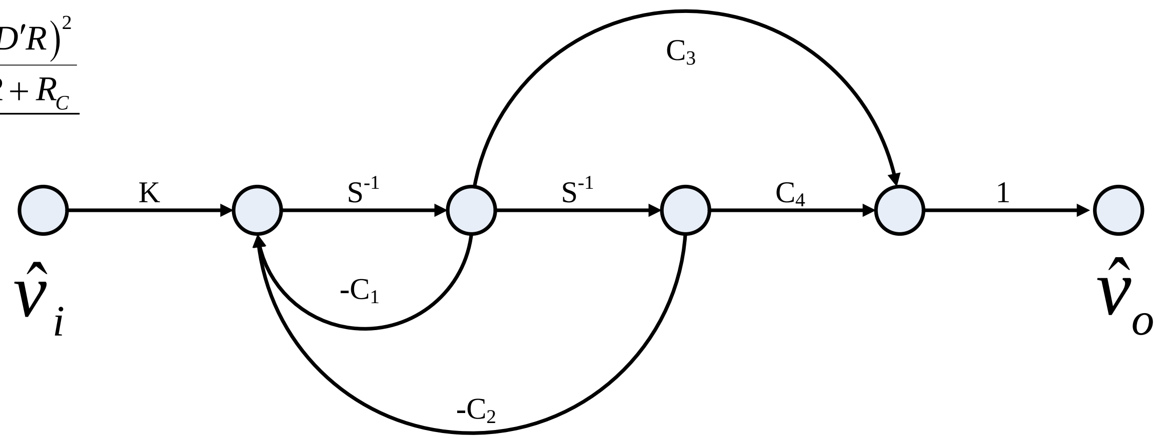
$$C_1 = \frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+R\parallel R_C\right)\right)}{\left(R+R_C\right)LC}$$

$$C_2 = \frac{R_L+DR_{ON}+D'\left(R_D+R\parallel R_C\right)+\dfrac{\left(D'R\right)^2}{R+R_C}}{\left(R+R_C\right)LC}$$

$$C_3 = R_C C$$

$$C_4 = 1$$

$$K = \frac{D'R}{LC\left(R+R_C\right)}$$



Boost (10/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{V}_I(s)=0} = \frac{\alpha_1\left(S\left(R_C C - \frac{R_L I_L L}{D'\alpha_1}\right) + \frac{\alpha_1-\alpha_2}{\alpha_1}\right)}{\Delta\left(R+R_C\right)LC}$$

$$\alpha_1 = \left(-R_{ON} + R_D + R \parallel R_C\right) I_L + \frac{V_C}{R+R_C}$$

$$\alpha_2 = \left(R_L + DR_{ON} + D'R_D + DD'(R \parallel R_C)\right) I_L$$

$$\Delta = S^2 + S\left(\frac{L + C\left(R+R_C\right)\left(R_L + DR_{ON} + D'\left(R_D + R\parallel R_C\right)\right)}{\left(R+R_C\right)LC}\right) + \left(\frac{R_L + DR_{ON} + D'\left(R_D + R\parallel R_C\right) + \frac{\left(D'R\right)^2}{R+R_C}}{\left(R+R_C\right)LC}\right)$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{V}_I(s)=0} = \frac{K(C_3S^{-1} + C_4S^{-2})}{1-(-C_1S^{-1} - C_2S^{-2})}$$

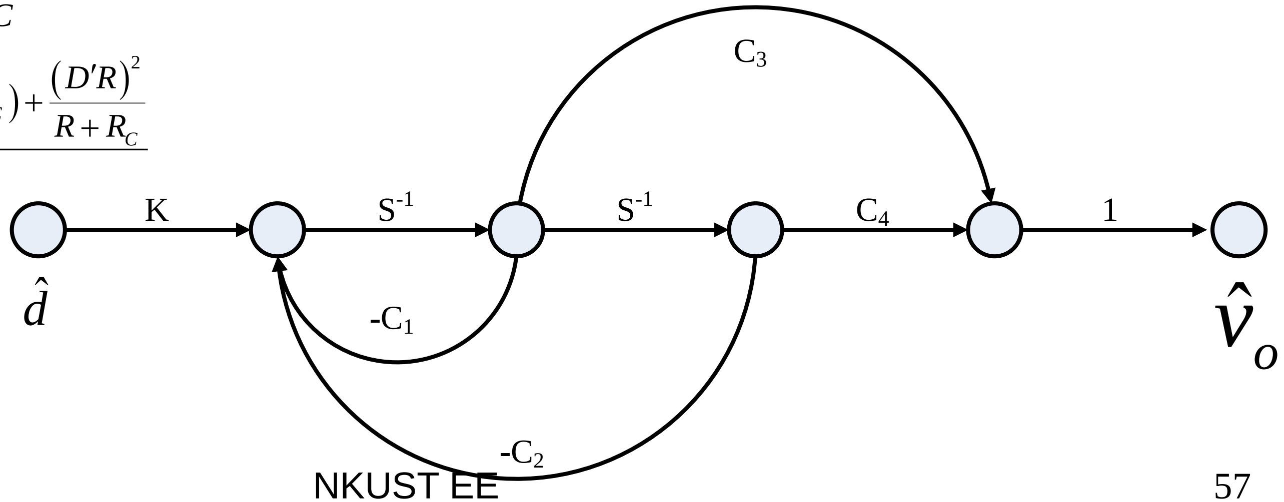
$$C_1 = \frac{L + C\left(R+R_C\right)\left(R_L + DR_{ON} + D'\left(R_D + R\parallel R_C\right)\right)}{\left(R+R_C\right)LC}$$

$$C_2 = \frac{R_L + DR_{ON} + D'\left(R_D + R\parallel R_C\right) + \frac{\left(D'R\right)^2}{R+R_C}}{\left(R+R_C\right)LC}$$

$$C_3 = R_C C - \frac{R_L I_L L}{D'\alpha_1}$$

$$C_4 = \frac{\alpha_1 - \alpha_2}{\alpha_1}$$

$$K = \frac{\alpha_1}{\left(R+R_C\right)LC}$$



Type	Boost
$G_{vg}(S)$	$\frac{1}{\Delta}\left(\frac{D'R\left(SR_C C+1\right)}{LC\left(R+R_C\right)}\right)$
$G_{vd}(S)$	$\left(\frac{\alpha_1}{\Delta\left(R+R_C\right)LC}\right)\left(\mathrm{S}\left(R_C C-\frac{R_L I_L L}{D'\alpha_1}\right)+\frac{\alpha_1-\alpha_2}{\alpha_1}\right)$
$Zout(s)$	$\left(1-\frac{\left(D'\right)^2 R\left(SR_C C+1\right)}{\Delta LC\left(R+R_C\right)}\right)\left(\frac{R\left(SR_C C+1\right)}{S\left(R+R_C\right)C+1}\right)$
ω_Z	$\frac{D'\left(\alpha_1-\alpha_2\right)}{R_L I_L L-D'\alpha_1 R_C C}$

$$\alpha_1=\left(-R_{ON}+R_D+R\parallel R_C\right)I_L+\frac{V_C}{R+R_C}$$

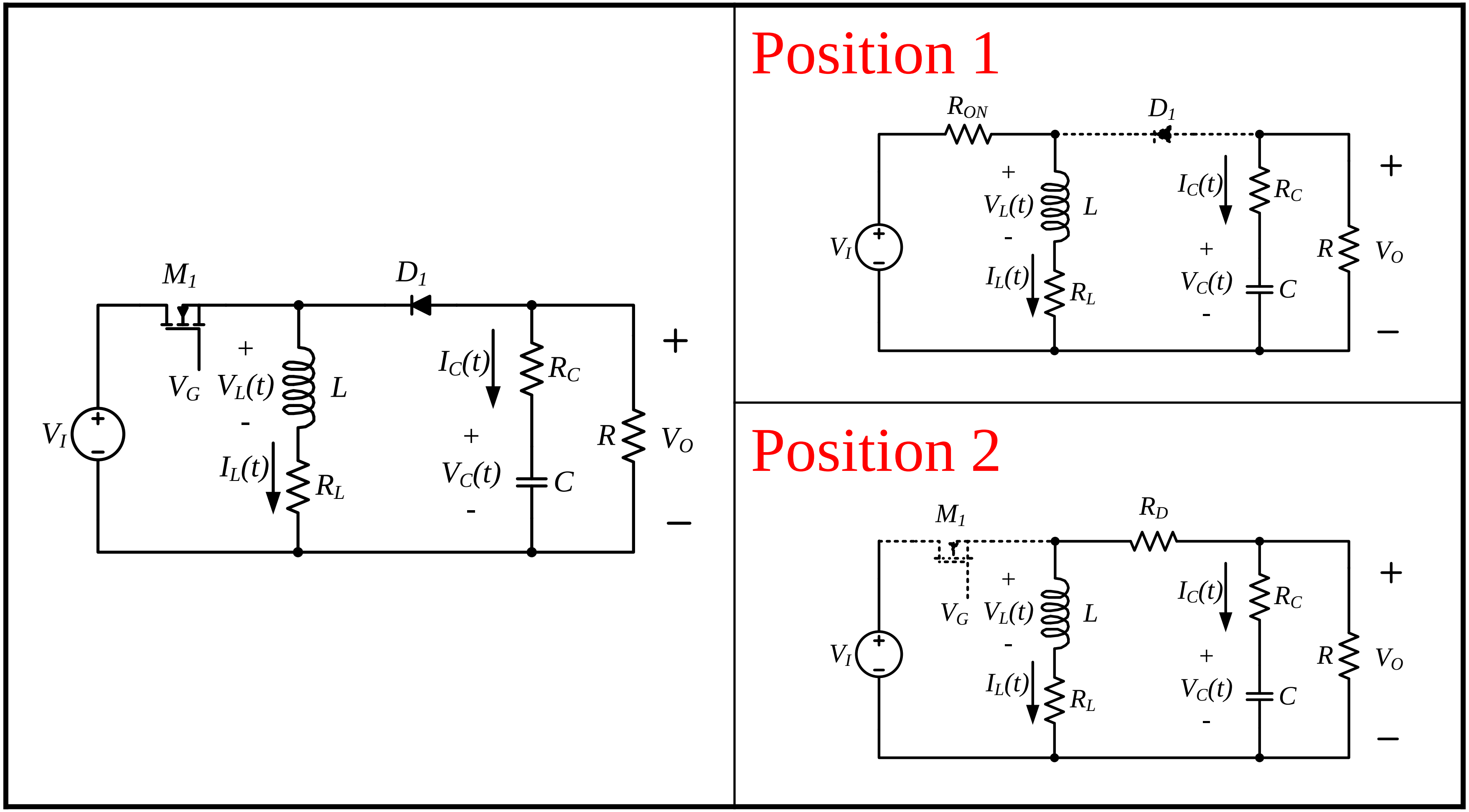
$$\alpha_2=\left(R_L+DR_{ON}+D'R_D+DD'(R\parallel R_C)\right)I_L$$

$$\Delta=S^2+S\left(\frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+R\parallel R_C\right)\right)}{\left(R+R_C\right)LC}\right)+\left(\frac{R_L+DR_{ON}+D'\left(R_D+R\parallel R_C\right)+\frac{\left(D'R\right)^2}{R+R_C}}{\left(R+R_C\right)LC}\right)$$

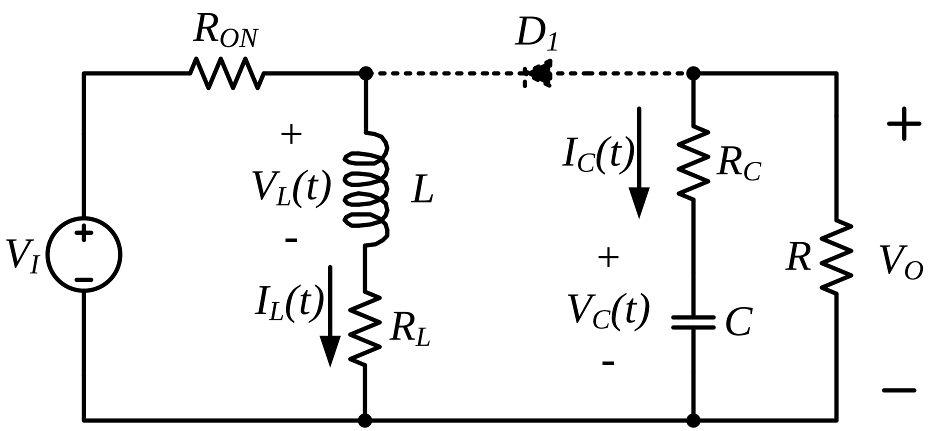
Outline

- 1-1. Steady-state analysis
  1) Buck converter
  2) Boost converter
  3) Buck-Boost converter
- 1-2. Transient-state analysis
  1) Buck converter
  2) Boost converter
  3) Buck-Boost converter

Buck-Boost (1/10)



Buck-Boost (2/10)



Position 1

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_{ON}+R_L}{L} & 0 \\ \frac{R}{(R+R_C)C} & -\frac{1}{(R+R_C)C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_I$$

$$[v_O] = \begin{bmatrix} 0 & \frac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$[i_i] = [1 \quad 0] \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

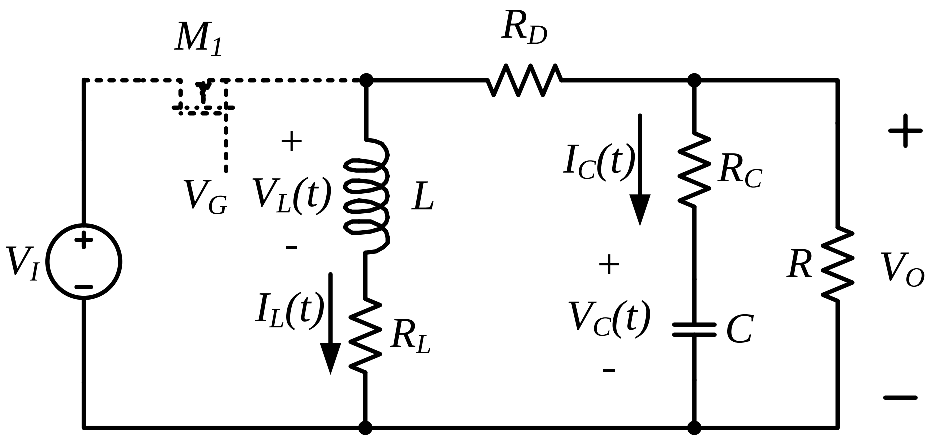
$$A_1 = \begin{bmatrix} -\frac{R_{ON}+R_L}{L} & 0 \\ 0 & -\frac{1}{(R+R_C)C} \end{bmatrix} +$$

$$B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 0 & \frac{R}{R+R_C} \end{bmatrix}$$

$$C_1' = \begin{bmatrix} 0 & \frac{1}{R_C} \end{bmatrix}$$

Buck-Boost (3/10)



Position 2

$$\begin{bmatrix} \frac{di_L}{dt} \\ \frac{dv_C}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R_D + R_L + \frac{R_C}{R+R_C}}{L} & \frac{R}{(R+R_C)L} \\ -\frac{R}{(R+R_C)C} & -\frac{1}{(R+R_C)C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V_I$$

$$\begin{bmatrix} v_O \end{bmatrix} = \begin{bmatrix} -\frac{R_C}{R+R_C} & \frac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$\begin{bmatrix} i_i \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{R_D + R_L + \frac{R_C}{R+R_C}}{L} & \frac{R}{(R+R_C)L} \\ -\frac{R}{(R+R_C)C} & -\frac{1}{(R+R_C)C} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} -R \parallel R_C & \frac{R}{R+R_C} \end{bmatrix}$$

$$C_2' = \begin{bmatrix} -1 & \frac{1}{R_C} \end{bmatrix}$$

Buck-Boost (4/10)

$$A = DA_1 + D'A_2 = \begin{bmatrix} -\dfrac{R_L + DR_{ON} + D'\left(R_D + \dfrac{R_C}{R+R_C}\right)}{L} & \dfrac{D'R}{(R+R_C)L} \\ -\dfrac{D'R}{(R+R_C)C} & -\dfrac{1}{(R+R_C)C} \end{bmatrix}$$

$$B = DB_1 + D'B_2 = \begin{bmatrix} \dfrac{D}{L} \\ 0 \end{bmatrix}$$

$$C = DC_1 + D'C_2 = \begin{bmatrix} -\dfrac{D'R_C}{R+R_C} & \dfrac{R}{R+R_C} \end{bmatrix}$$

$$C' = DC_1' + D'C_2' = \begin{bmatrix} D & 0 \end{bmatrix}$$

$$A_1 - A_2 = \begin{bmatrix} \dfrac{-R_{ON} + R_D + \dfrac{R_C}{R+R_C}}{L} & -\dfrac{R}{\left(R+R_C\right)L} \\ \dfrac{R}{\left(R+R_C\right)C} & 0 \end{bmatrix}$$

$$B_1 - B_2 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}$$

$$C_1 - C_2 = \begin{bmatrix} \dfrac{R_C}{R+R_C} & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} \dfrac{-R_{ON} + R_D + \dfrac{R_C}{R+R_C}}{L} & -\dfrac{R}{\left(R+R_C\right)L} \\ \dfrac{R}{\left(R+R_C\right)C} & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \begin{bmatrix} V_I \end{bmatrix}$$

$$G = \begin{bmatrix} \dfrac{R_C}{R+R_C} & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix}$$

Buck-Boost (5/10)

$$(SI-A)^{-1}=\frac{adj(SI-A)}{\left|SI-A\right|}$$

$$(SI-A)^{-1}=\begin{bmatrix} S+\dfrac{R_L+DR_{ON}+D'\left(R_D+\dfrac{R_C}{R+R_C}\right)}{L} & -\dfrac{D'R}{\left(R+R_C\right)L} \\ \dfrac{D'R}{\left(R+R_C\right)C} & S+\dfrac{1}{\left(R+R_C\right)C} \end{bmatrix}^{-1}$$

$$adj(SI-A)=\begin{bmatrix} S+\dfrac{1}{\left(R+R_C\right)C} & \dfrac{D'R}{\left(R+R_C\right)L} \\ -\dfrac{D'R}{\left(R+R_C\right)C} & S+\dfrac{R_L+DR_{ON}+D'\left(R_D+\dfrac{R_C}{R+R_C}\right)}{L} \end{bmatrix}$$

$$\Delta=\left|SI-A\right|=S^2+S\left(\frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+\dfrac{R_C}{R+R_C}\right)\right)}{LC\left(R+R_C\right)}\right)+\left(\frac{R_L+DR_{ON}+D'\left(R_D+\dfrac{R_C}{R+R_C}\right)+\dfrac{\left(D'R\right)^2}{R+R_C}}{LC\left(R+R_C\right)}\right)$$

Buck-Boost (6/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = C(SI - A)^{-1}B$$

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \begin{bmatrix} -\frac{D'R_C}{R+R_C} & \frac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} S + \frac{R_L + DR_{ON} + D'\left(R_D + \frac{R_C}{R+R_C}\right)}{L} & -\frac{D'R}{\left(R+R_C\right)L} \\ \frac{D'R}{\left(R+R_C\right)C} & S + \frac{1}{\left(R+R_C\right)C} \end{bmatrix}^{-1} \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}$$

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{1}{\Delta}\left(-\frac{DD'R\left(SR_C C + 1\right)}{\left(R+R_C\right)LC} \right)$$

Buck-Boost (7/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0} = C(SI-A)^{-1}F+G$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0} = \begin{bmatrix} -\frac{D'R_C}{R+R_C} & \frac{R}{R+R_C} \end{bmatrix} \begin{bmatrix} S+\frac{R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)}{L} & -\frac{D'R}{(R+R_C)L} \\ \frac{D'R}{(R+R_C)C} & S+\frac{1}{(R+R_C)C} \end{bmatrix}^{-1} F+G$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0} = \frac{1}{\Delta}\begin{bmatrix} SR_C+\frac{R^2+R_C}{(R+R_C)C} & S+\frac{R_L+DR_{ON}+D'R_D+D'D\left(\frac{R_C}{R+R_C}\right)}{L} \end{bmatrix} \begin{bmatrix} \frac{-D'}{R+R_C} & 0 \\ 0 & \frac{R}{R+R_C} \end{bmatrix} F+G$$

$$\alpha_1 = \left(-R_{ON}+R_D+\frac{R_C}{R+R_C}\right)I_L - \left(\frac{R_C}{R+R_C}\right)V_C$$

$$\alpha_2 = \left(\frac{R^2}{R+R_C}\right)\left(R_L+DR_{ON}+D'R_D+DD'\left(\frac{R_C}{R+R_C}\right)\right)I_L$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0} = \frac{(D'V_I+\alpha_1)}{\Delta}\left(S\left(\frac{R^2 I_L L}{(R+R_C)(D'V_I+\alpha_1)}-R_C C\right)+\left(\frac{\alpha_2}{(D'V_I+\alpha_1)}-\frac{R^2+R_C}{R+R_C}\right)\right)$$

Buck-Boost (8/10)

$$\left.\frac{\hat{i}_i(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = C'(SI-A)^{-1}F$$

$$\left.\frac{\hat{i}_i(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \begin{bmatrix} D & 0 \end{bmatrix} \begin{bmatrix} S+\dfrac{R_L+DR_{ON}+D'\left(R_D+\dfrac{R_C}{R+R_C}\right)}{L} & -\dfrac{D'R}{\left(R+R_C\right)L} \\ \dfrac{D'R}{\left(R+R_C\right)C} & S+\dfrac{1}{\left(R+R_C\right)C} \end{bmatrix}^{-1} \begin{bmatrix} \dfrac{D}{L} \\ 0 \end{bmatrix}$$

$$\left.\frac{\hat{i}_i(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{1}{\Delta}\left(\frac{D^2}{L}\right)\left(\frac{S\left(R+R_C\right)C+1}{\left(R+R_C\right)C}\right)$$

$$Z'_{out}(S) = \left(-\frac{D}{D'}\right)^2 Z'_{out}(S) = \left(\frac{1}{D'}\right)^2\left(\frac{\Delta\left(R+R_C\right)LC}{S\left(R+R_C\right)C+1}\right)$$

$$Z_{out}(S) = \left(1-\frac{\left(D'\right)^2 R\left(SR_CC+1\right)}{\Delta\left(R+R_C\right)LC}\right)\left(\frac{R\left(SR_CC+1\right)}{S\left(R+R_C\right)C+1}\right)$$

Buck-Boost (9/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{1}{\Delta}\left(\left(-\frac{DD'R\left(SR_C C+1\right)}{\left(R+R_C\right)LC}\right)\right)$$

$$\Delta = S^2 + S\left(\frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)\right)}{LC\left(R+R_C\right)}\right)+\left(\frac{R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)+\frac{\left(D'R\right)^2}{R+R_C}}{LC\left(R+R_C\right)}\right)$$

$$\left.\frac{\hat{v}_O(S)}{\hat{v}_i(S)}\right|_{\hat{d}(s)=0} = \frac{K(C_3S^{-1}+C_4S^{-2})}{1-(-C_1S^{-1}-C_2S^{-2})}$$

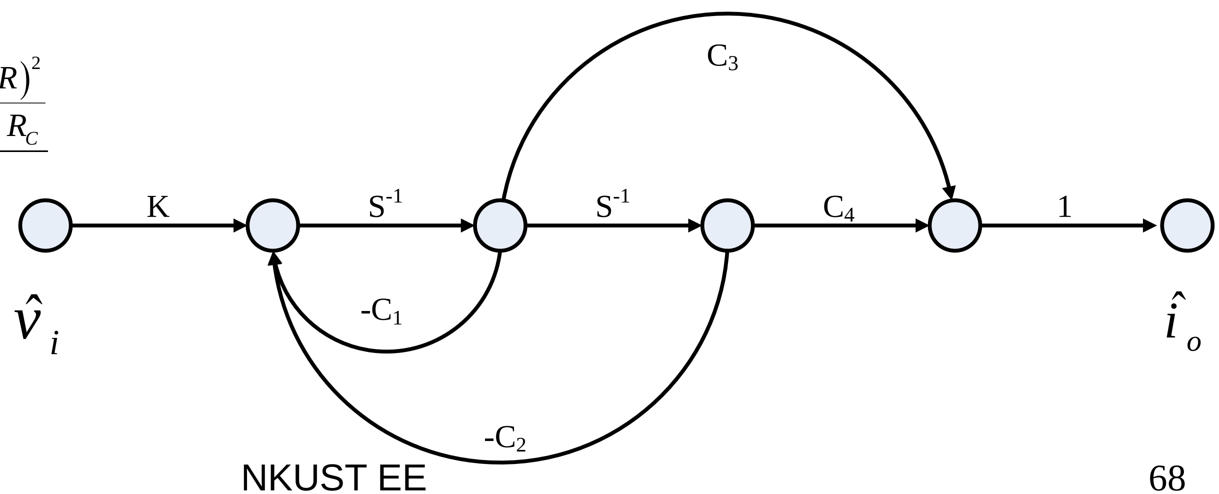
$$C_1 = \frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)\right)}{LC\left(R+R_C\right)}$$

$$C_2 = \frac{R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)+\frac{\left(D'R\right)^2}{R+R_C}}{LC\left(R+R_C\right)}$$

$$C_3 = R_C C$$

$$C_4 = 1$$

$$K = -\frac{DD'R}{\left(R+R_C\right)LC}$$



Buck-Boost (10/10)

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{u}(s)=0}=\frac{\left(D'V_I+\alpha_1\right)}{\Delta}\left(S\left(\frac{R^2I_LL}{\left(R+R_C\right)\left(D'V_I+\alpha_1\right)}-R_CC\right)+\left(\frac{\alpha_2}{\left(D'V_I+\alpha_1\right)}-\frac{R^2+R_C}{R+R_C}\right)\right)$$

$$\Delta=S^2+S\left(\frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)\right)}{LC\left(R+R_C\right)}\right)+\left(\frac{R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)+\frac{\left(D'R\right)^2}{R+R_C}}{LC\left(R+R_C\right)}\right)$$

$$\left.\frac{\hat{v}_O(S)}{\hat{d}(S)}\right|_{\hat{V}_I(s)=0}=\frac{K(C_3S^{-1}+C_4S^{-2})}{1-(-C_1S^{-1}-C_2S^{-2})}$$

$$\alpha_1=\left(-R_{ON}+R_D+\frac{R_C}{R+R_C}\right)I_L-\left(\frac{R_C}{R+R_C}\right)V_C$$

$$\alpha_2=\left(\frac{R^2}{R+R_C}\right)\left(R_L+DR_{ON}+D'R_D+DD'\left(\frac{R_C}{R+R_C}\right)\right)I_L$$

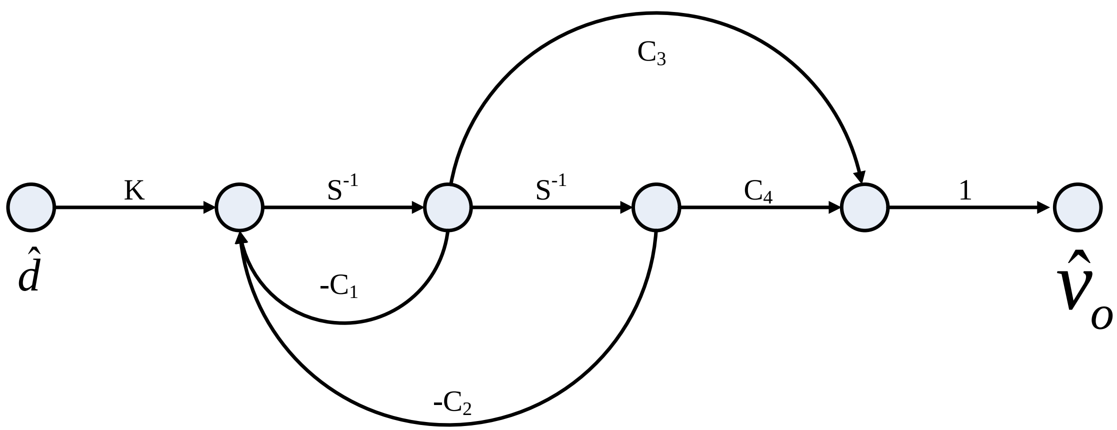
$$C_1=\frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)\right)}{LC\left(R+R_C\right)}$$

$$C_2=\frac{R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)+\frac{\left(D'R\right)^2}{R+R_C}}{LC\left(R+R_C\right)}$$

$$C_3=\frac{R^2I_LL}{\left(R+R_C\right)\left(D'V_I+\alpha_1\right)}-R_CC$$

$$C_4=\frac{\alpha_2}{\left(D'V_I+\alpha_1\right)}-\frac{R^2+R_C}{R+R_C}$$

$$K=\left(D'V_I+\alpha_1\right)$$



Type	Buck-Boost
$G_{vg}(S)$	$\frac{1}{\Delta}\left(\left(-\frac{DD'R\left(SR_C C+1\right)}{\left(R+R_C\right)LC}\right)\right)$
$G_{vd}(S)$	$\frac{\left(D'V_I+\alpha_1\right)}{\Delta}\left(S\left(\frac{R^2 I_L L}{\left(R+R_C\right)\left(D'V_I+\alpha_1\right)}-R_C C\right)+\left(\frac{\alpha_2}{\left(D'V_I+\alpha_1\right)}-\frac{R^2+R_C}{R+R_C}\right)\right)$
$Zout(s)$	$\left(1-\frac{\left(D'\right)^2 R\left(SR_C C+1\right)}{\Delta\left(R+R_C\right)LC}\right)\left(\frac{R\left(SR_C C+1\right)}{S\left(R+R_C\right)C+1}\right)$
ω_Z	$-\left(\frac{\alpha_2}{\left(D'V_I+\alpha_1\right)}-\frac{R^2+R_C}{R+R_C}\right)\Big/\left(\frac{R^2 I_L L}{\left(R+R_C\right)\left(D'V_I+\alpha_1\right)}-R_C C\right)$

$$\Delta = S^2 + S\left(\frac{L+C\left(R+R_C\right)\left(R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)\right)}{LC\left(R+R_C\right)}\right)+\left(\frac{R_L+DR_{ON}+D'\left(R_D+\frac{R_C}{R+R_C}\right)+\frac{\left(D'R\right)^2}{R+R_C}}{LC\left(R+R_C\right)}\right)$$

$$\alpha_1 = \left(-R_{ON}+R_D+\frac{R_C}{R+R_C}\right)I_L-\left(\frac{R_C}{R+R_C}\right)V_C$$

$$\alpha_2 = \left(\frac{R^2}{R+R_C}\right)\left(R_L+DR_{ON}+D'R_D+DD'\left(\frac{R_C}{R+R_C}\right)\right)I_L$$

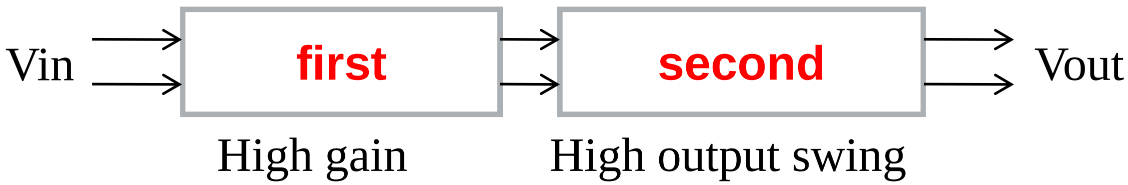
Chapter 2. Operational Amplifier Design

Outline

- 2-1. Introduction
- 2-2. Schematic of OP
- 2-3. Design flow
- 2-4. Layout of OP
- 2-5. Simulation of OP

OP

- Chose the two-stage OP with high gain and high output swing.

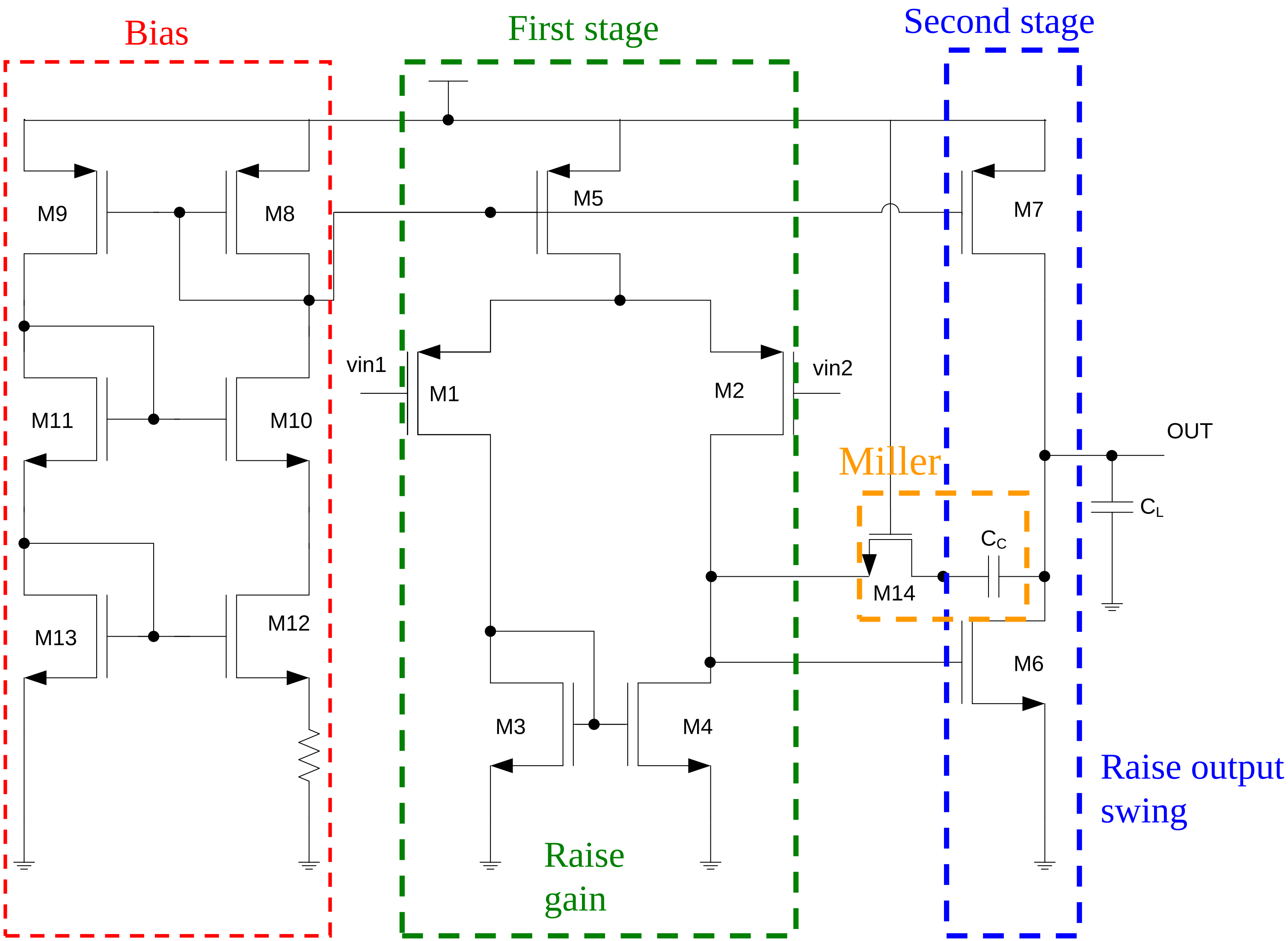


	Gain	Output swing	Speed	Power dissipation	Noise
Telescopic	Medium	Medium	High	Low	Low
Folded cascode	Medium	Medium	High	Medium	Medium
Two stage	High	High	Low	Medium	Low
Gain boosting	High	Medium	Medium	High	Medium

OP (Spec.)

	Specification	Pre-sim	Post-sim
Av	Ao >= 70 db	71 db	71 db
Phase margin	ΦM>=45^o near 70 ^{o}	67 ^{o}	68 ^{o}
Unit-gain frequency	Fo <= 200 MHz	64 MHz	57 MHz
Cc	< 4 pf	0.4 pf	0.4 pf
Slew rate	<= 250 v/μs	75 v/μs	62 v/μs
Setting time	<=100 ns	81 ns	70 ns
Offset	0v	0.2v	0.18v
ICMR	0v – 5v	0.2v – 4.45v	0.18v – 4.48v
PSRR	>60 db	75 db	77db
CMRR	>60 db	64 db	68 db
Power Dissipation	< 10mv	3.009 mv	2.92 mv

Schematic of OP



Design Flow (1/4)

From equation (1) use SR and Cc find I_{D5}：

$$Let \quad SR = 250v/\mu s \qquad C_c = 0.4pF$$

$$\Rightarrow I_{D5} = SR \times C_c = 100\mu A \qquad (1)$$

$$I_8 = I_9 = I_{10} = I_{11} = \frac{I_{D5}}{2} = 50\mu A$$

From equation (2) for M6：

$$\because g_{mi} = g_{m1} = g_{m2} = \omega_o C_c = 2\pi f_0 C_c \qquad (2)$$

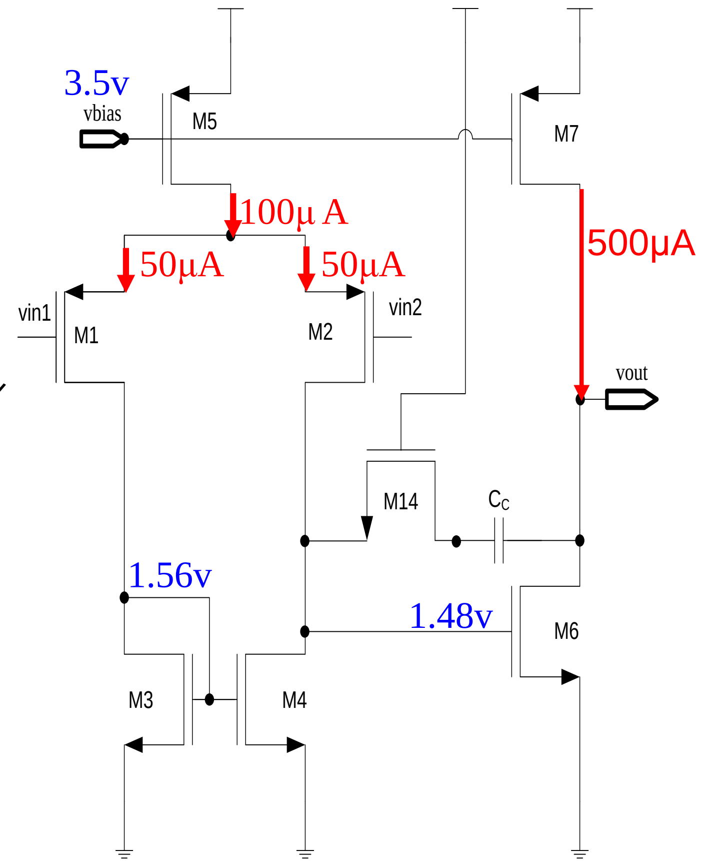
$$= 2 \times 3.14 \times 200 \times 10^6 \times 0.4 \times 10^{-12} = 500\,{}^{\mu A}\!/_{v}$$

$$\therefore g_{m6} = 3g_{m1} = 3 \times 500 = 1500\,{}^{\mu A}\!/_{v}$$

From equation (3) For M3 and M4：

$$\because g_m = \frac{2I_D}{(V_{gs} - V_{th})} \qquad (3)$$

$$\therefore g_{m3} = g_{m4} = \frac{I_o/2}{I_{bias}} g_{m6} = \frac{50\mu A}{500\mu A} g_{m6} = 150\,\mu A/v$$



Design Flow (2/4)

- Assume | V_{GS5} – V_{tp} | = | V_{GS7} - V_{tp} | = 0.5v :

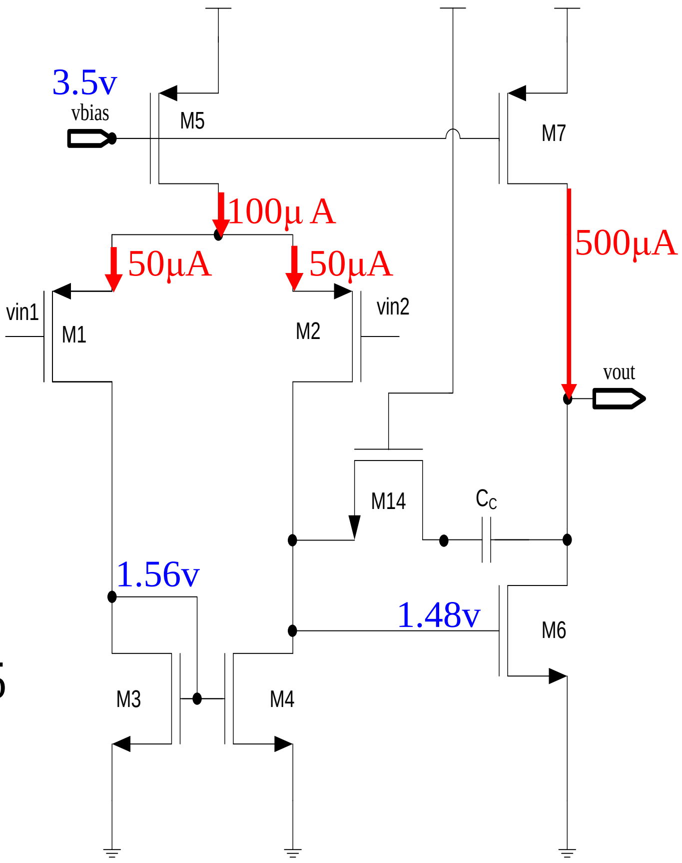
$$\left(\frac{W}{L}\right)_5 = \frac{I_{D5}}{\frac{1}{2}\mu_p C_{OX}\left(V_{SG5}-\left|V_{tp}\right|\right)} = \frac{100\times10^{-6}}{\frac{1}{2}\times26.271\times(0.5)^2} = 30$$

$$\left(\frac{W}{L}\right)_7 = \frac{I_{D7}}{\frac{1}{2}\mu_p C_{OX}\left(V_{SG7}-\left|V_{tp}\right|\right)} = \frac{500\times10^{-6}}{\frac{1}{2}\times26.271\times(0.5)^2} = 150$$

$$\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = \frac{(gm_3)^2}{2\mu_n C_{OX} I_{D3}} = \frac{\left(150\times10^{-6}\right)^2}{2\times42.801\times10^{-6}\times50\times10^{-6}} = 5$$

$$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = \frac{(gm_1)^2}{2\mu_n C_{OX} I_{D1}} = \frac{\left(500\times10^{-6}\right)^2}{2\times26.271\times10^{-6}\times50\times10^{-6}} = 95$$

$$\left(\frac{W}{L}\right)_6 = \frac{I_{D6}}{I_{D3}}\left(\frac{W}{L}\right)_3 = \frac{500\mu A}{50\mu A}\times5 = 50$$



Design Flow (3/4)

- Find V_{G5} from equation (4)：

$$I_{D5}=\frac{1}{2}\mu_p C_{OX}\left(\frac{W}{L}\right)_5\left(V_{GS5}-\left|V_{tp}\right|\right)^2 \qquad (4)$$

$$V_{G5}=V_{S5}-\left[\frac{2I_{D5}}{\mu_P C_{OX}}\left(\frac{L}{W}\right)_5\right]^{\frac{1}{2}}+V_{tp}$$

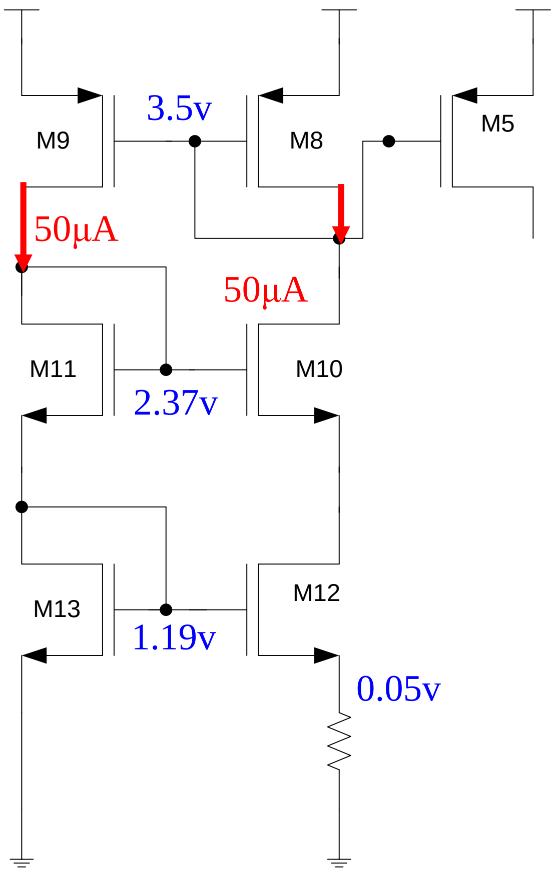
$$=5-\left[\frac{2\times100\times10^{-6}}{26.271\times10^{-6}}\times\frac{1}{30}\right]^{\frac{1}{2}}-1.009=3.491\approx3.5$$

- M8 and M9 can be ：

$$\left(\frac{W}{L}\right)_8=\frac{2\times I_{D5}}{\mu_P C_{OX}\left(V_{SG8}-\left|V_{tp}\right|\right)^2}$$

$$=\frac{2\times50\times10^{-6}}{26.271\times10^{-6}\times\left(5-3.5-1.009\right)^2}=15.8$$

$$\left(\frac{W}{L}\right)_8=\left(\frac{W}{L}\right)_9=\left(\frac{W}{L}\right)_{10}=\left(\frac{W}{L}\right)_{11}=\left(\frac{W}{L}\right)_{13}=15.8$$



Design Flow (4/4)

- V_{G13} can be find from equation ：

$$V_{G13}=\left(\frac{2I_{D5}}{\mu_n C_{OX}\left(\frac{W}{L}\right)_{13}}\right)^{\frac{1}{2}}+V_{tn} \qquad (5)$$

$$=\left(\frac{2\times 50}{42.801\times 15.8}\right)^{\frac{1}{2}}+0.799=1.19$$

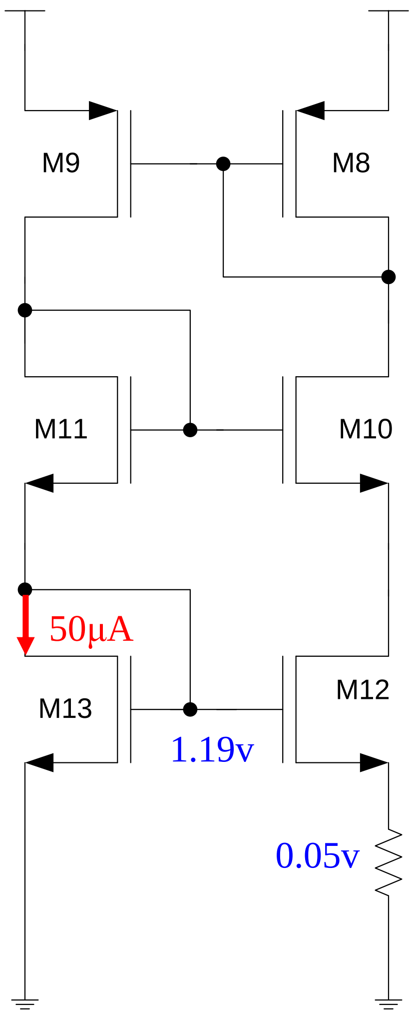
- $(W/L)_{13}$ can be find ：

$$I_{D5}=\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_{12}(V_{GS12}-V_{tn})^2$$

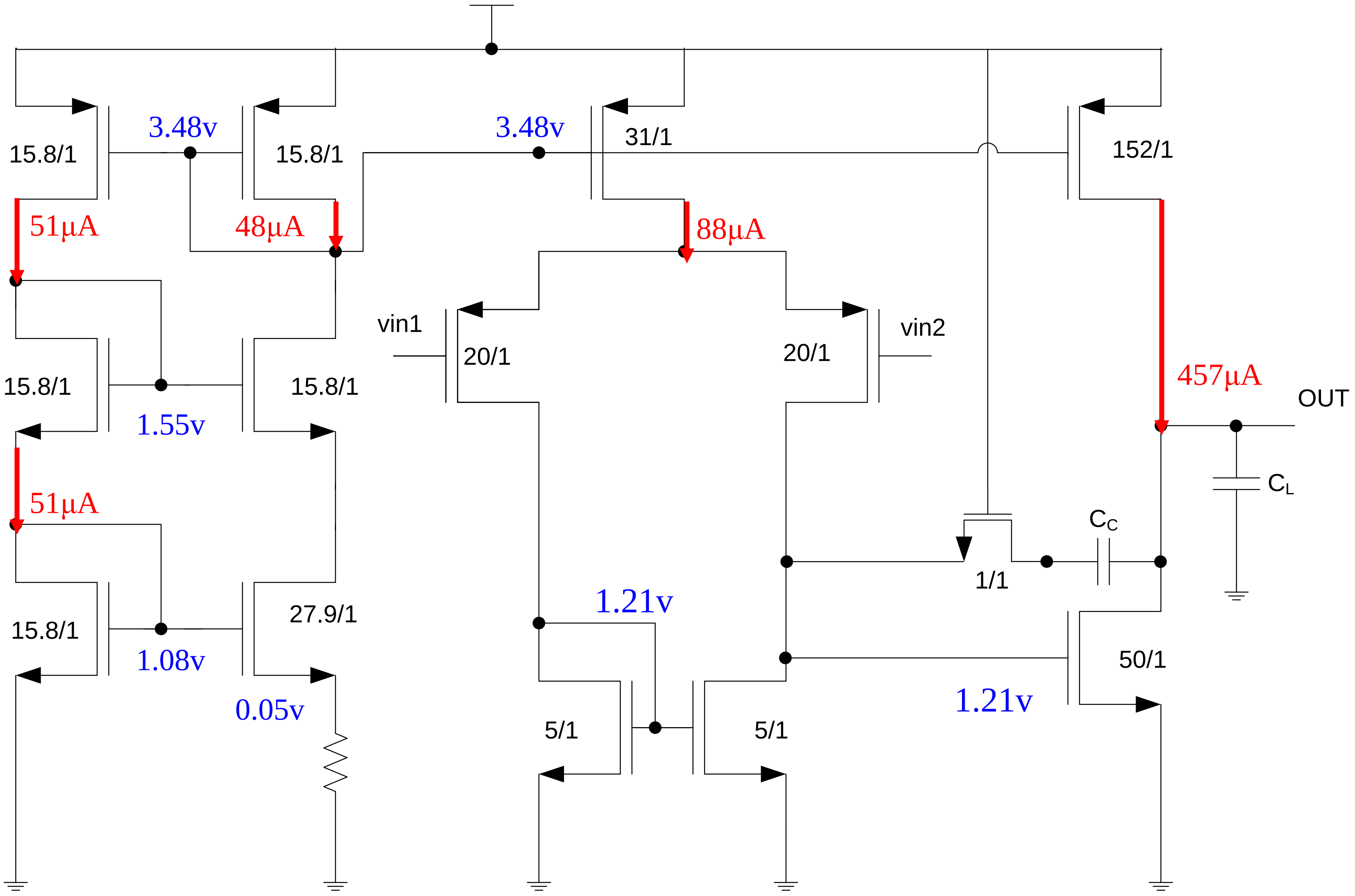
$$\Rightarrow\left(\frac{W}{L}\right)_{12}=\frac{2I_{D5}}{\mu_n C_{ox}(V_{GS12}-V_{tn})^2}\text{, } where\ V_{S12}=I_{D5}R_b$$

$$\Rightarrow\left(\frac{W}{L}\right)_{12}=\frac{2I_{D5}}{\mu_n C_{ox}(V_{G12}-I_{D5}R_b-V_{tn})^2}=\frac{2\times 50\times 10^{-6}}{42.801\times 10^{-6}\times(1.19-50\times 10^{-6}\times R_b-0.799)^2}$$

$$\text{let } R_b=1k\Omega\Rightarrow\left(\frac{W}{L}\right)_{12}=\frac{2\times 50\times 10^{-6}}{42.801\times 10^{-6}\times(1.19-50\times 10^{-6}\times 10^3-0.799)^2}=20$$

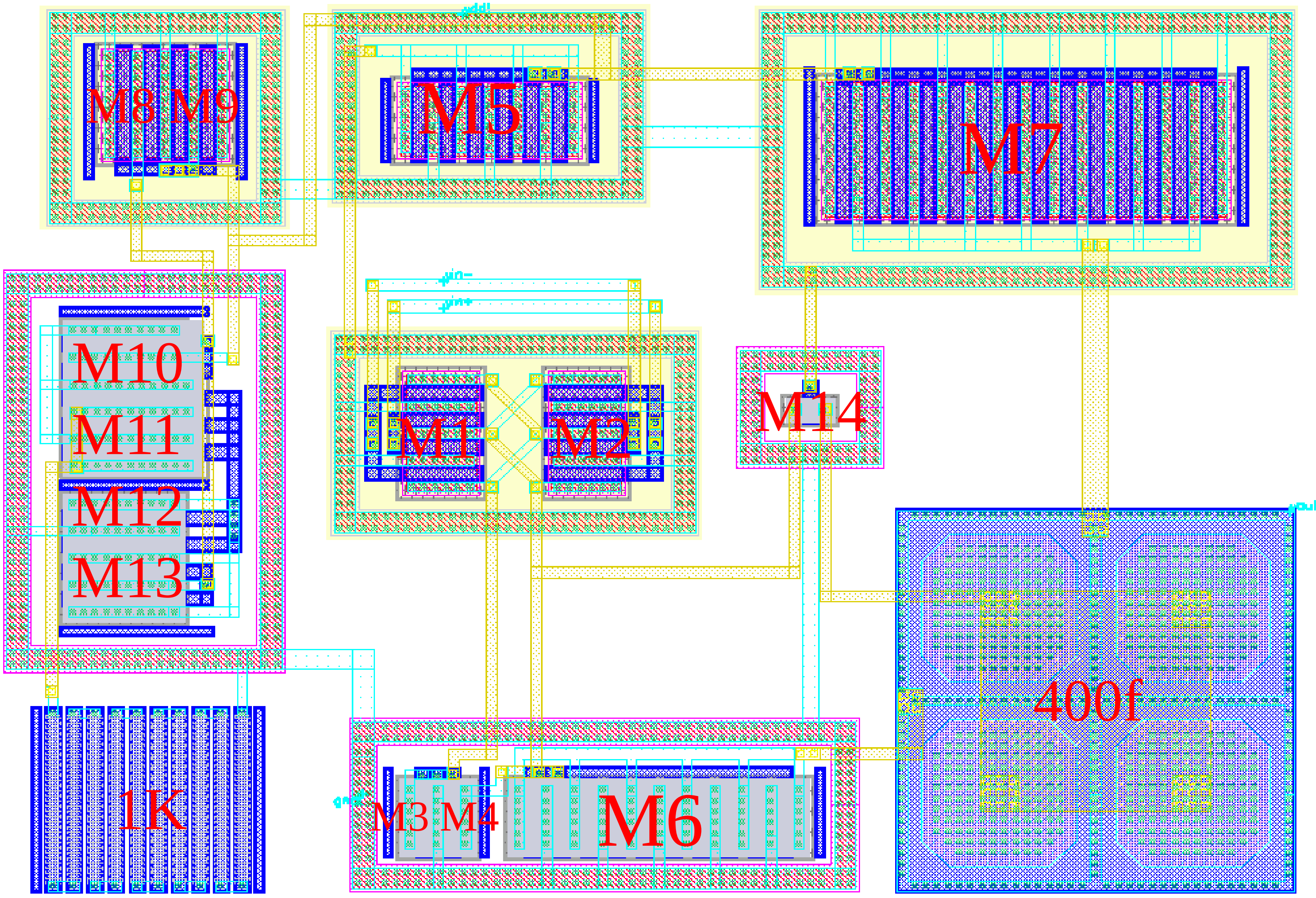


Size of OP

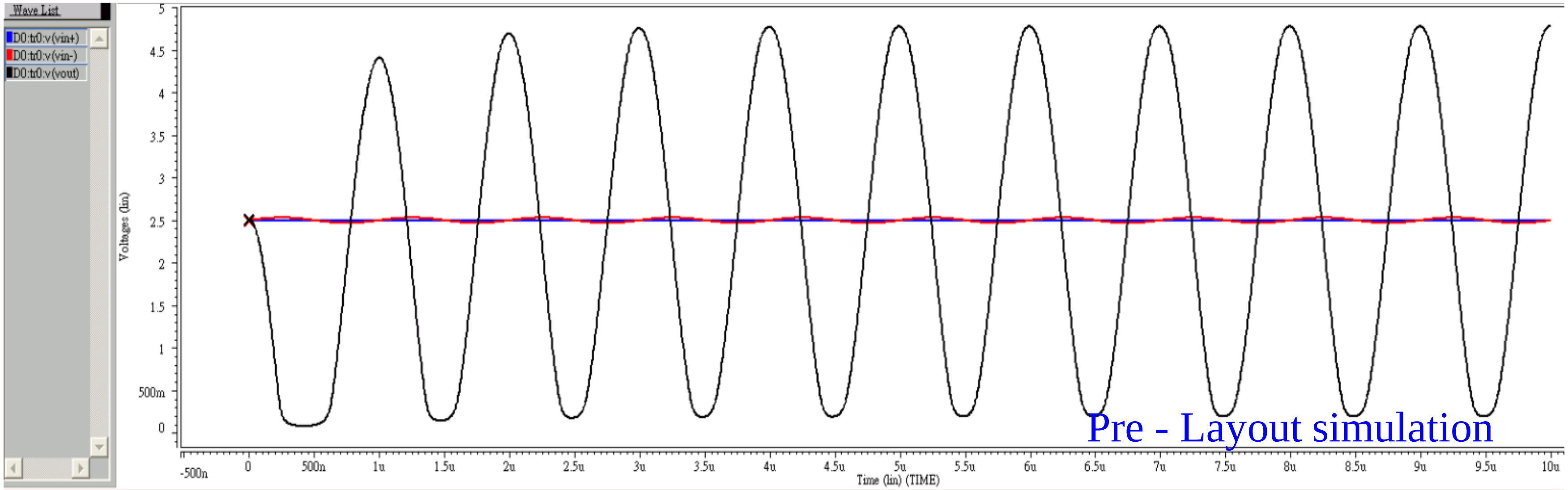


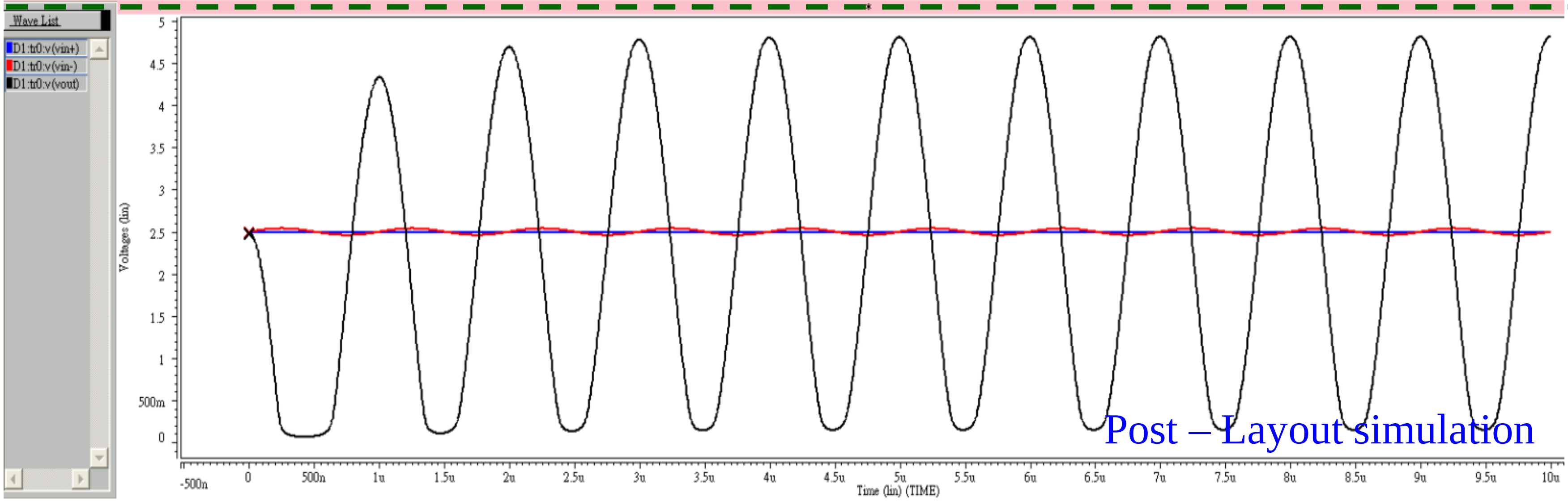
3.48v
3.48v
31/1
15.8/1
15.8/1
152/1
51μA
48μA
88μA
vin1
vin2
20/1
20/1
15.8/1
15.8/1
457μA
OUT
1.55v
51μA
C_L
C_C
1/1
1.21v
27.9/1
15.8/1
1.08v
50/1
0.05v
1.21v
5/1
5/1

Layout of OP

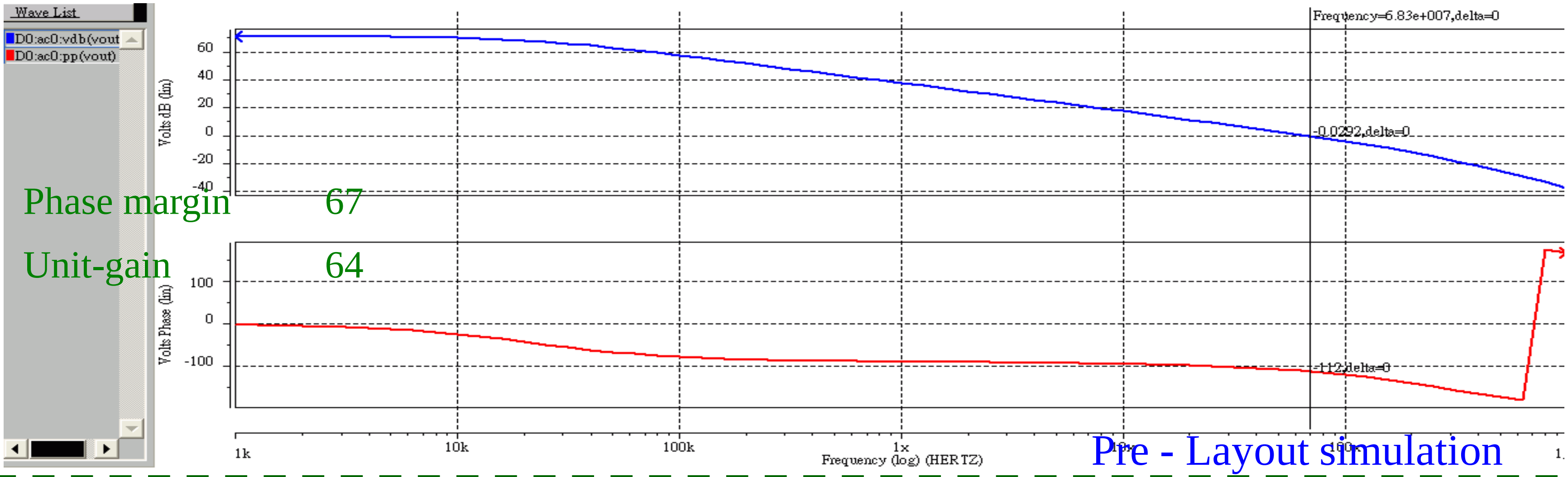

M8 M9
M5
M7
vdd!
vin-
vin+
M10
M11
M12
M13
M1
M2
M14
vout
400f
1K
gnd!
M3 M4
M6

Simulation of Output





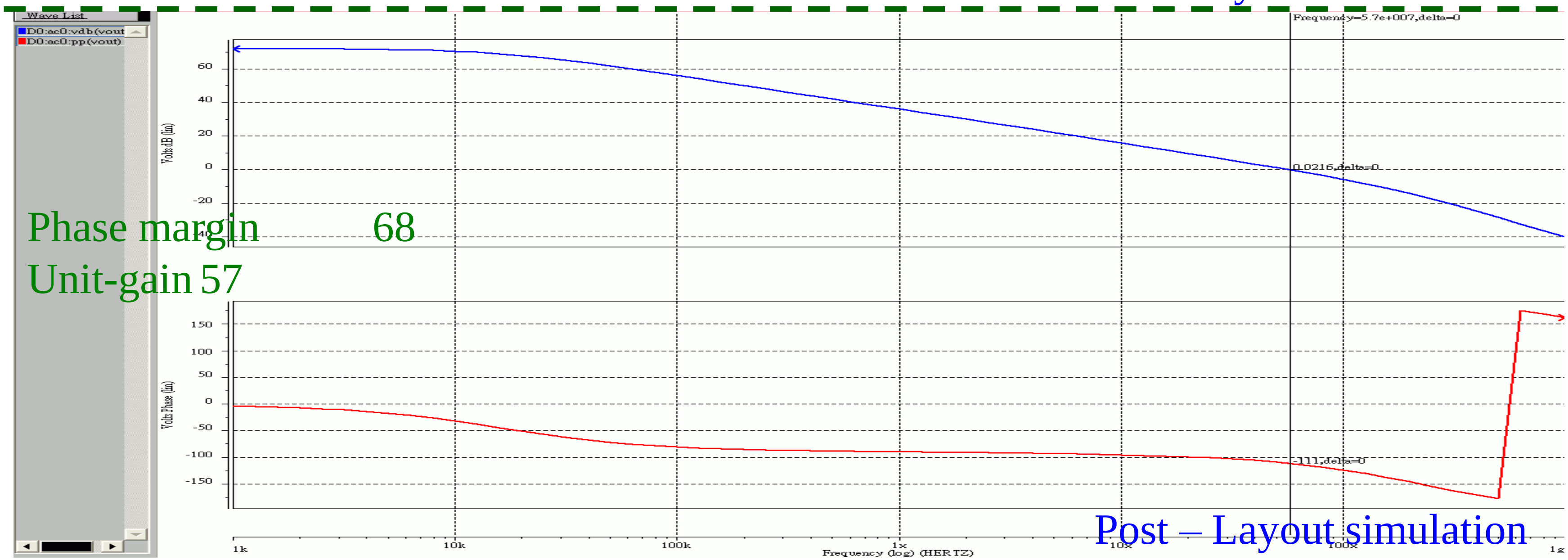
Simulation of Phase Margin



Phase margin 67

Unit-gain 64

Pre - Layout simulation

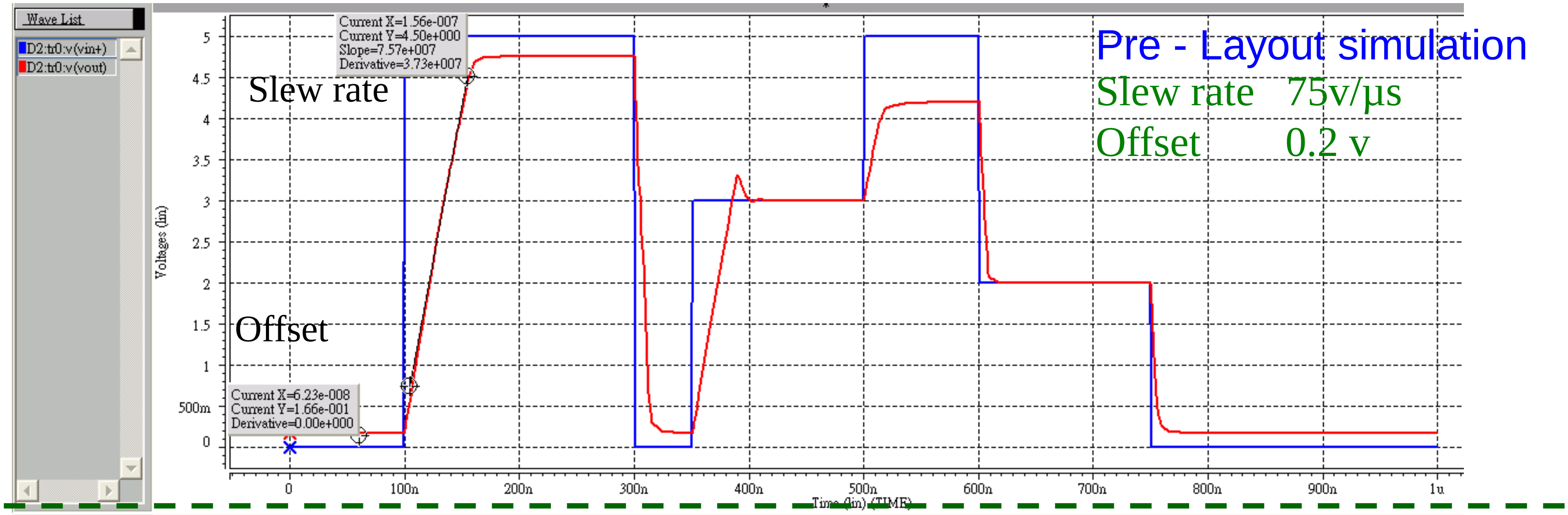


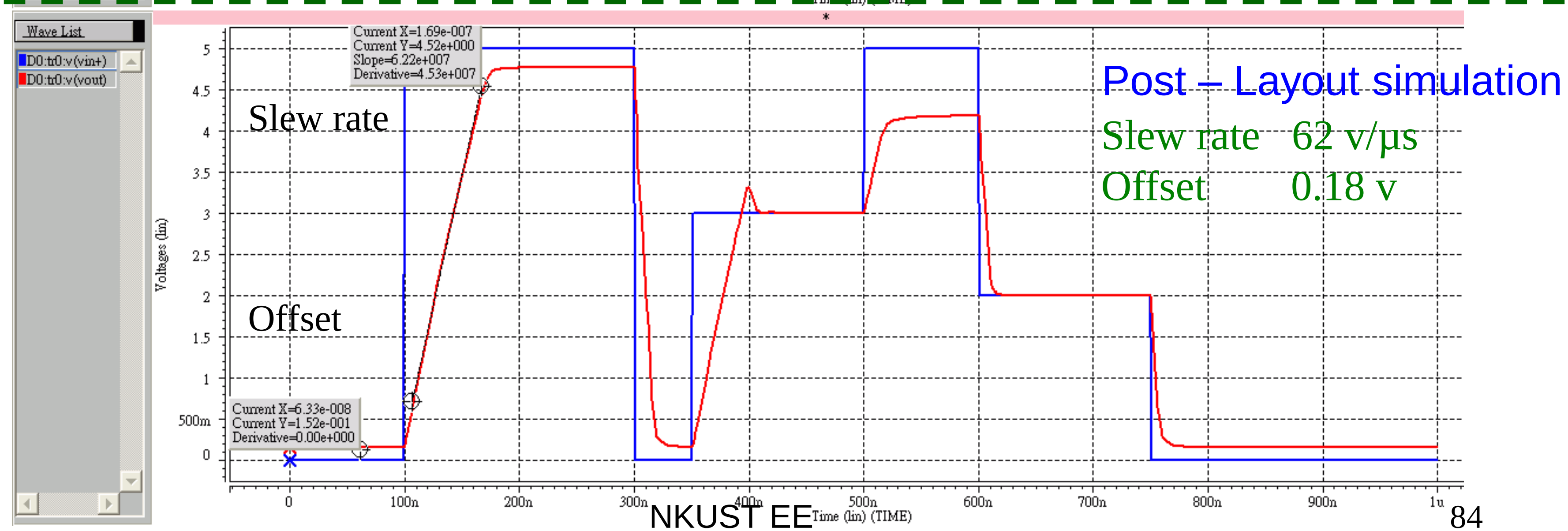
Phase margin 68

Unit-gain 57

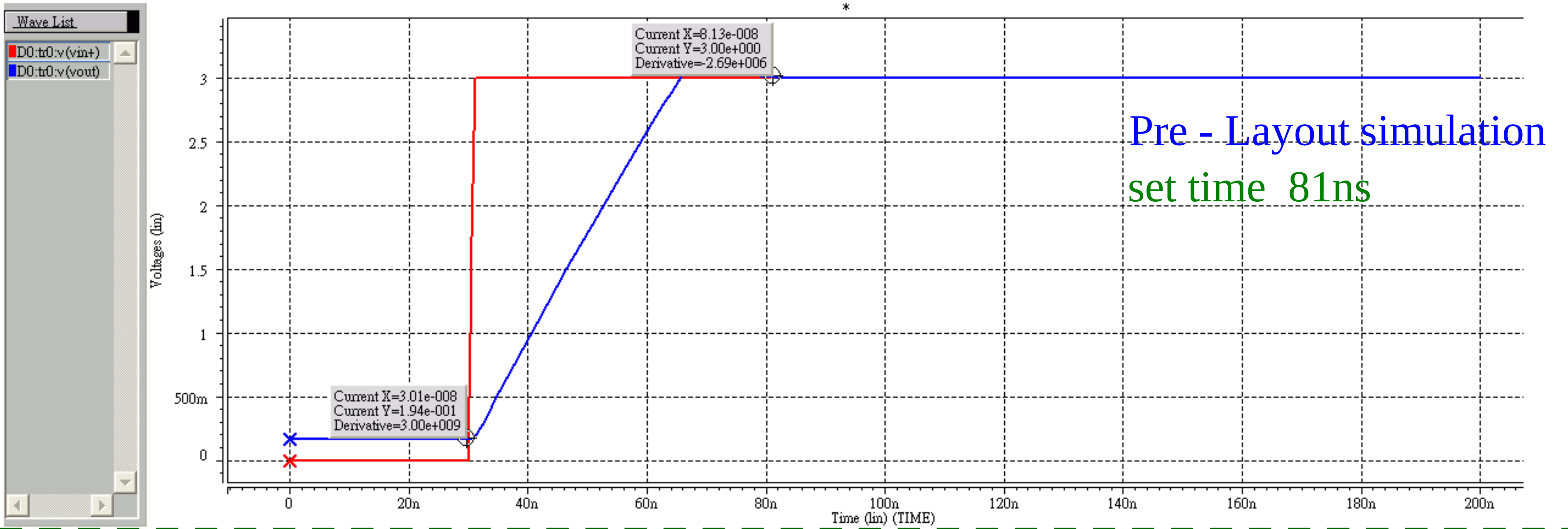
Post – Layout simulation

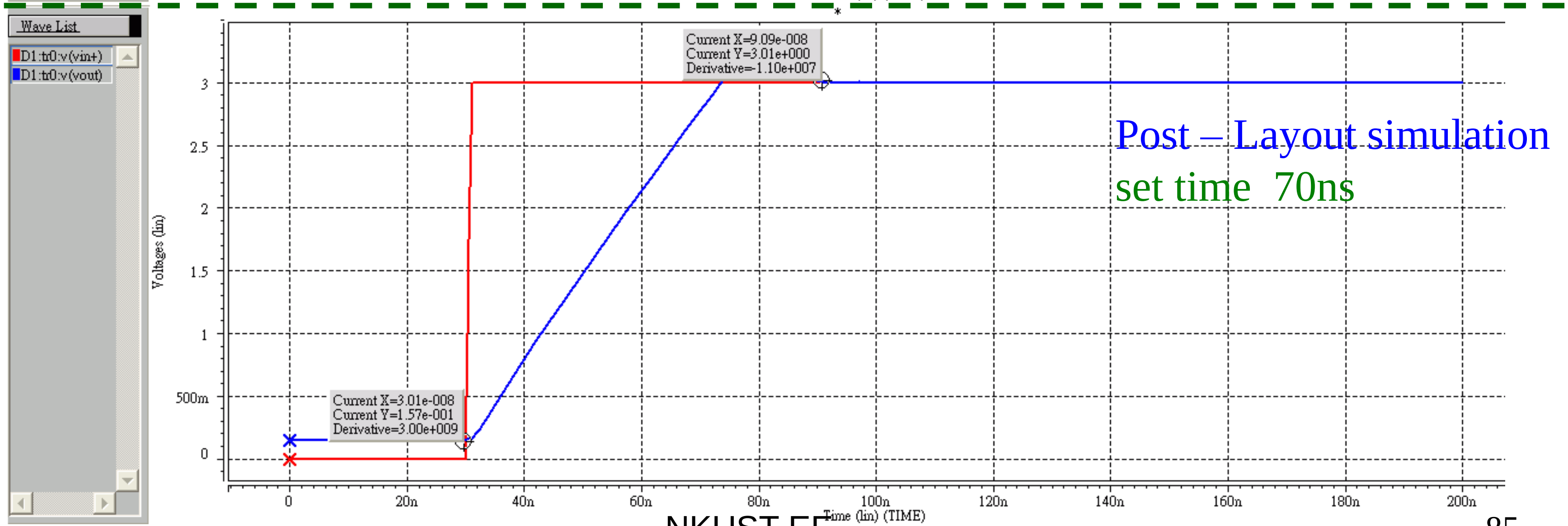
Simulation of Slew Rate



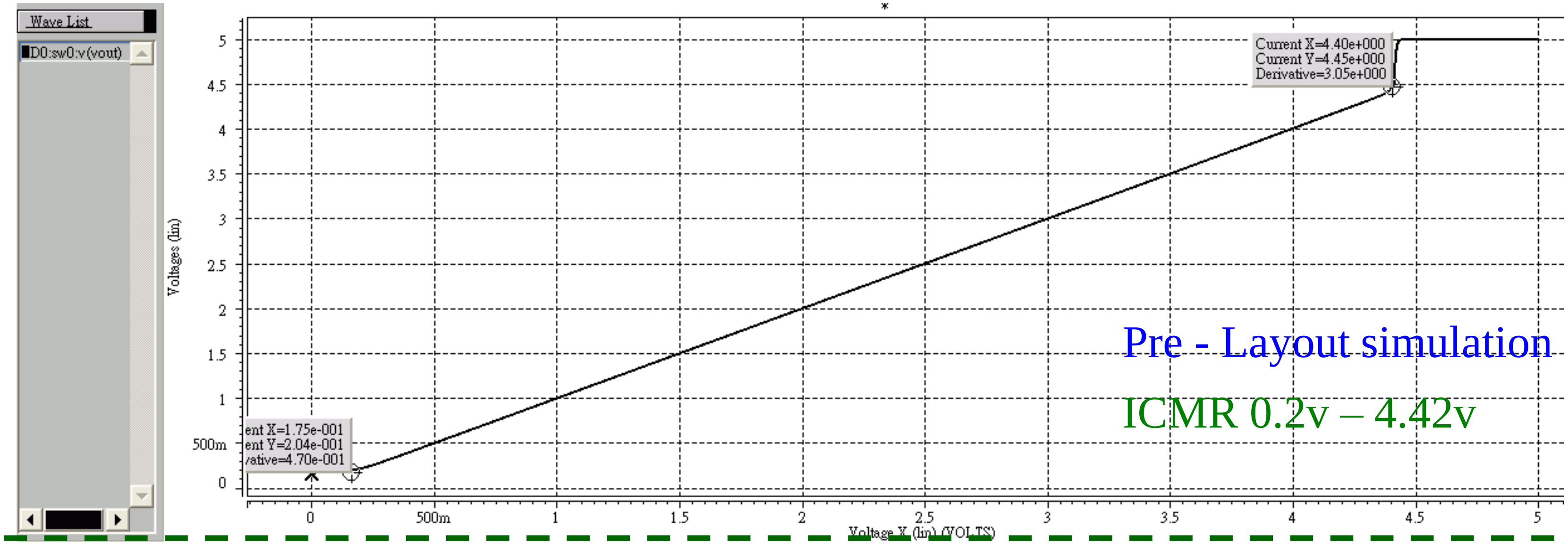


Simulation of Settling Time



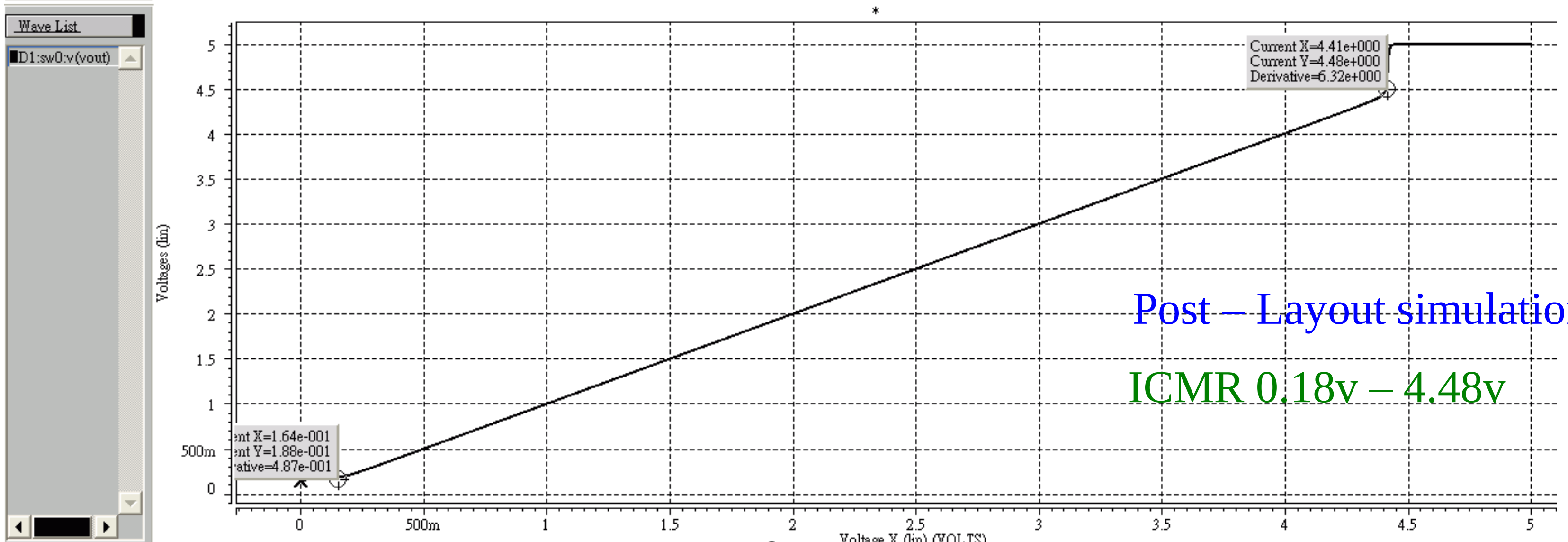


Simulation of ICMR



Pre - Layout simulation

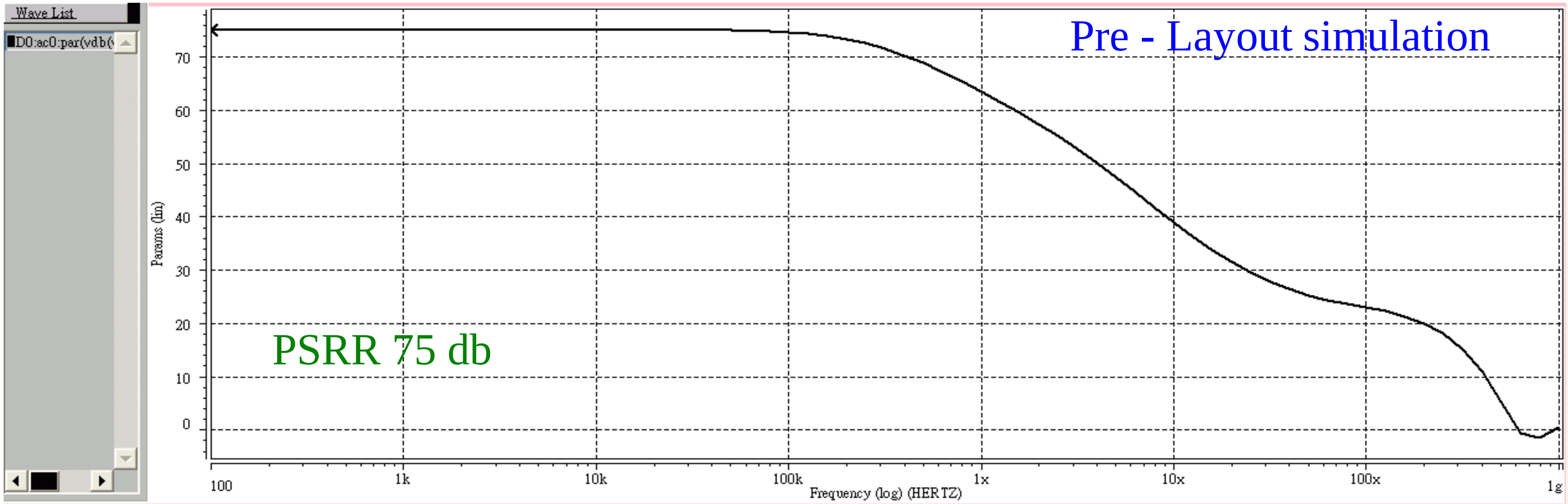
ICMR 0.2v – 4.42v



Post – Layout simulation

ICMR 0.18v – 4.48v

Simulation of PSRR

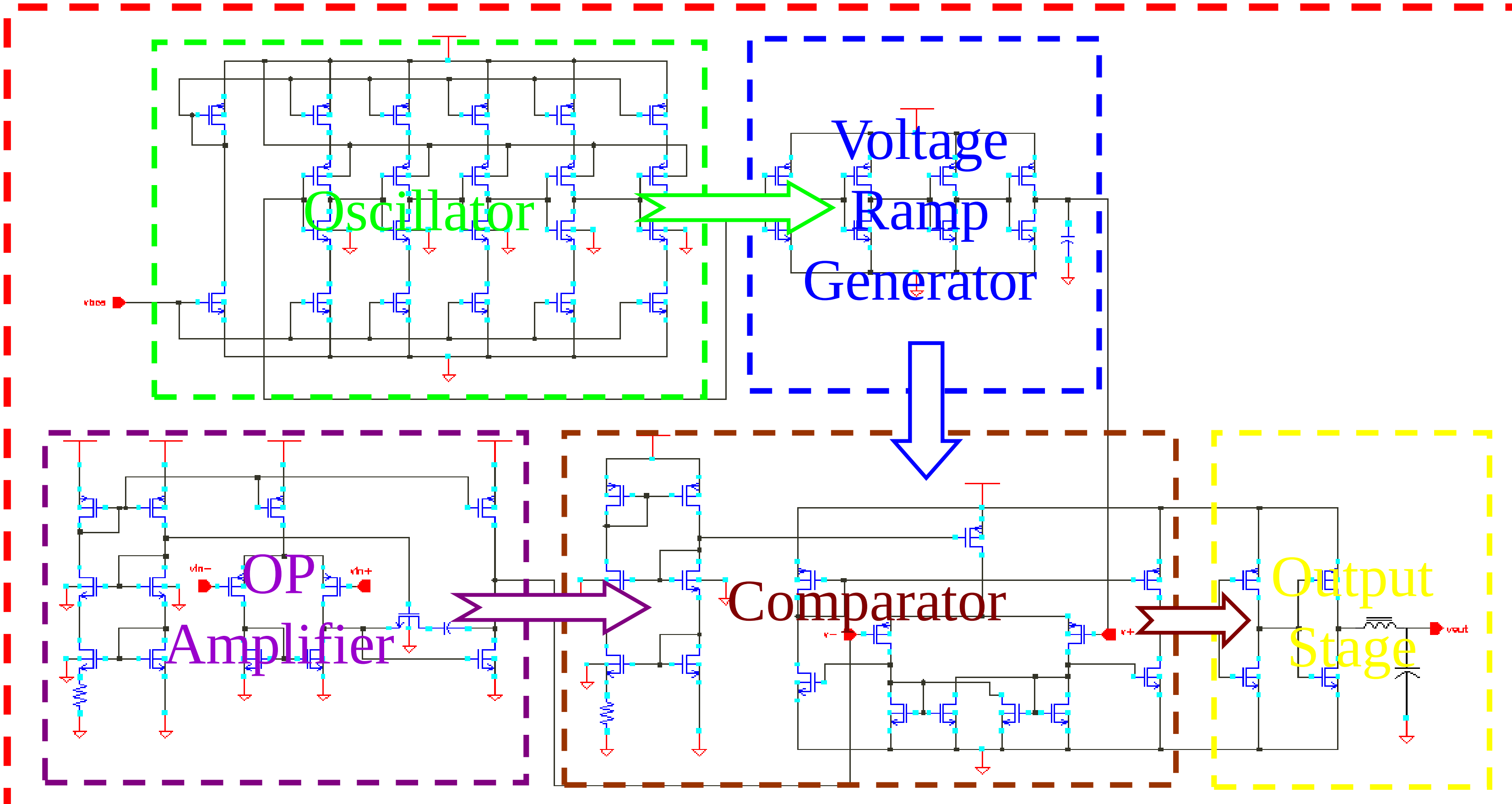


Chapter 3. PWM IC Implementation (Full-Custom Design Tool Operation : Hspice, Laker, and Calibre)

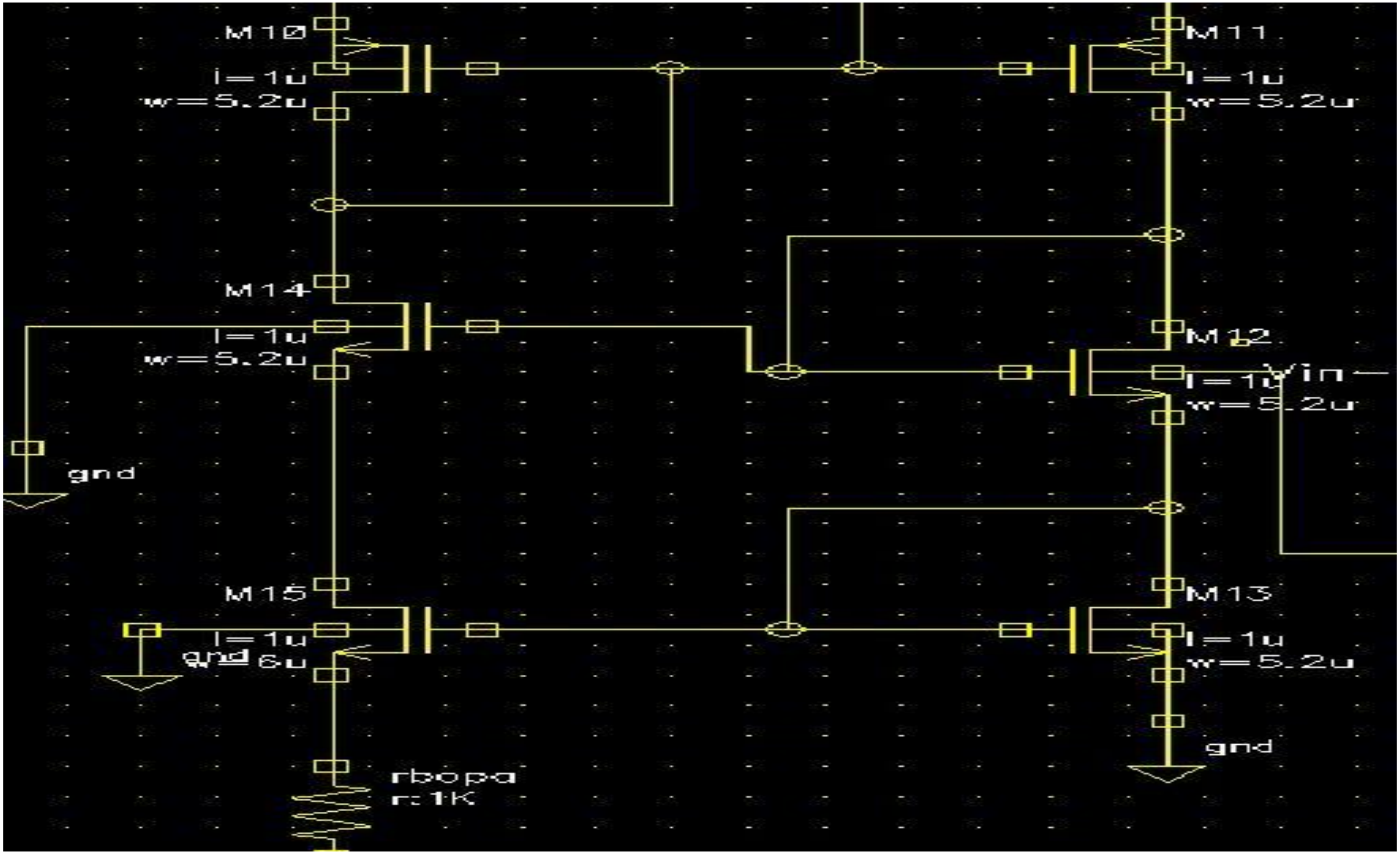
Outline

- 3-1. Operational Amplifier
- 3-2. Comparator
- 3-3. Oscillator
- 3-4. PWM (Pulse-Width Modulation)

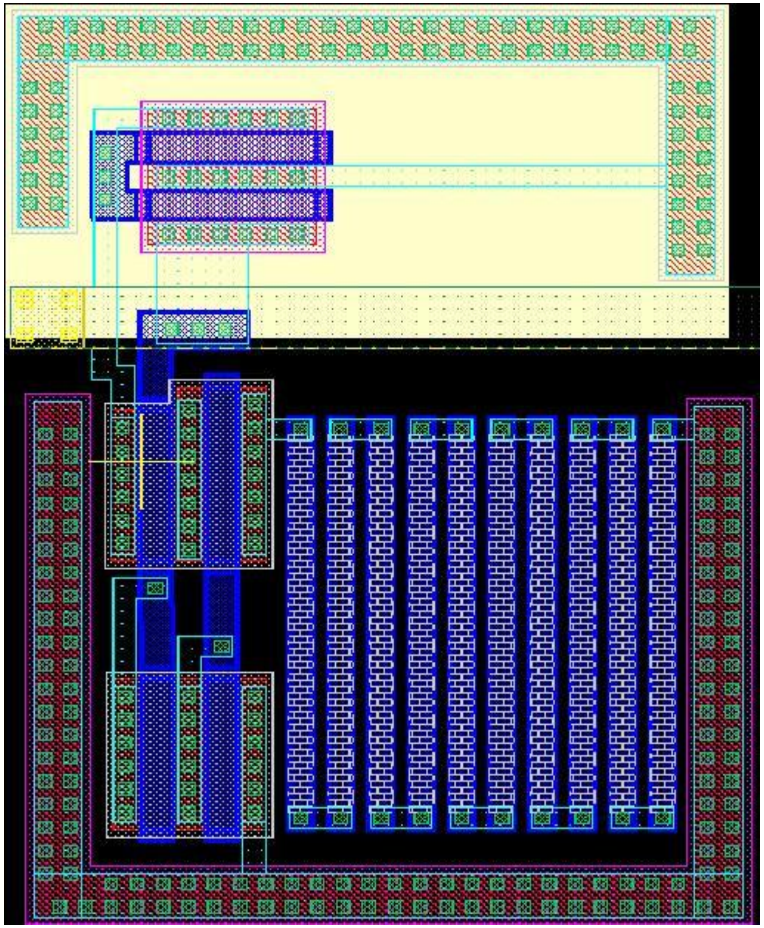
PWM Schematic



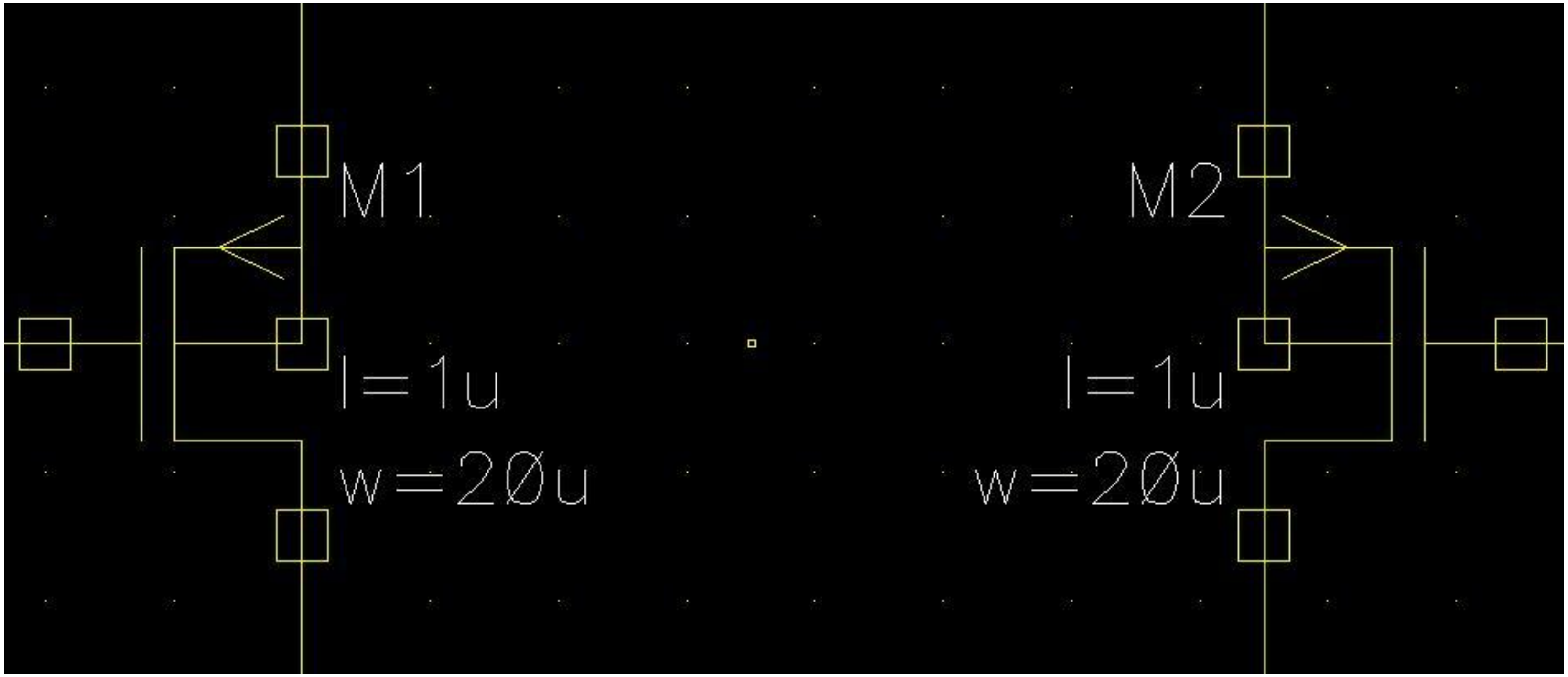
OP-Bias-Schematic



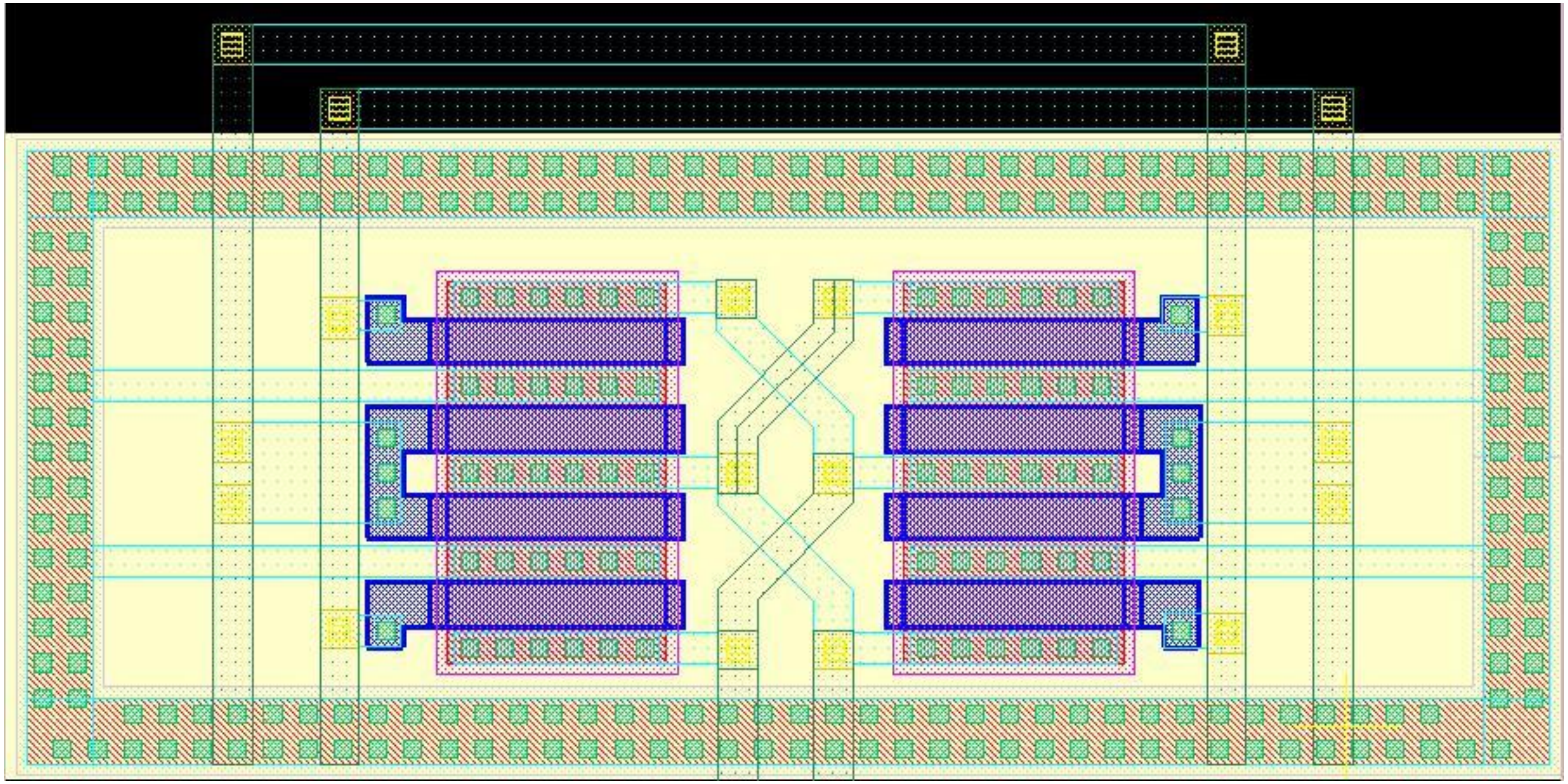
OP-Bias-Layout



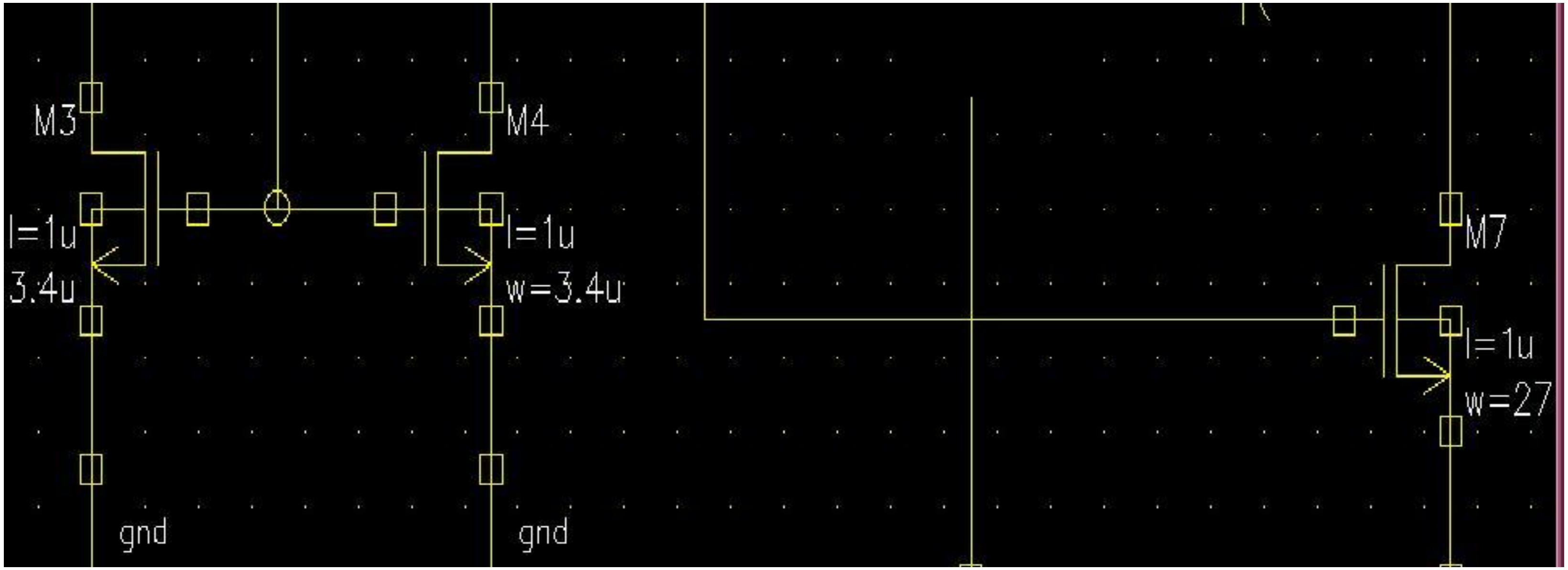
OP-M1~2-Schematic



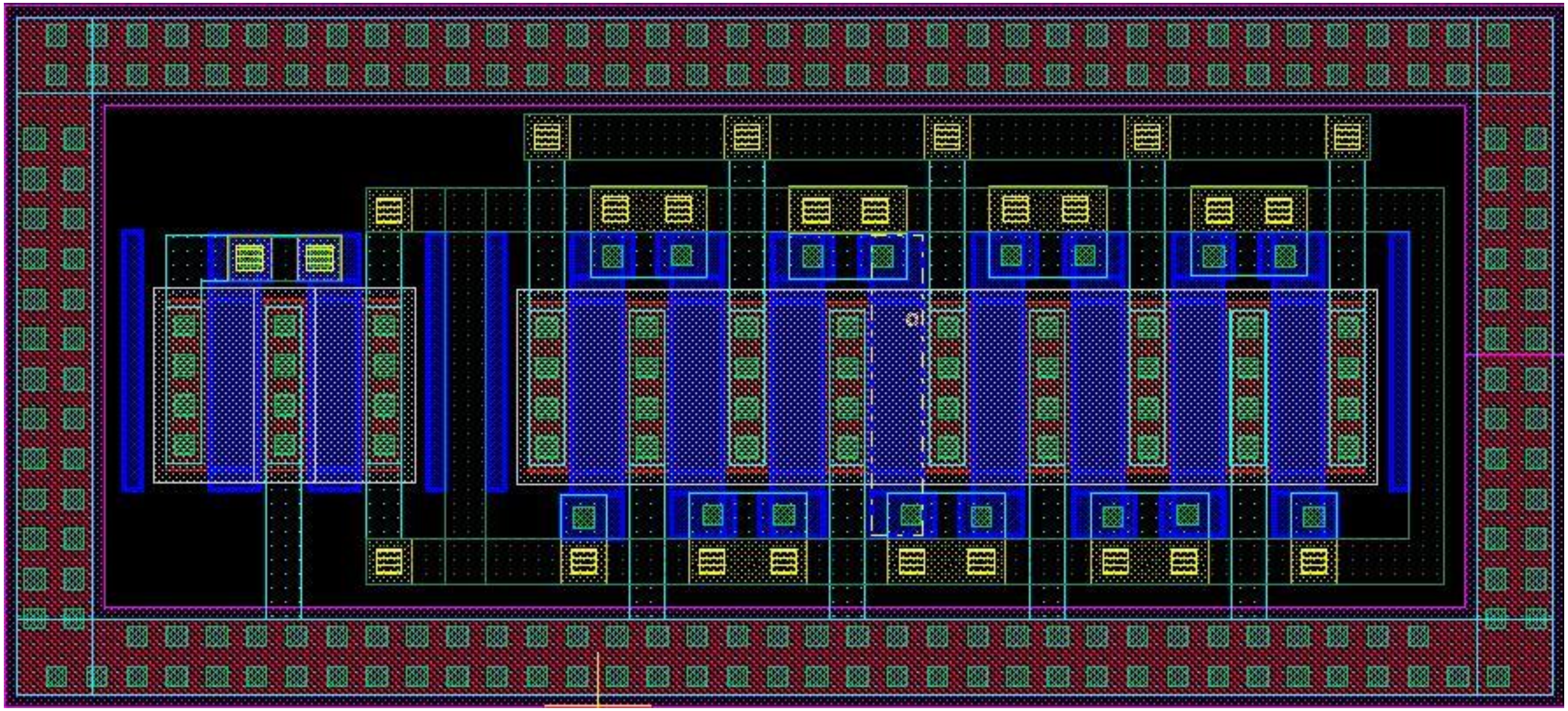
OP-M1~2 Layout



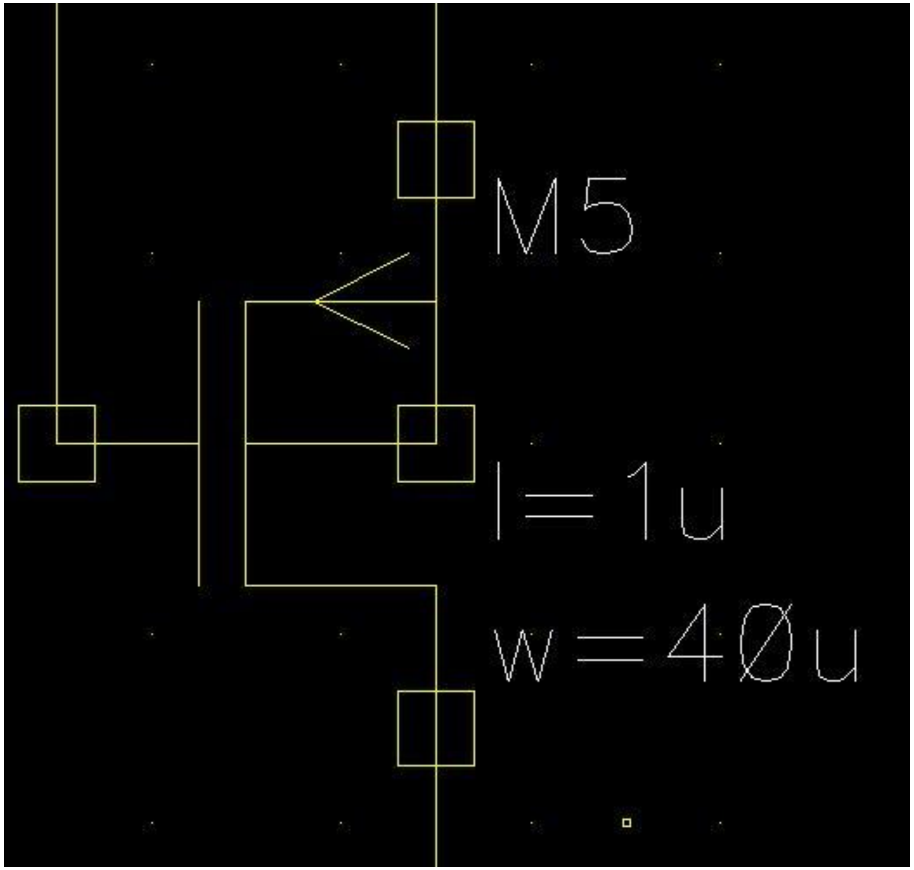
OP-M3,4,7-Schematic



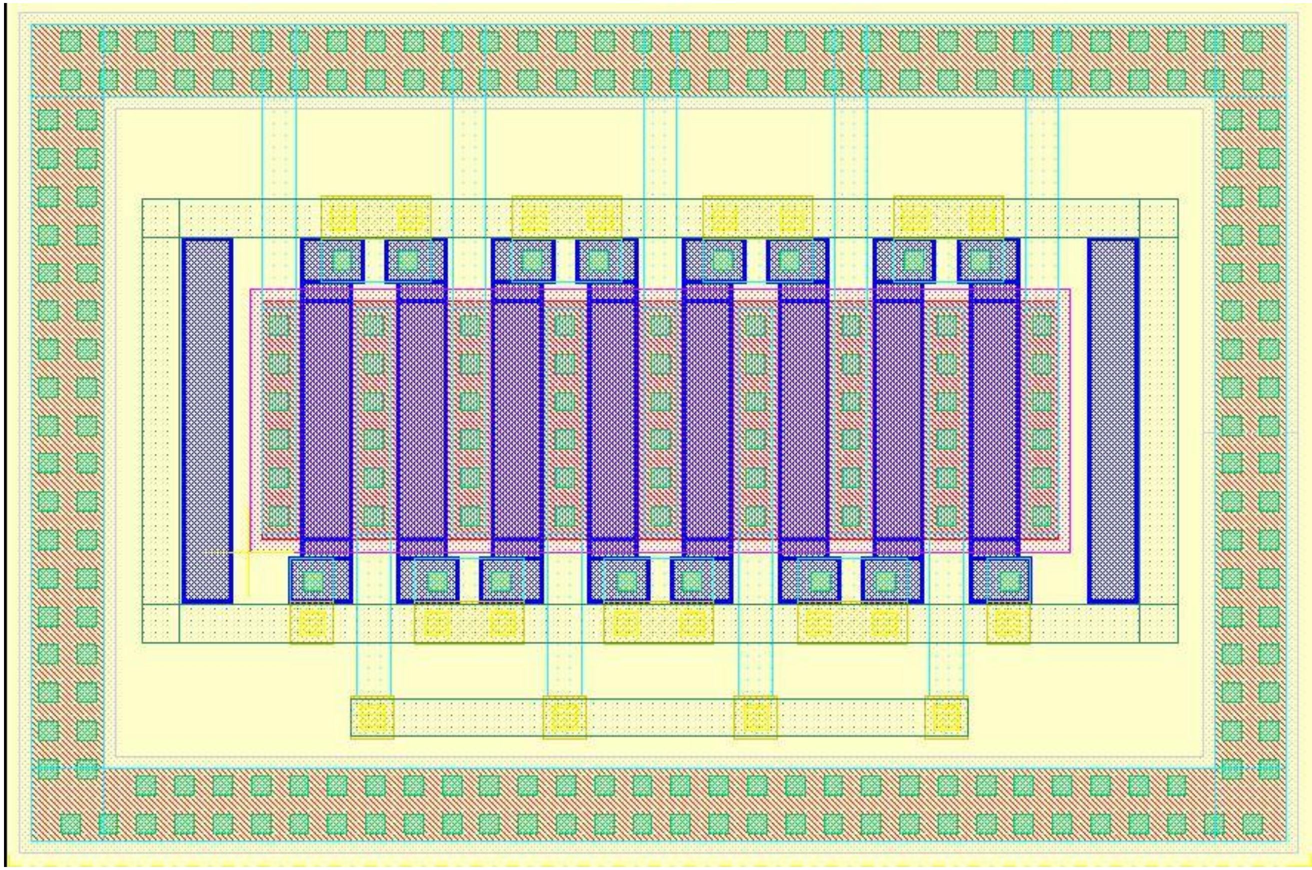
OP-M3,4,7-Layout



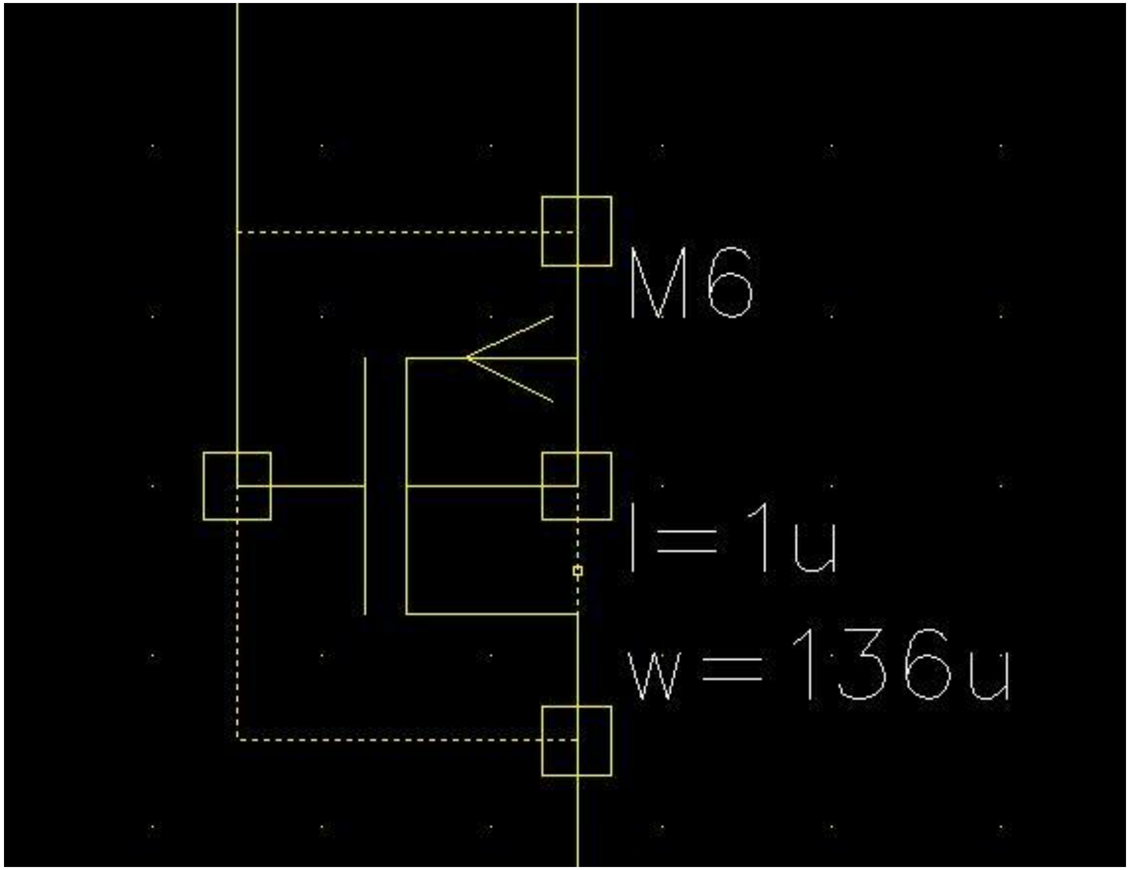
OP-M5-Schematic



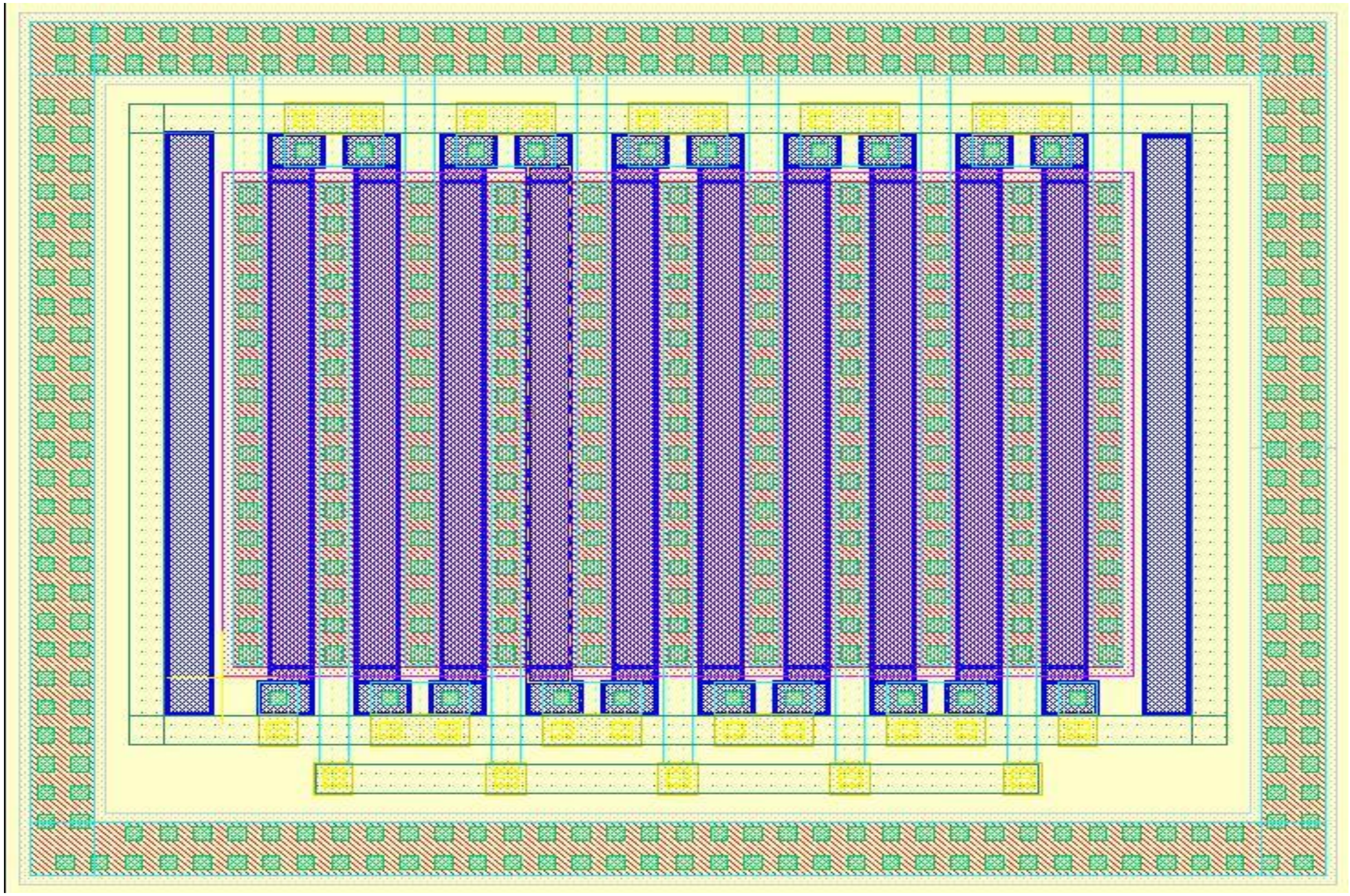
OP-M5-Layout



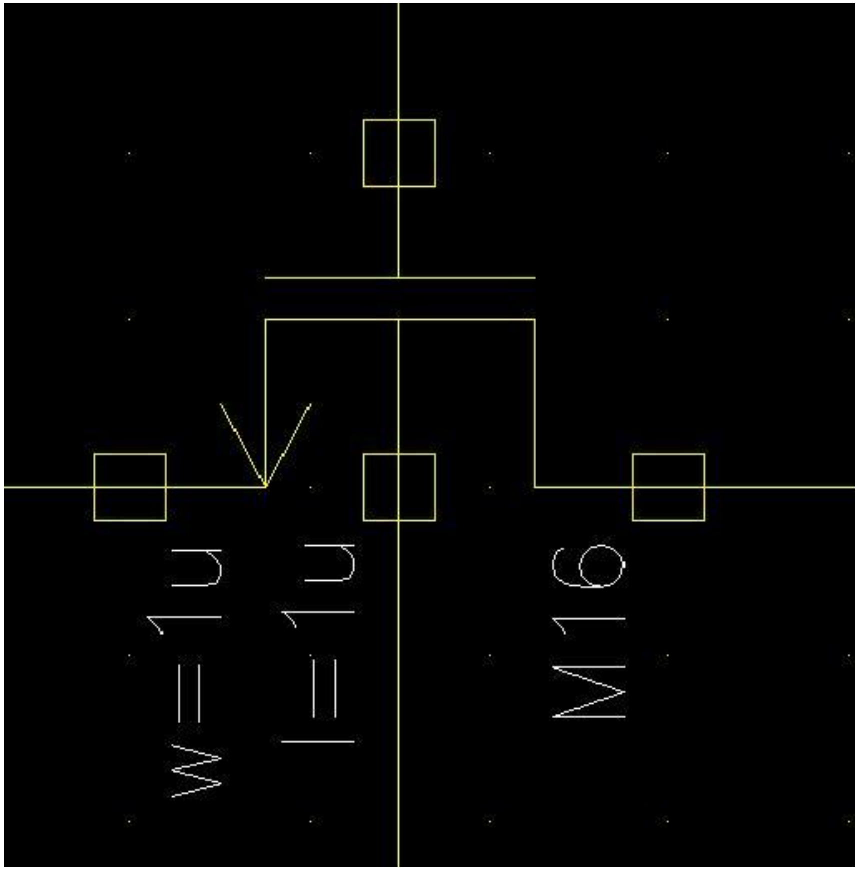
OP-M6-Schematic



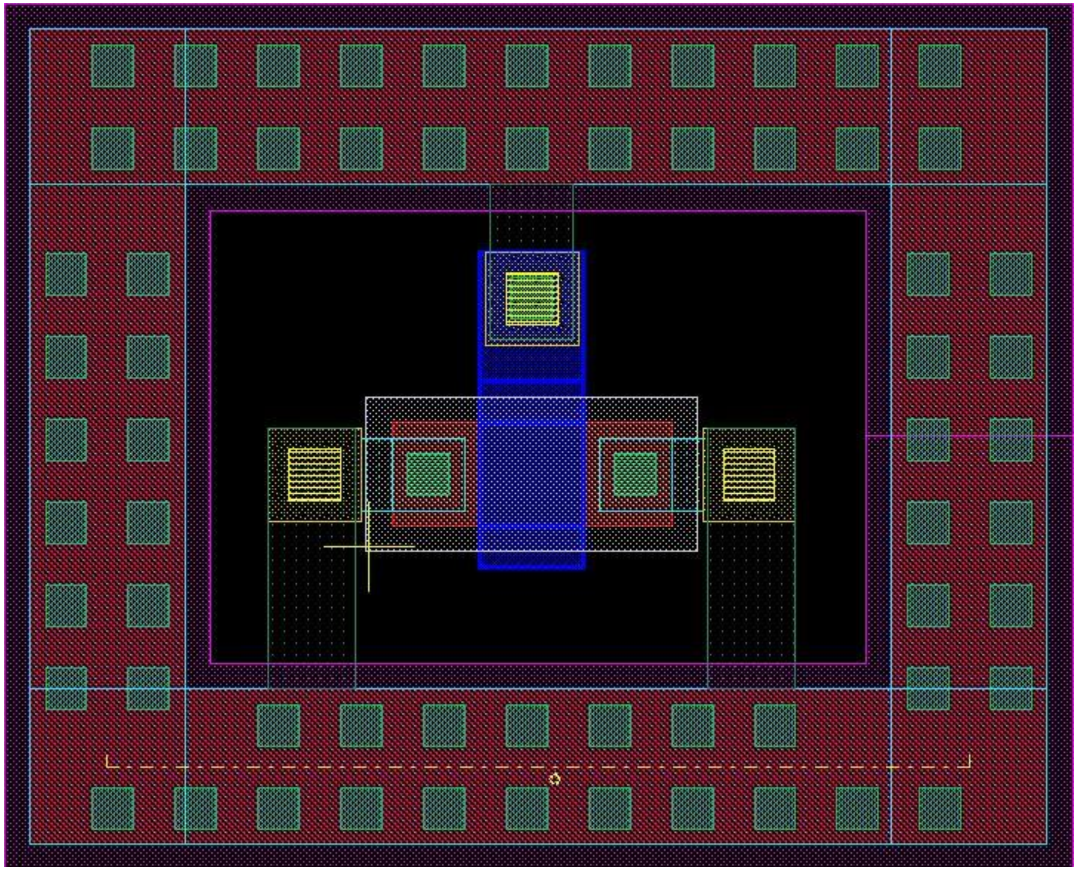
OP-M6-Layout



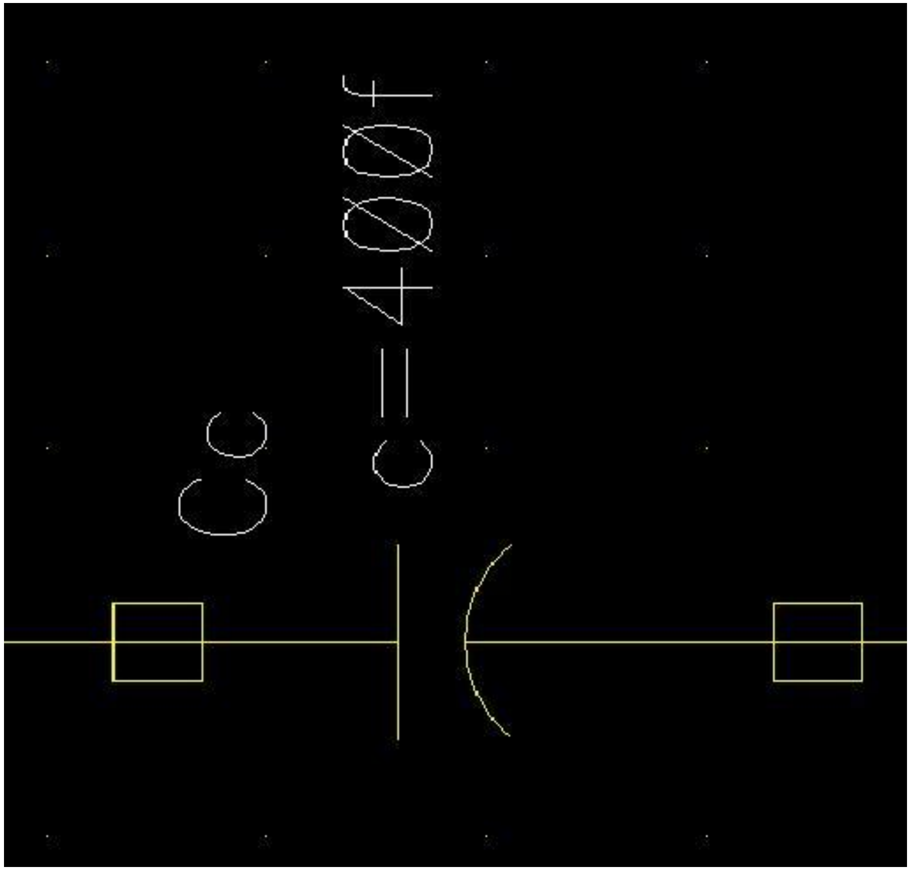
OP-M16-Schematic



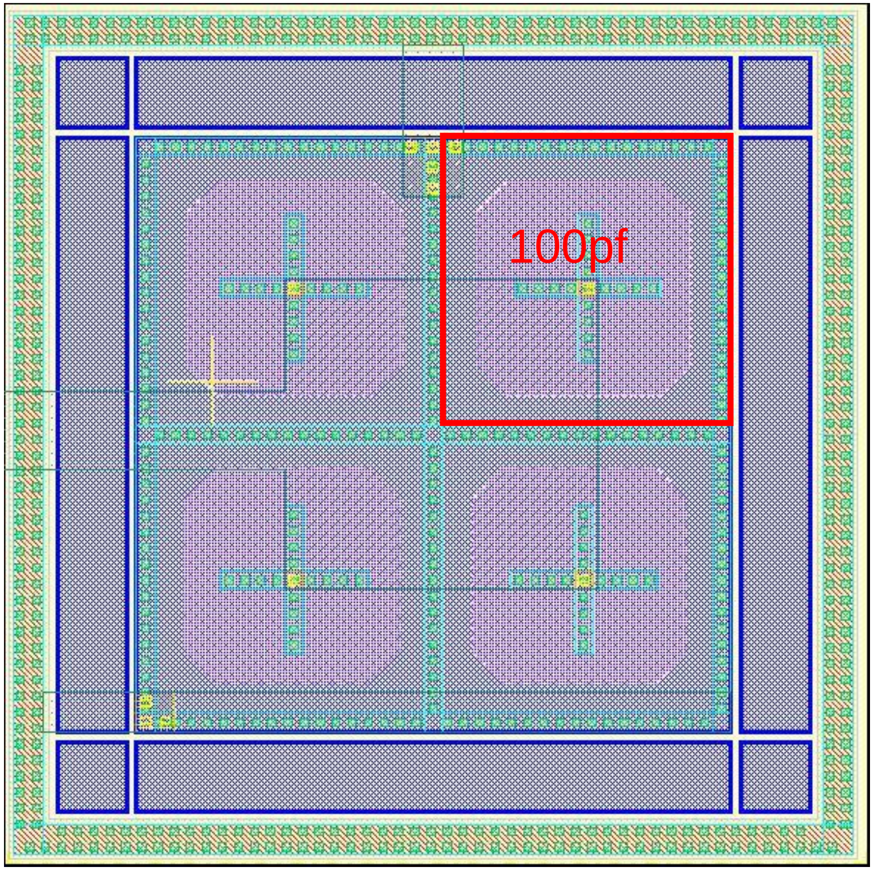
OP-M16-Layout



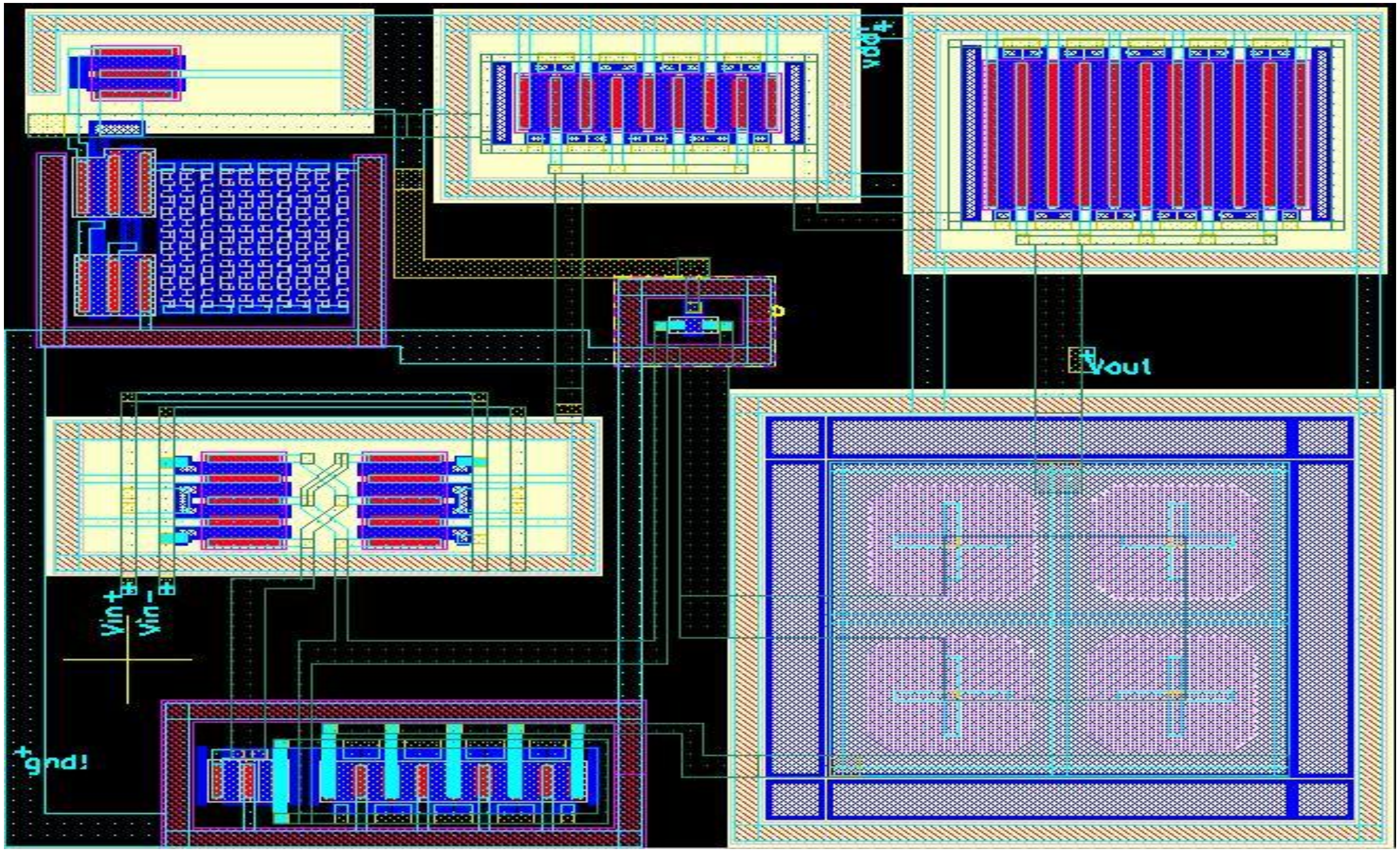
OP-Cc-Schematic



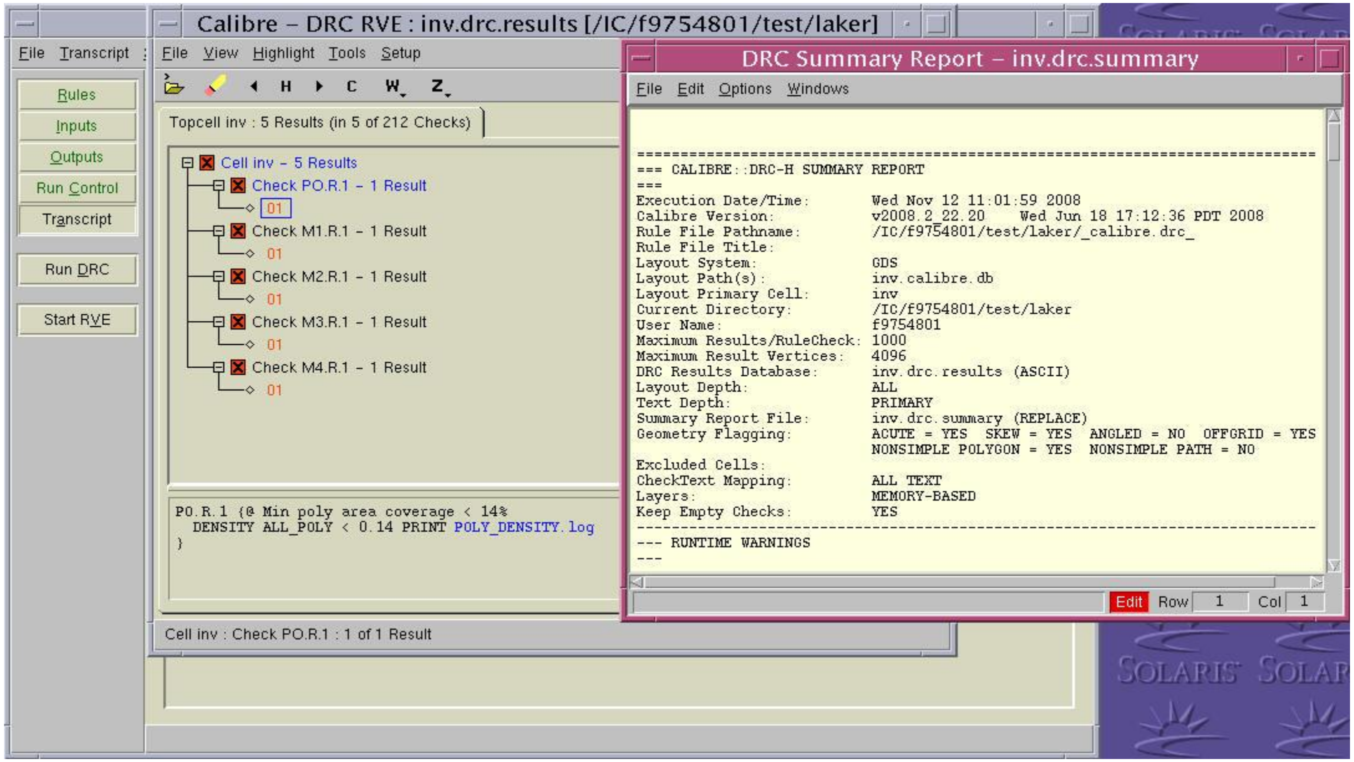
OP-Cc-Layout



OP-Layout



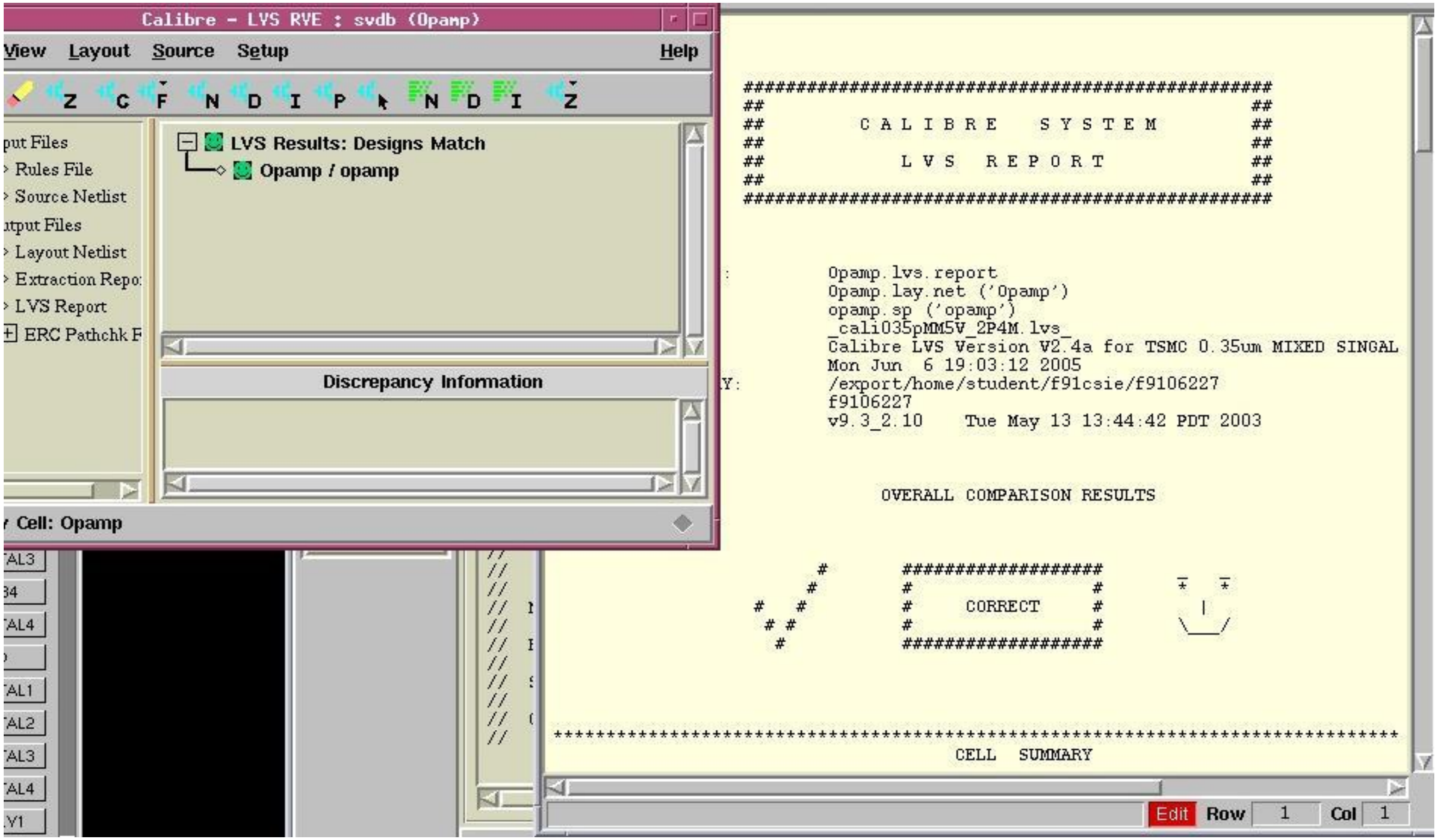
OP-Drc



File Transcript
Rules
Inputs
Outputs
Run Control
Transcript
Run DRC
Start RVE
Calibre – DRC RVE : inv.drc.results [/IC/f9754801/test/laker]
File View Highlight Tools Setup
Topcell inv : 5 Results (in 5 of 212 Checks)
Cell inv – 5 Results
Check PO.R.1 – 1 Result
01
Check M1.R.1 – 1 Result
01
Check M2.R.1 – 1 Result
01
Check M3.R.1 – 1 Result
01
Check M4.R.1 – 1 Result
01
PO.R.1 {@ Min poly area coverage < 14%
DENSITY ALL_POLY < 0.14 PRINT POLY_DENSITY.log
}
Cell inv : Check PO.R.1 : 1 of 1 Result
DRC Summary Report – inv.drc.summary
File Edit Options Windows
=== CALIBRE::DRC-H SUMMARY REPORT
===
Execution Date/Time: Wed Nov 12 11:01:59 2008
Calibre Version: v2008.2_22.20 Wed Jun 18 17:12:36 PDT 2008
Rule File Pathname: /IC/f9754801/test/laker/_calibre.drc_
Rule File Title:
Layout System: GDS
Layout Path(s): inv.calibre.db
Layout Primary Cell: inv
Current Directory: /IC/f9754801/test/laker
User Name: f9754801
Maximum Results/RuleCheck: 1000
Maximum Result Vertices: 4096
DRC Results Database: inv.drc.results (ASCII)
Layout Depth: ALL
Text Depth: PRIMARY
Summary Report File: inv.drc.summary (REPLACE)
Geometry Flagging: ACUTE = YES SKEW = YES ANGLED = NO OFFGRID = YES
NONSIMPLE POLYGON = YES NONSIMPLE PATH = NO
Excluded Cells:
CheckText Mapping: ALL TEXT
Layers: MEMORY-BASED
Keep Empty Checks: YES
--- RUNTIME WARNINGS

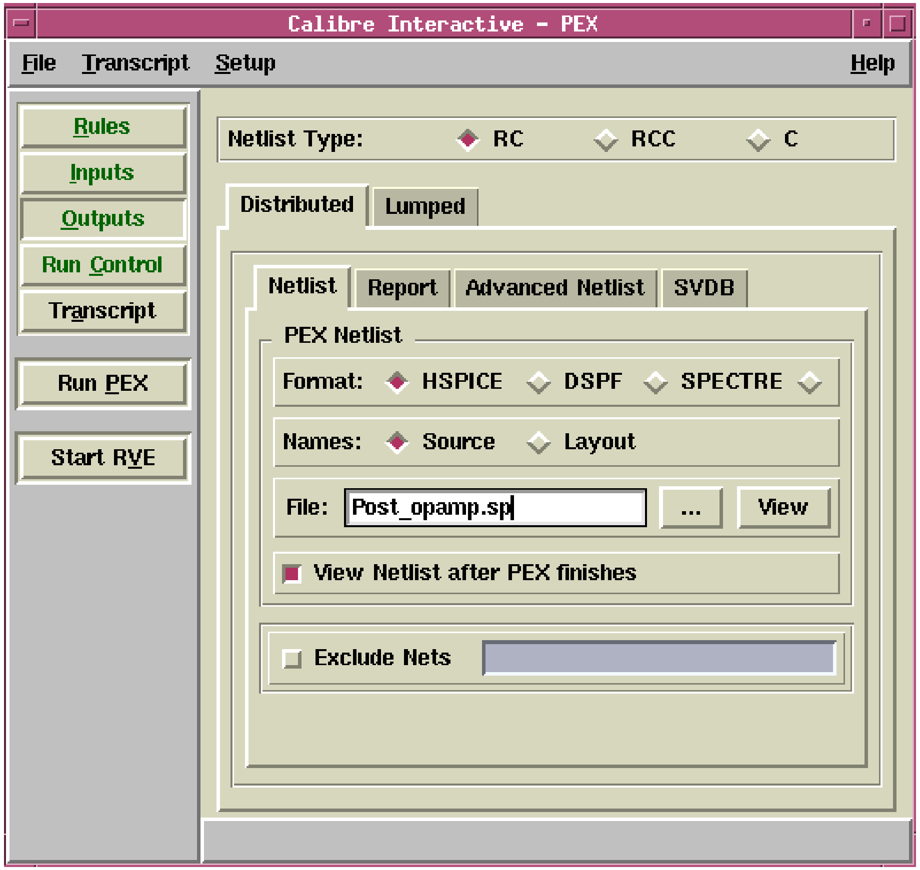
Edit Row 1 Col 1

OP-Lvs



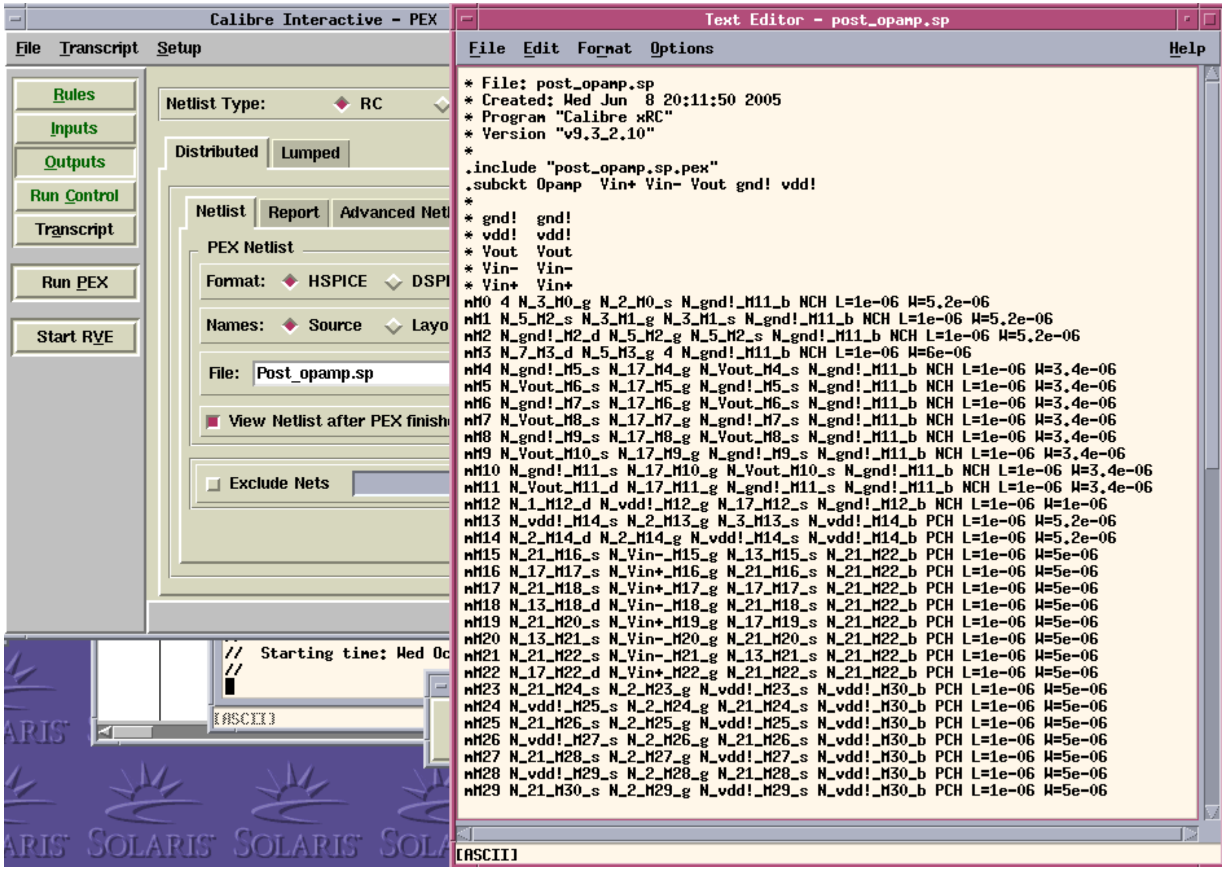
Calibre - LVS RVE : svdb (Opamp)
View Layout Source Setup Help
LVS Results: Designs Match
Opamp / opamp
Rules File
Source Netlist
Layout Netlist
LVS Report
Discrepancy Information
Cell: Opamp
C A L I B R E S Y S T E M
L V S R E P O R T
Opamp.lvs.report
Opamp.lay.net ('Opamp')
opamp.sp ('opamp')
_cali035pMM5V_2P4M.lvs_
Calibre LVS Version V2.4a for TSMC 0.35um MIXED SINGAL
Mon Jun 6 19:03:12 2005
/export/home/student/f91csie/f9106227
f9106227
v9.3_2.10 Tue May 13 13:44:42 PDT 2003
OVERALL COMPARISON RESULTS
CORRECT
CELL SUMMARY
Edit Row 1 Col 1

OP-R/C Run PEX

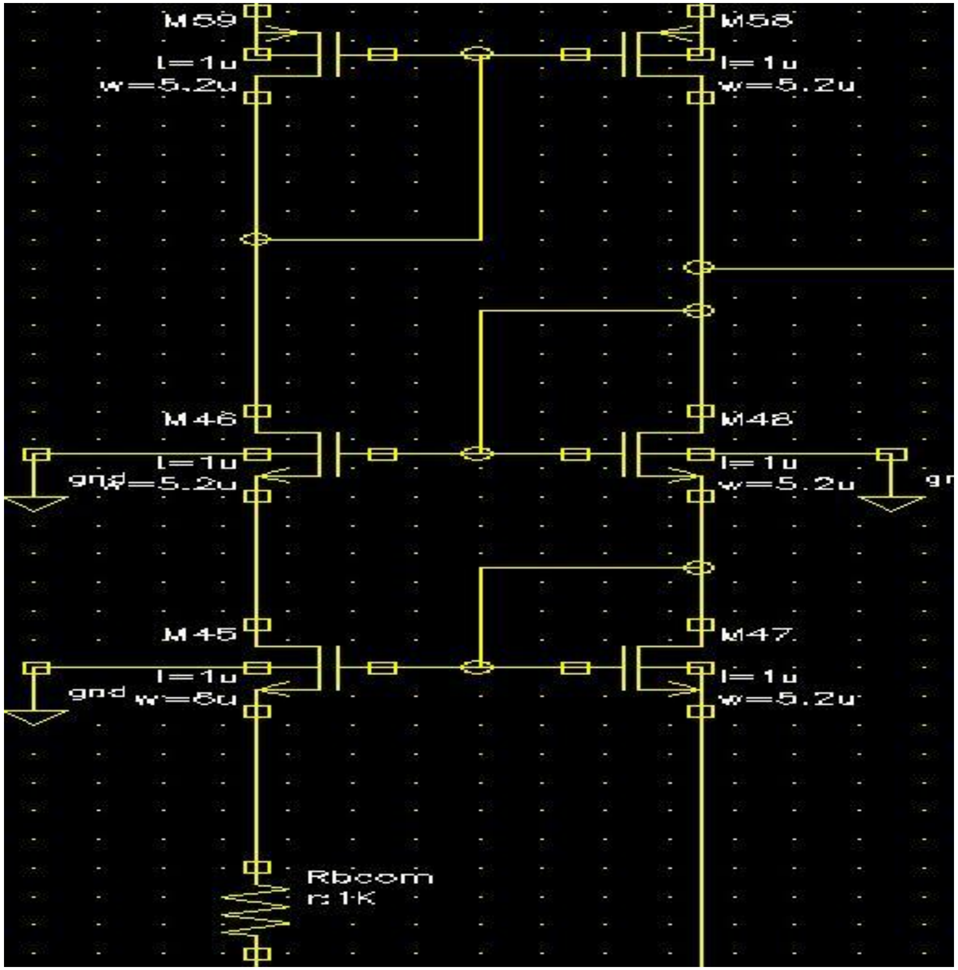


Calibre Interactive - PEX
File
Transcript
Setup
Help
Rules
Inputs
Outputs
Run Control
Transcript
Run PEX
Start RVE
Netlist Type:
RC
RCC
C
Distributed
Lumped
Netlist
Report
Advanced Netlist
SVDB
PEX Netlist
Format:
HSPICE
DSPF
SPECTRE
Names:
Source
Layout
File:
Post_opamp.sp
...
View
View Netlist after PEX finishes
Exclude Nets

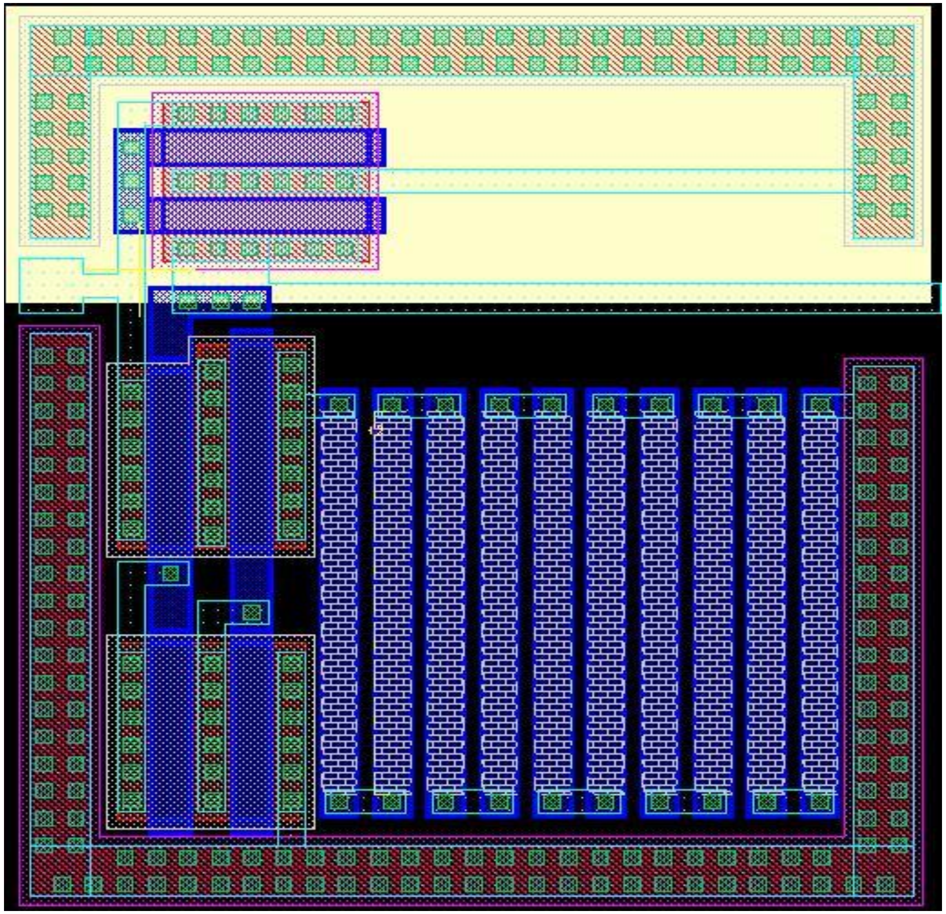
OP-R/C Run PEX Success



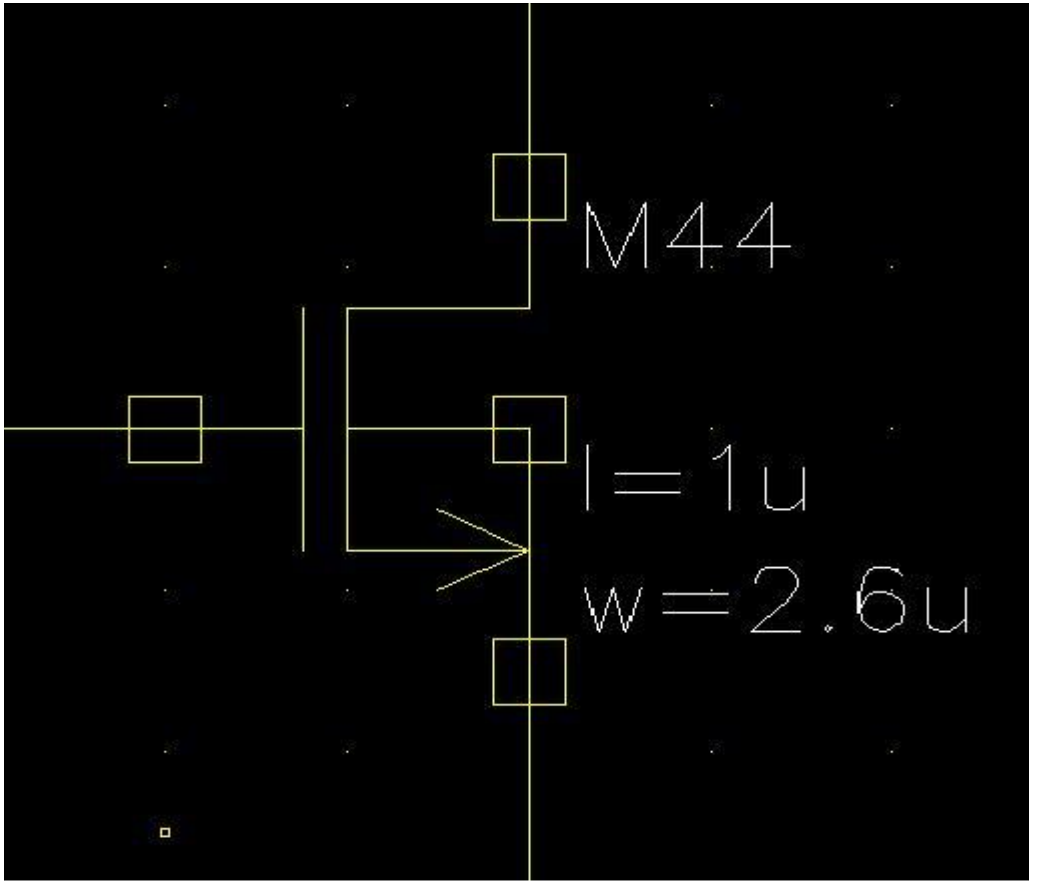
Comparator-Bias-Schematic



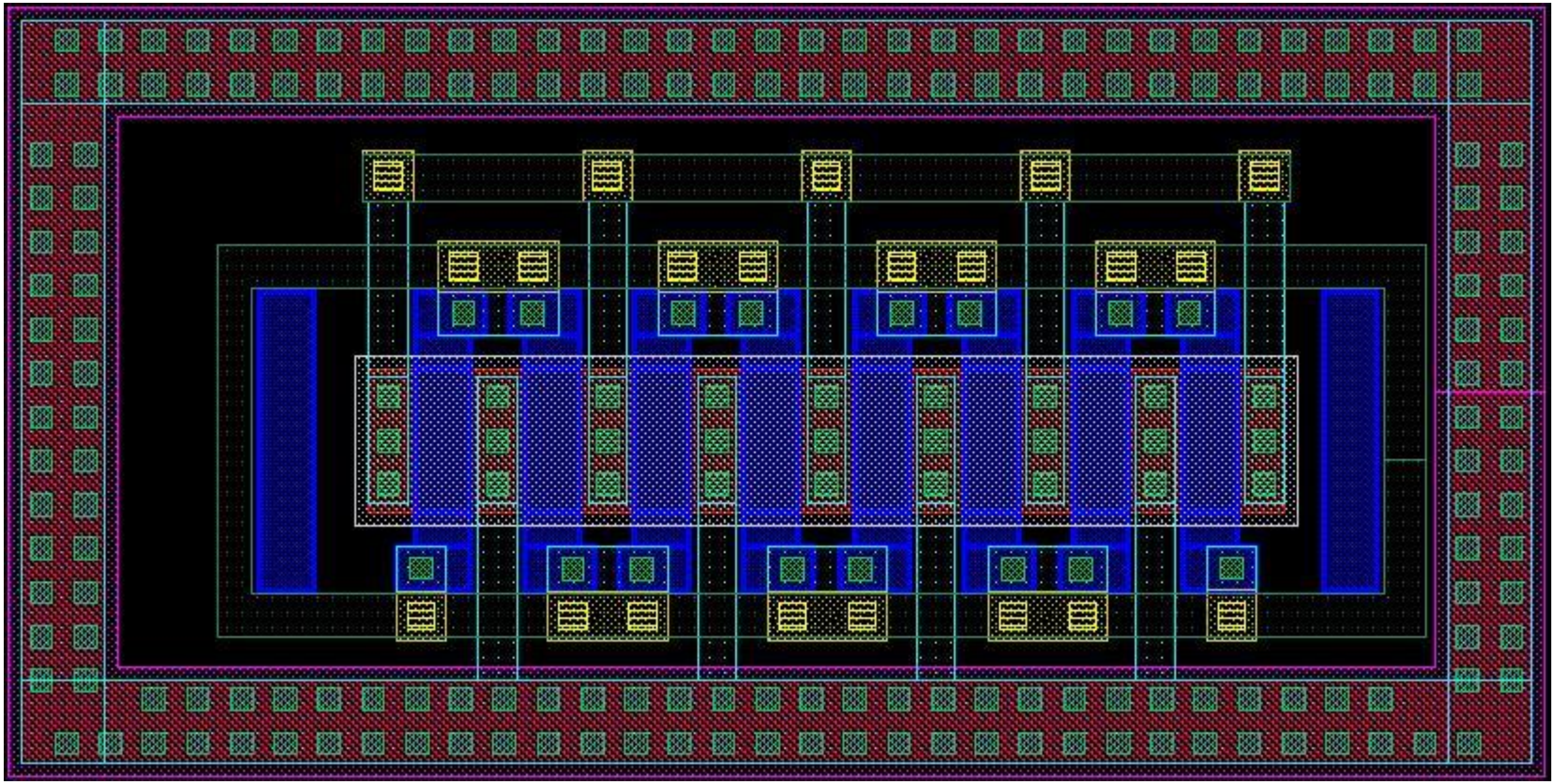
Comparator-Bias-Layout



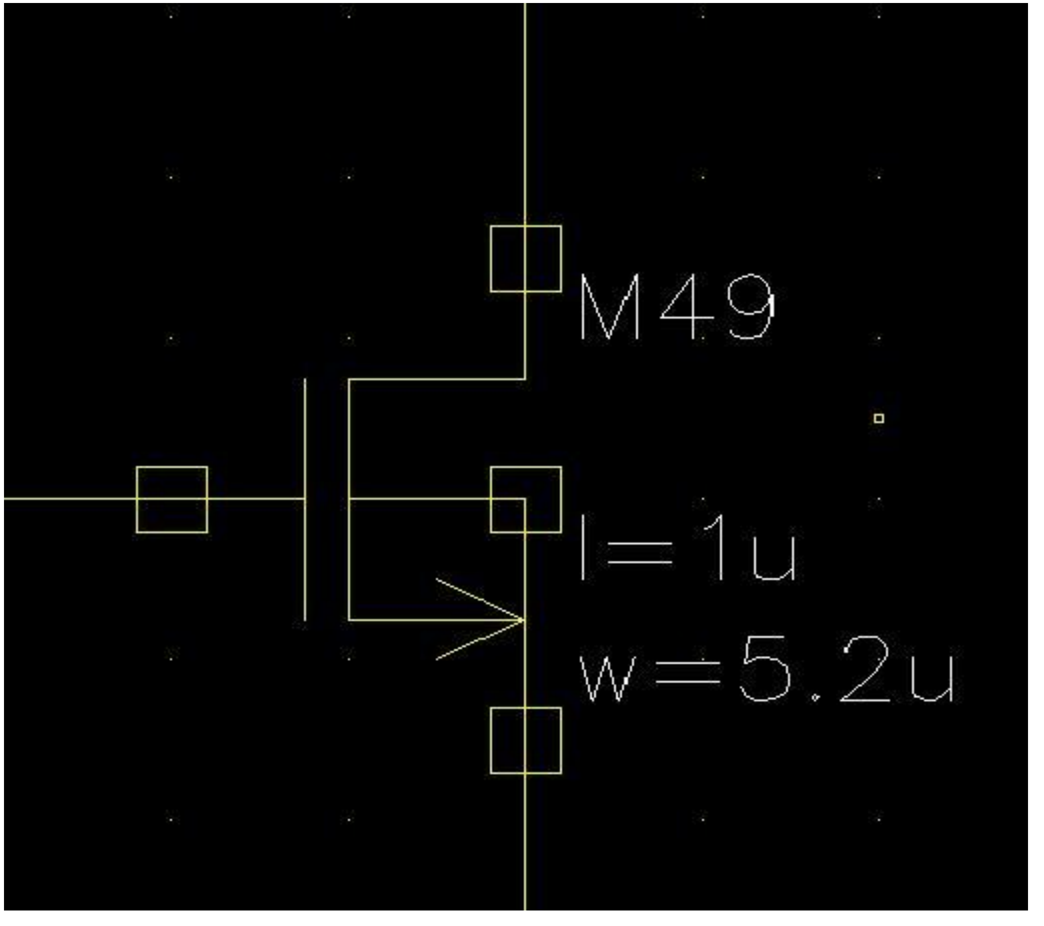
Comparator-M44-Schematic



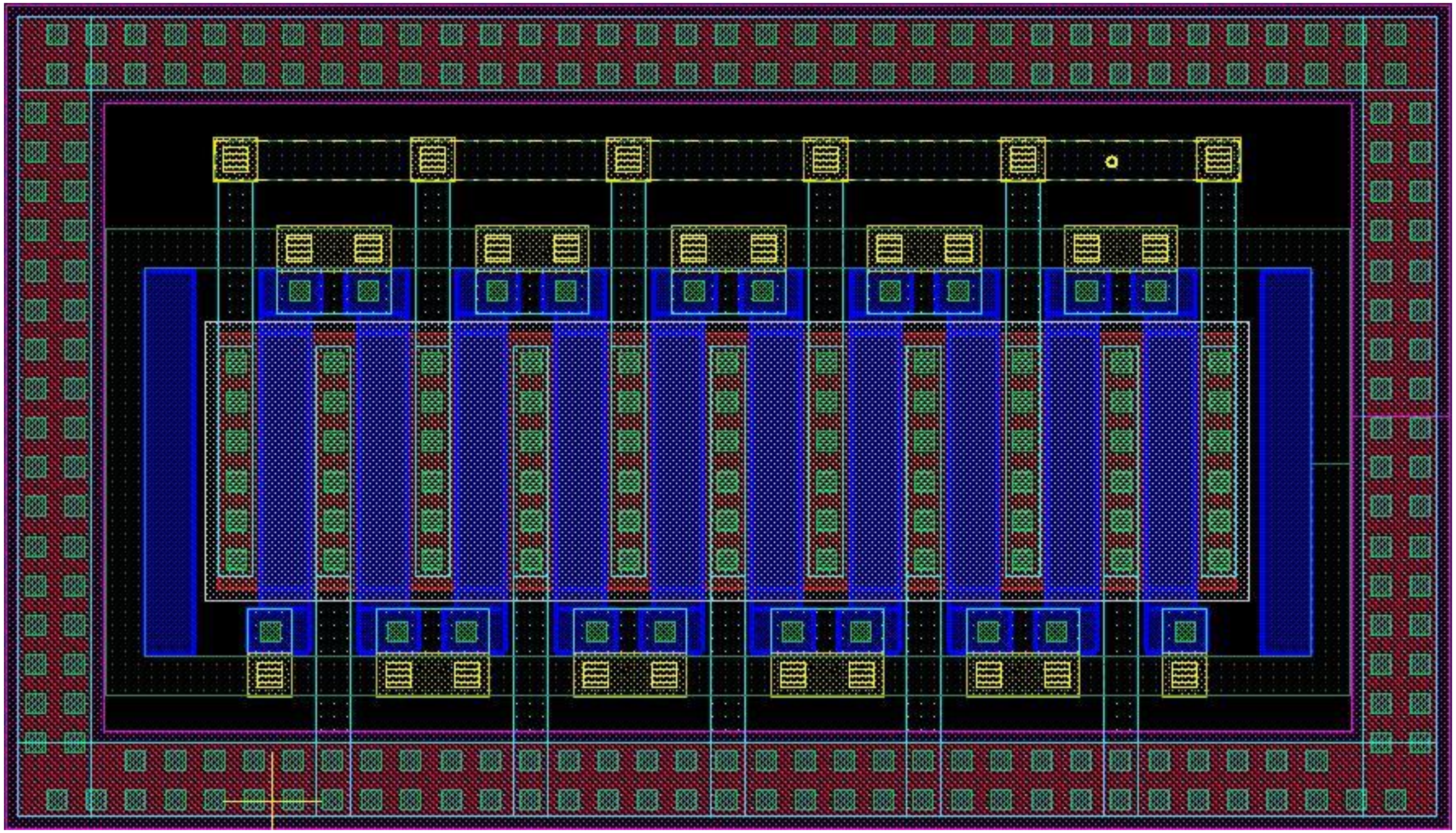
Comparator-M44-Layout



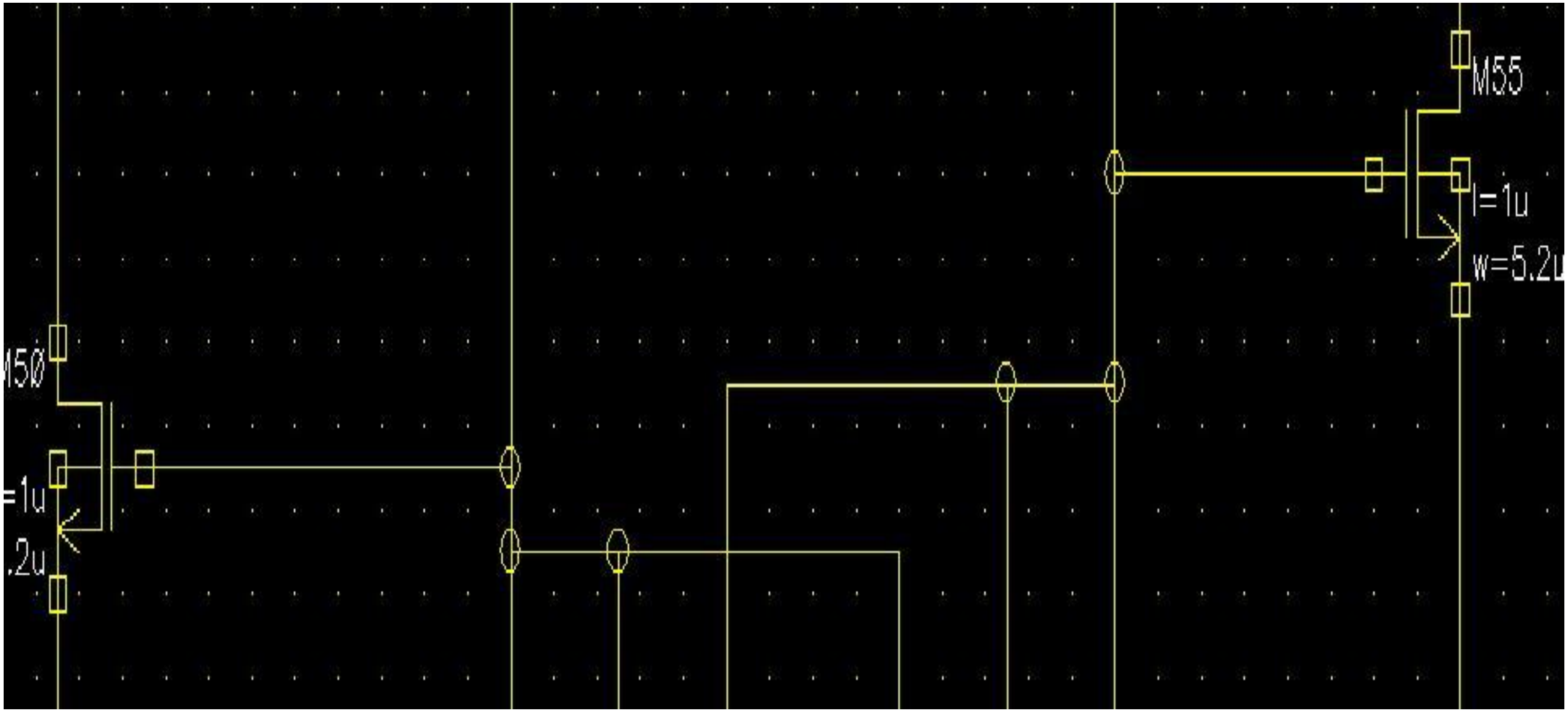
Comparator-M49-Schematic



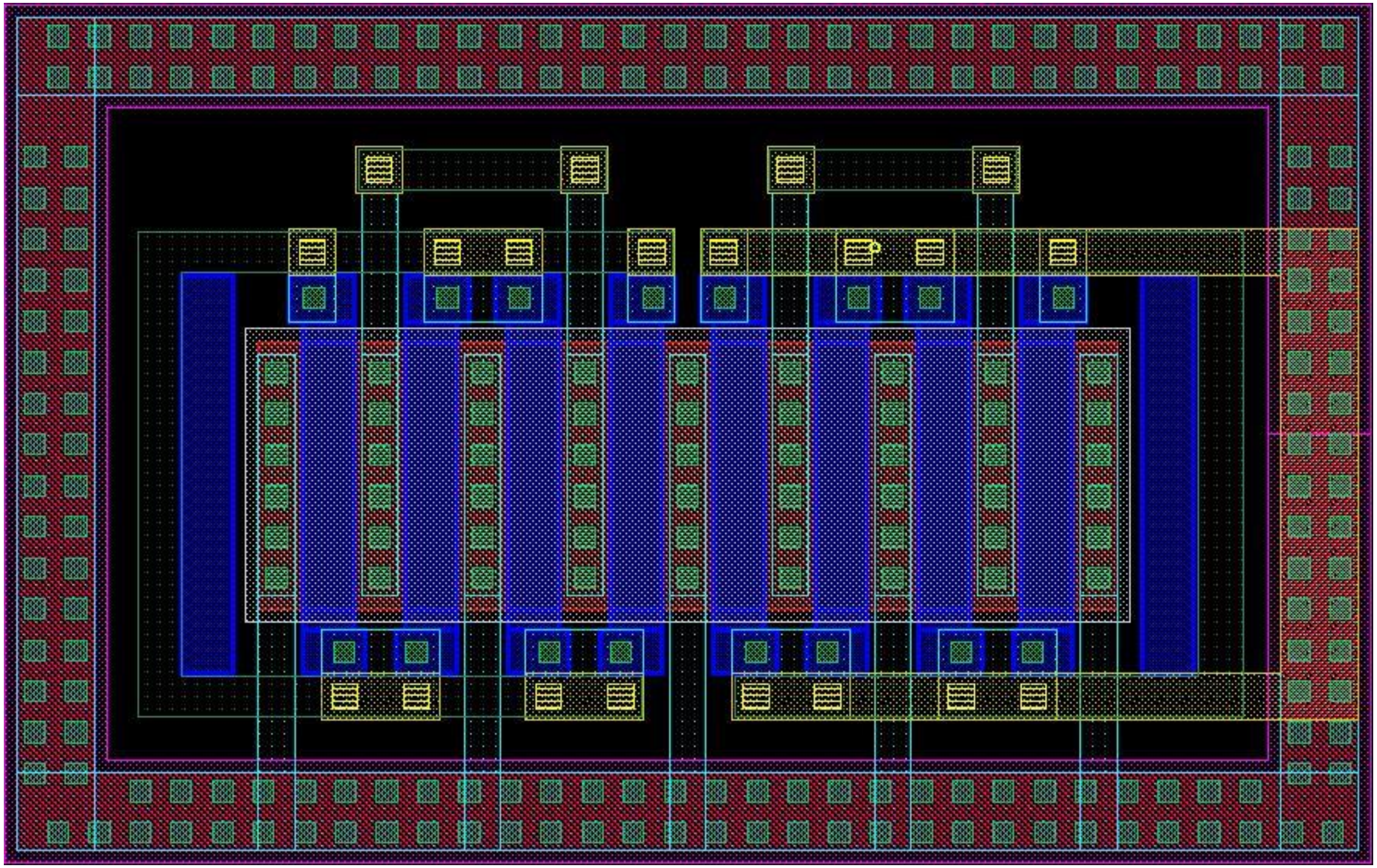
Comparator-M49-Layout



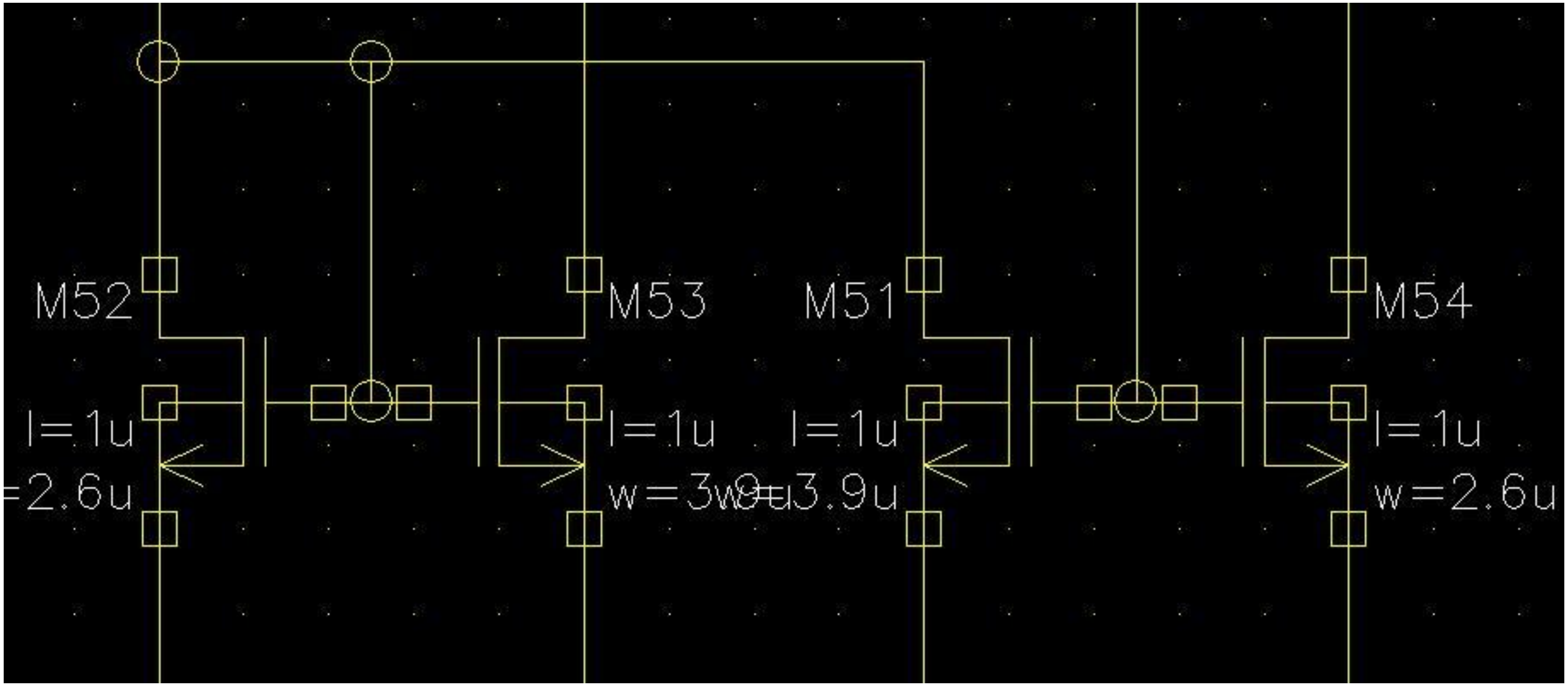
Comparator-M50,55-Schematic



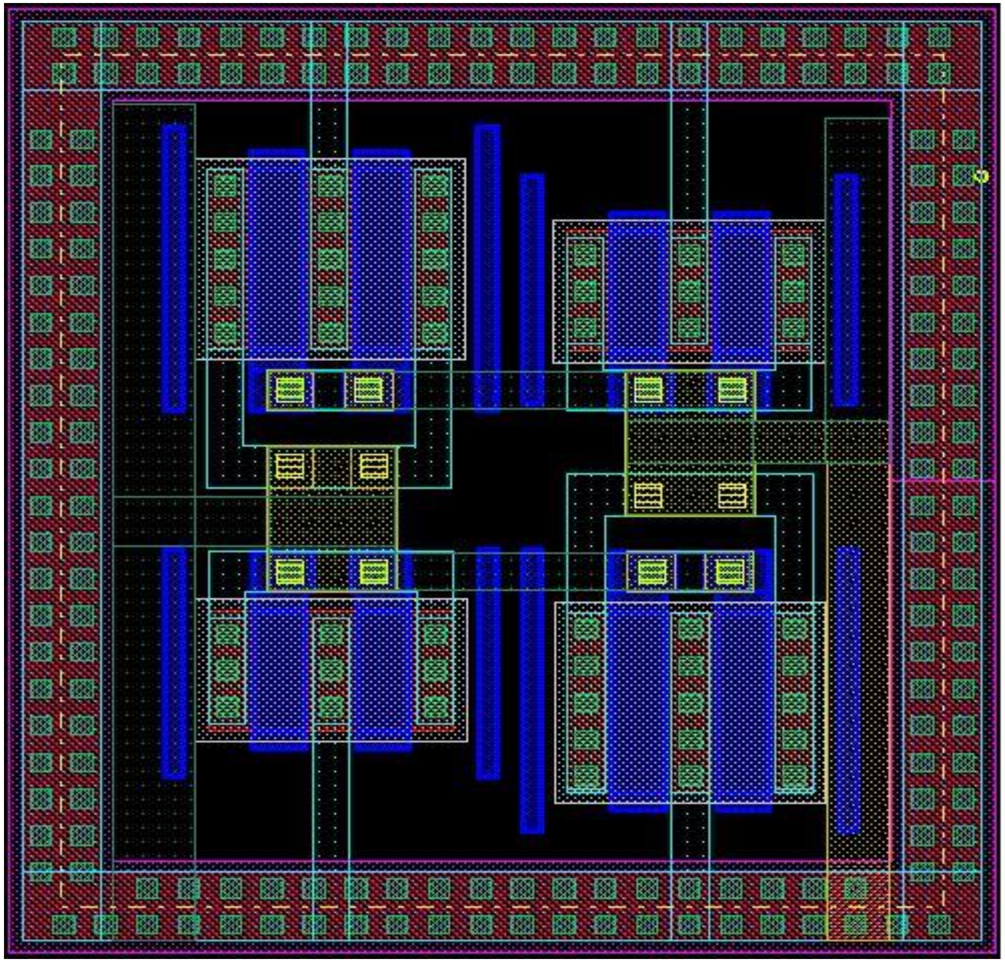
Comparator-M50,55-Layout



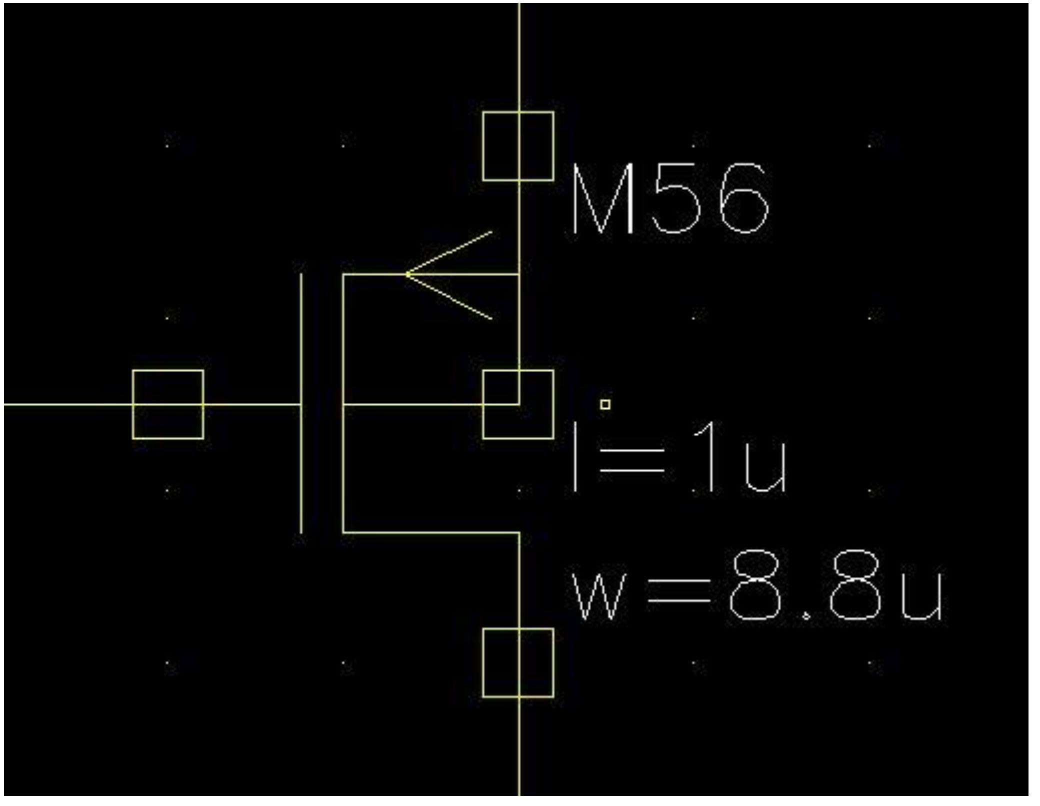
Comparator-M51~54-Schematic



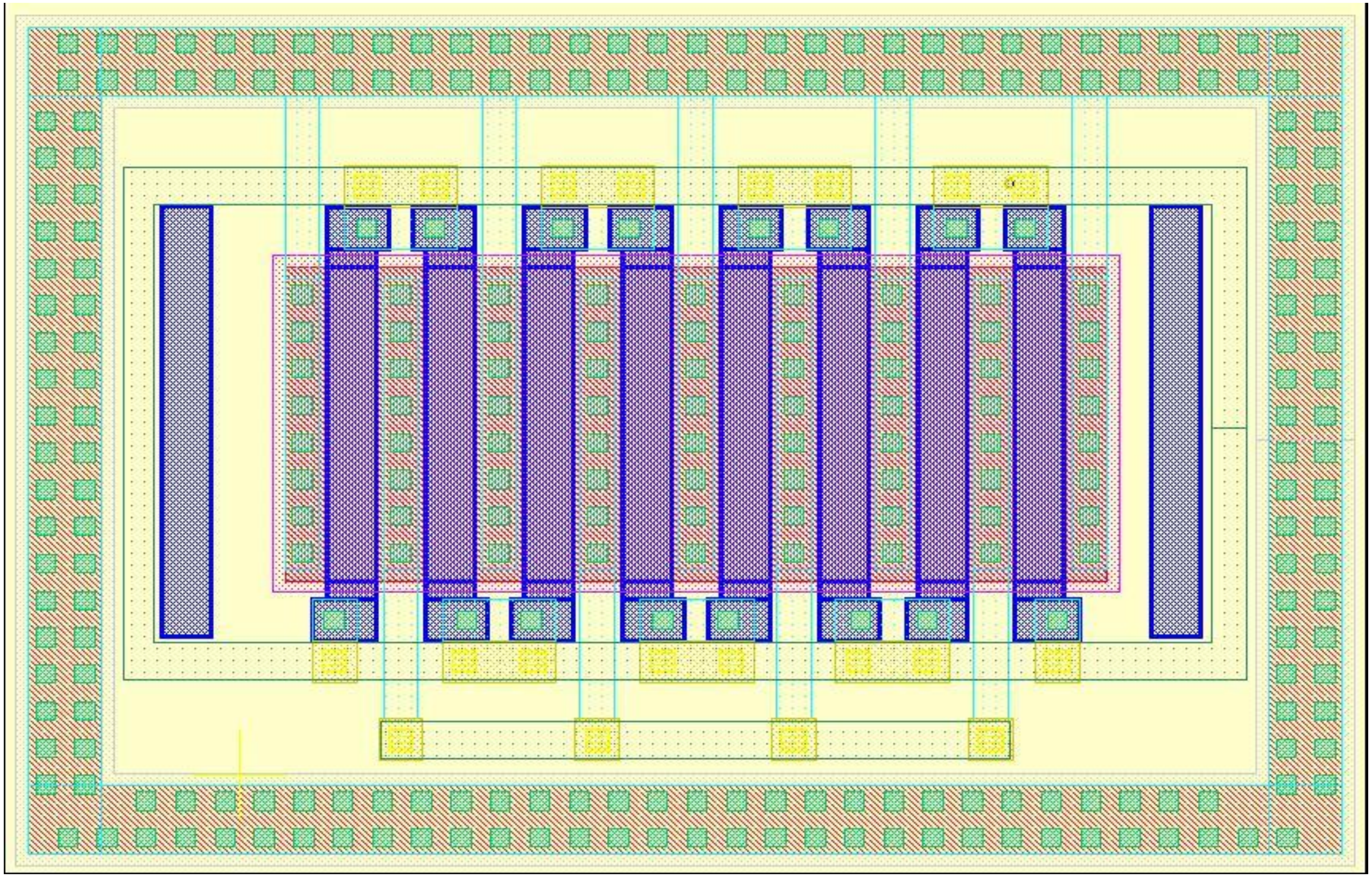
Comparator-M51~54-Layout



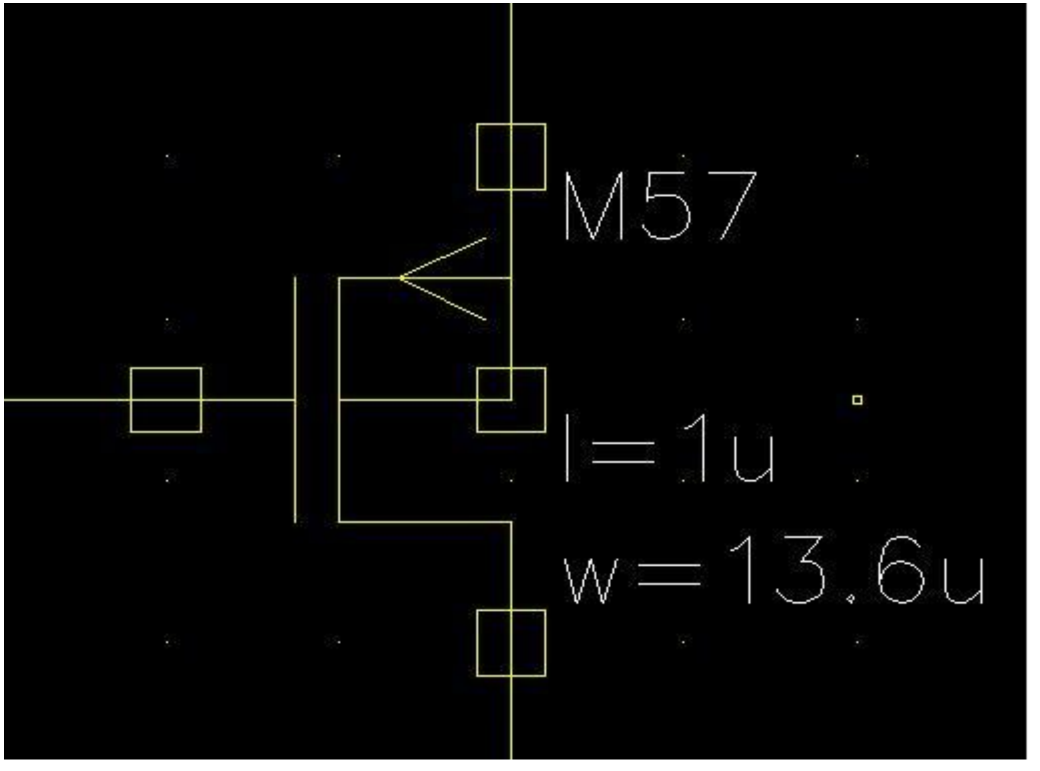
Comparator-M56-Schematic



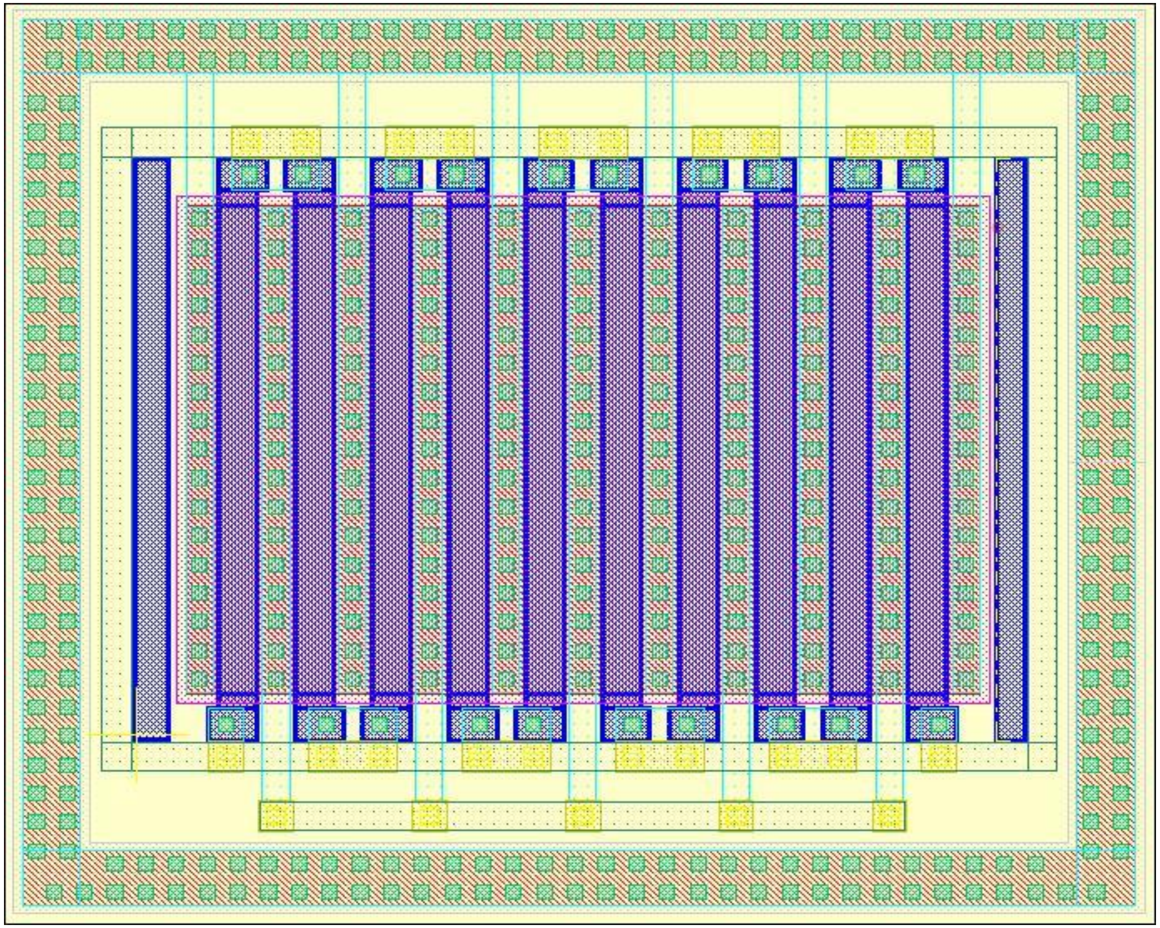
Comparator-M56-Layout



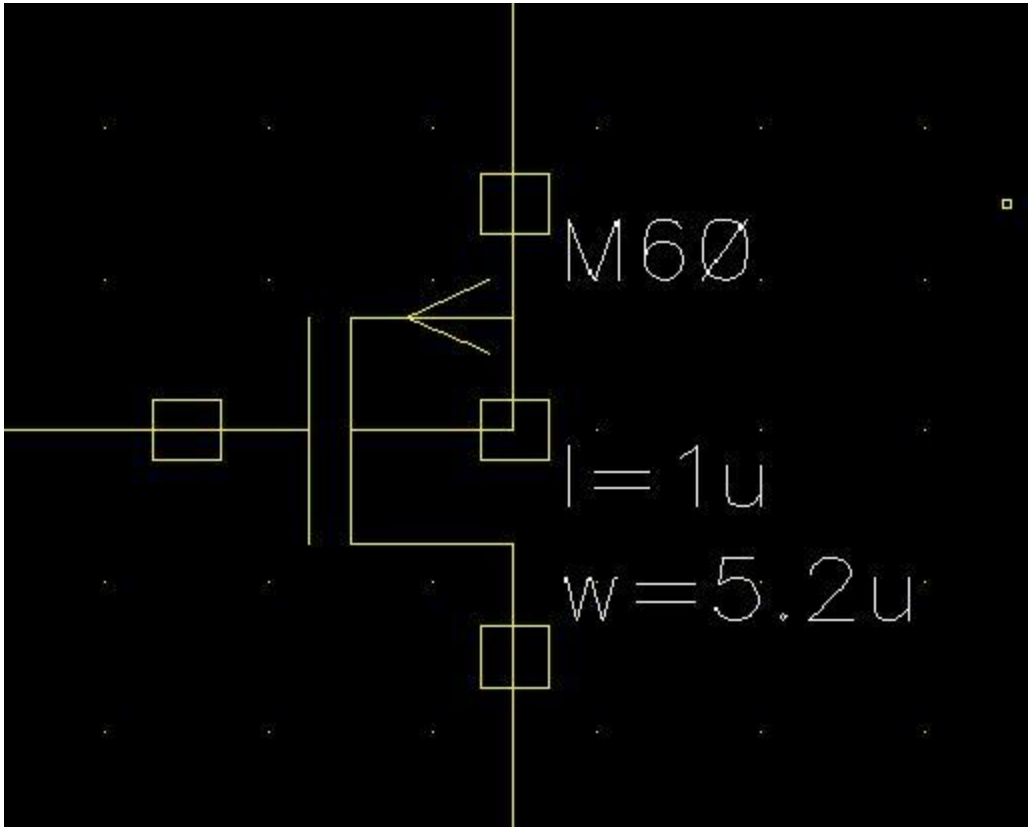
Compatator-M57-Schematic



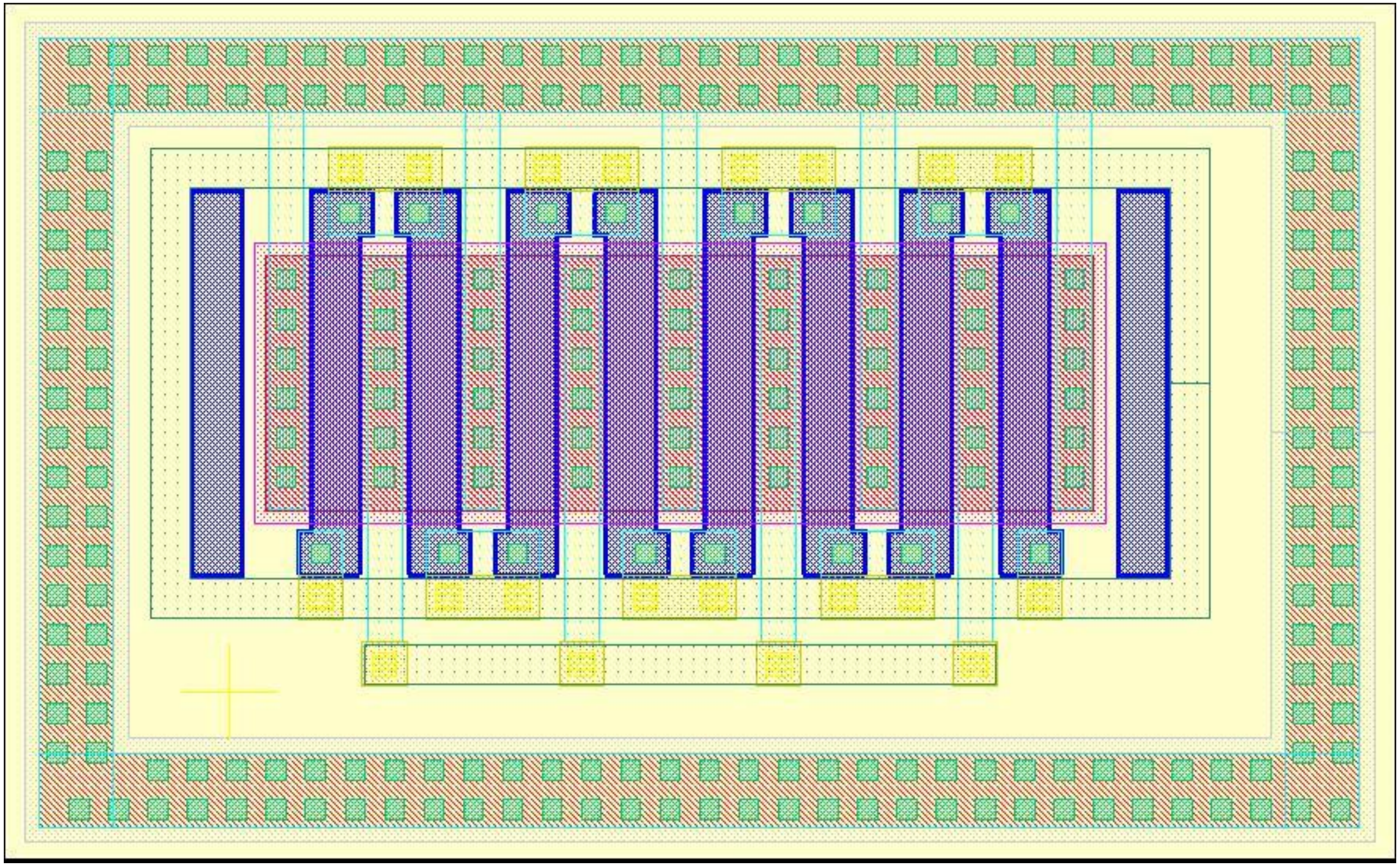
Compatator-M57-Layout



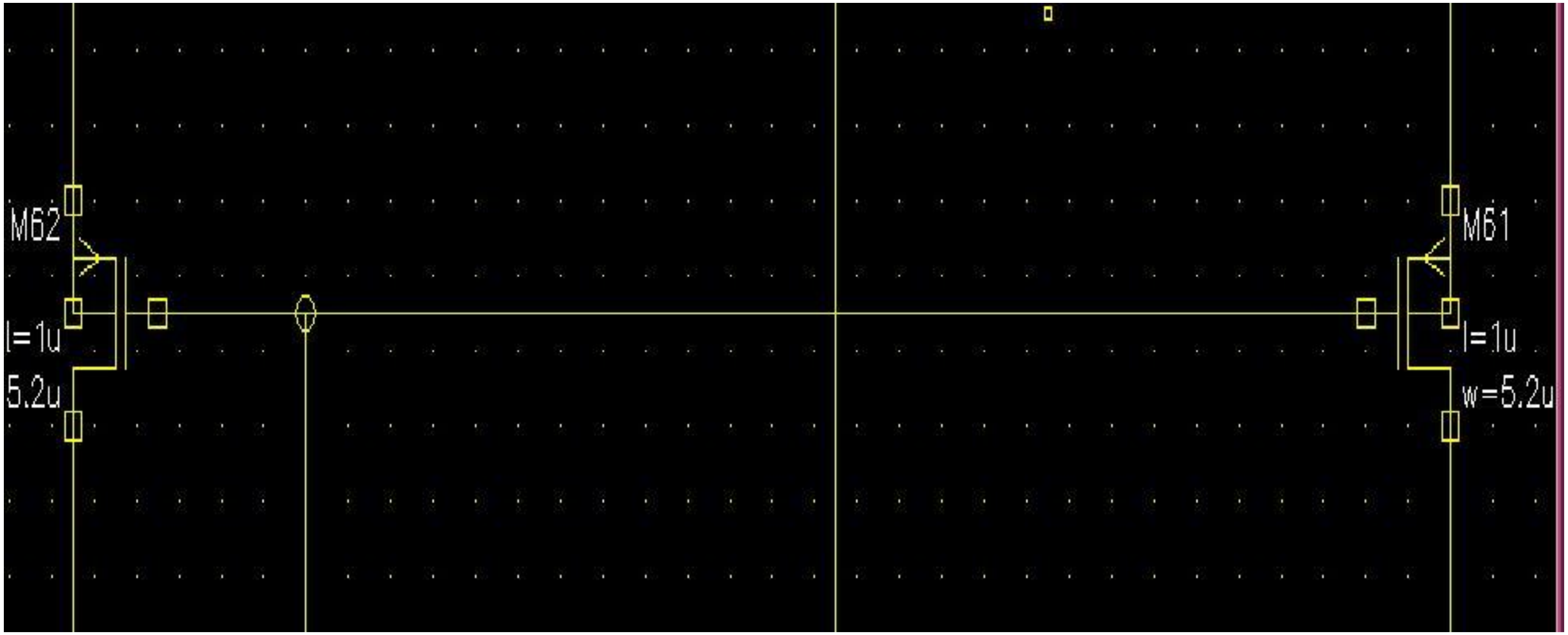
Comparator-M60-Schematic



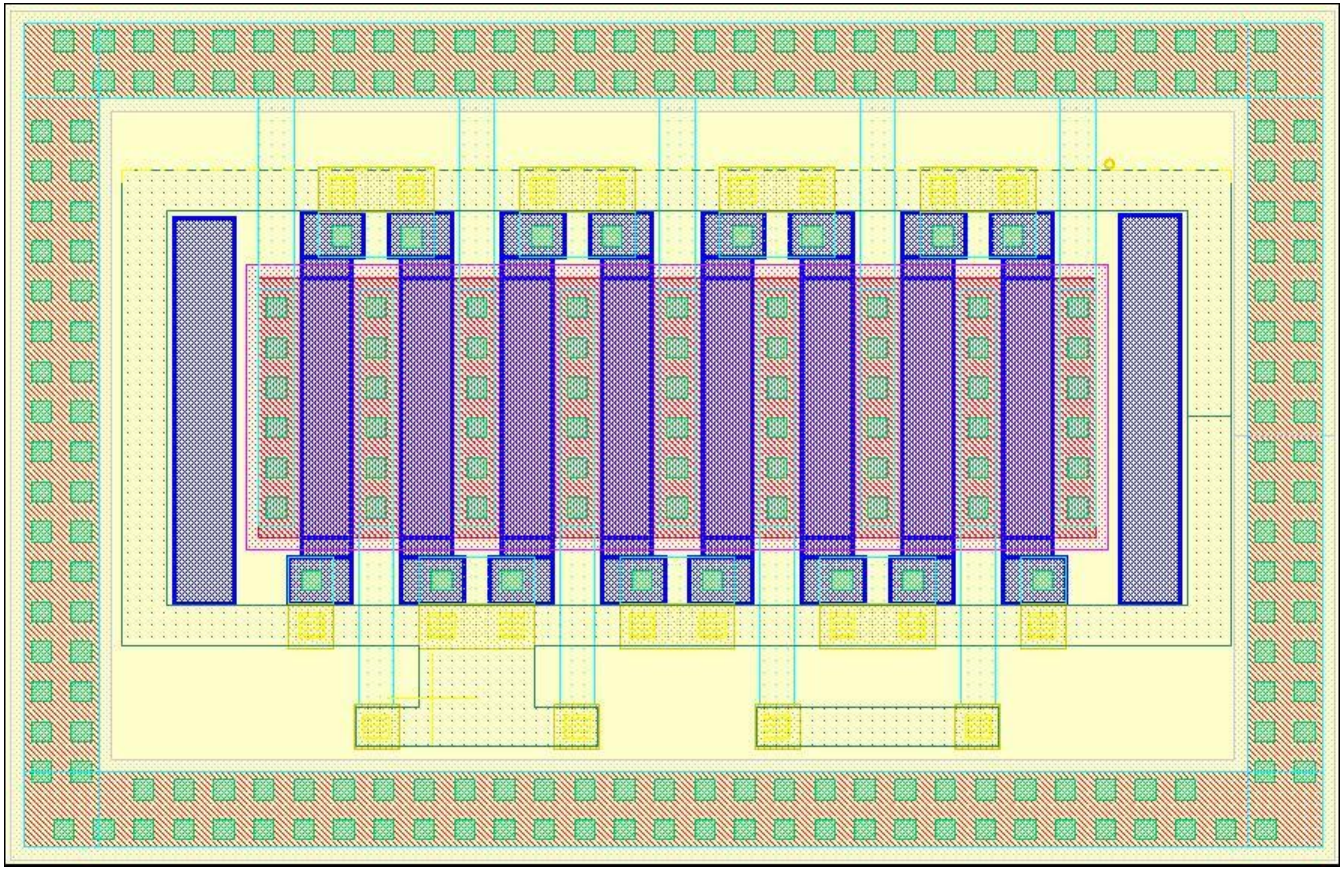
Comparator-M60-Layout



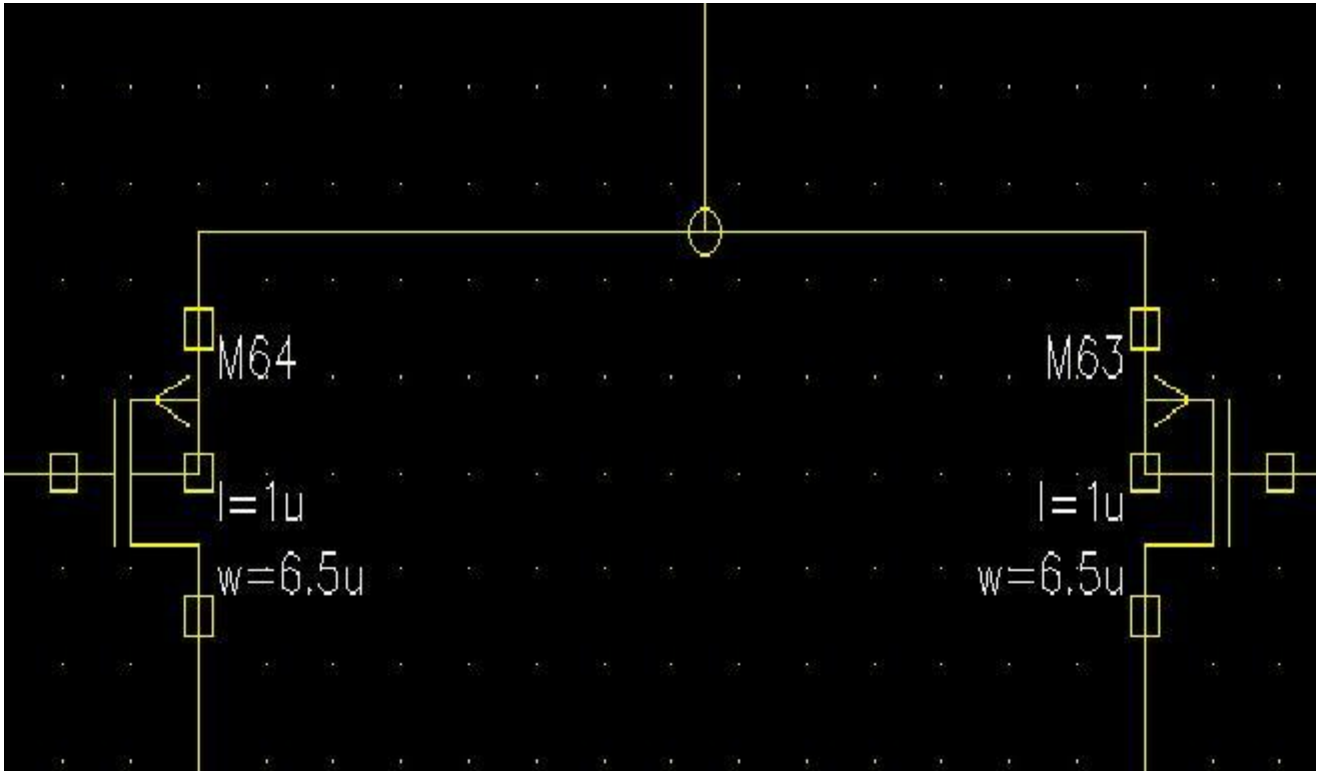
Comparator-M61~62-Schematic



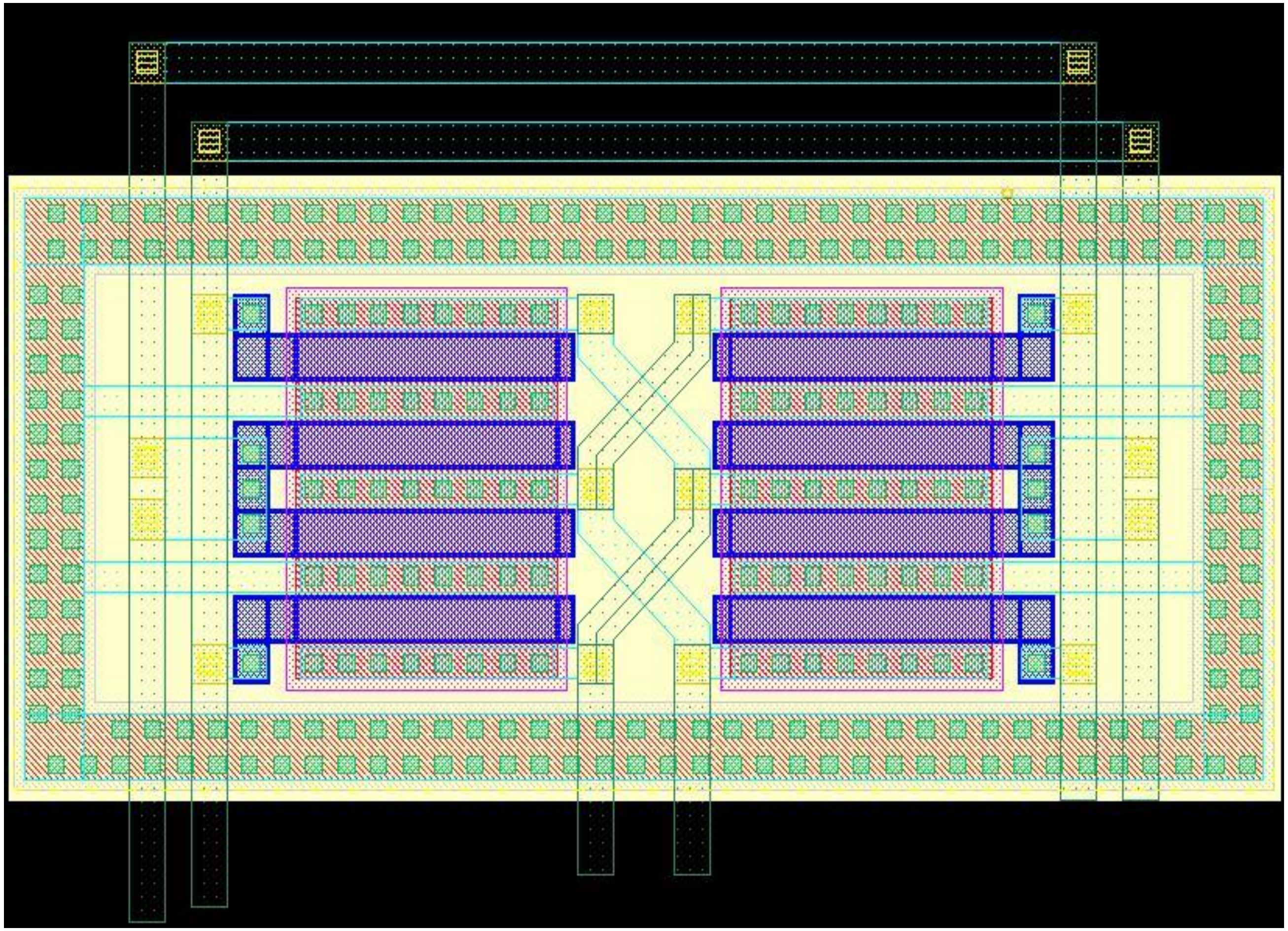
Comparator-M61~62-Layout



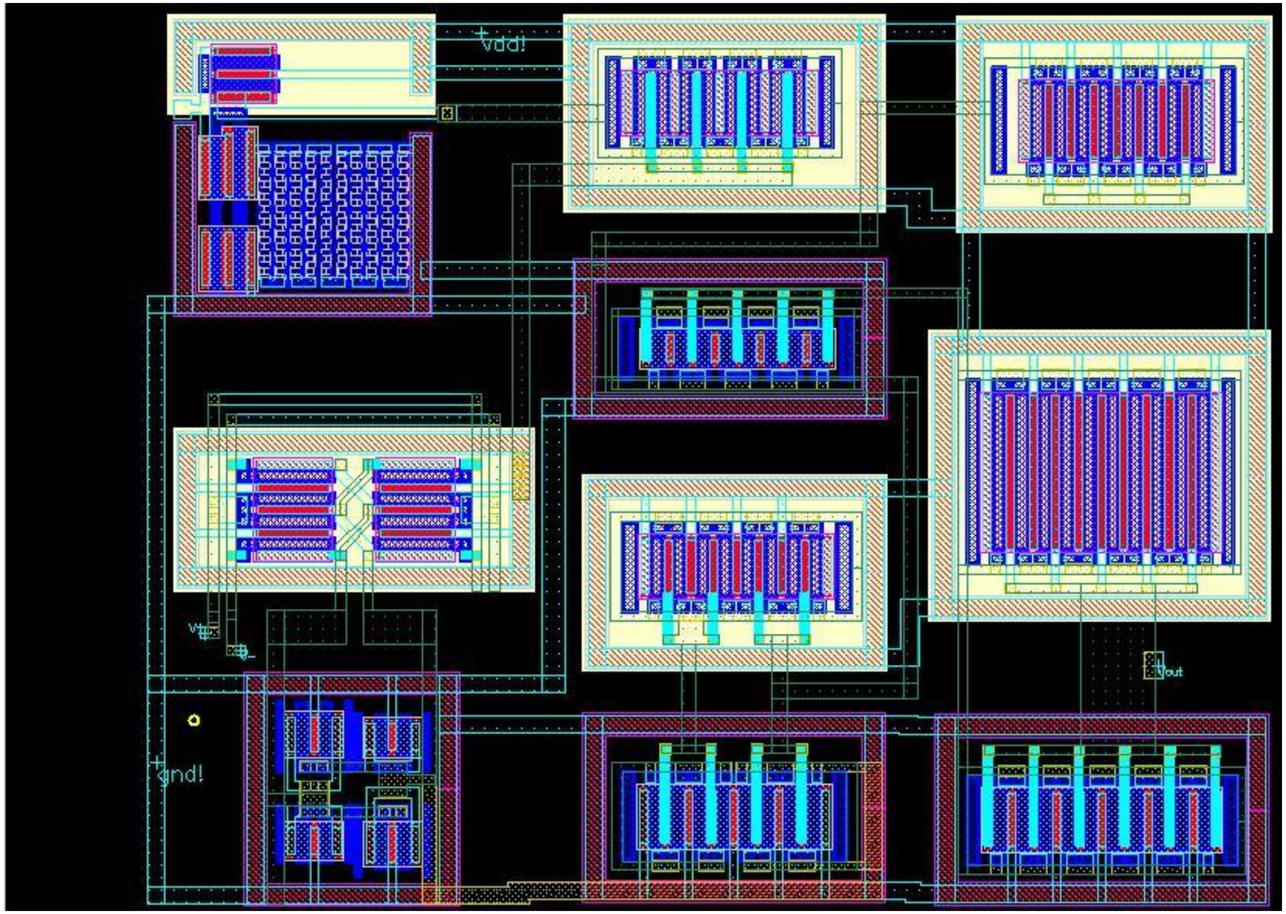
Comparator-M63~64-Schematic



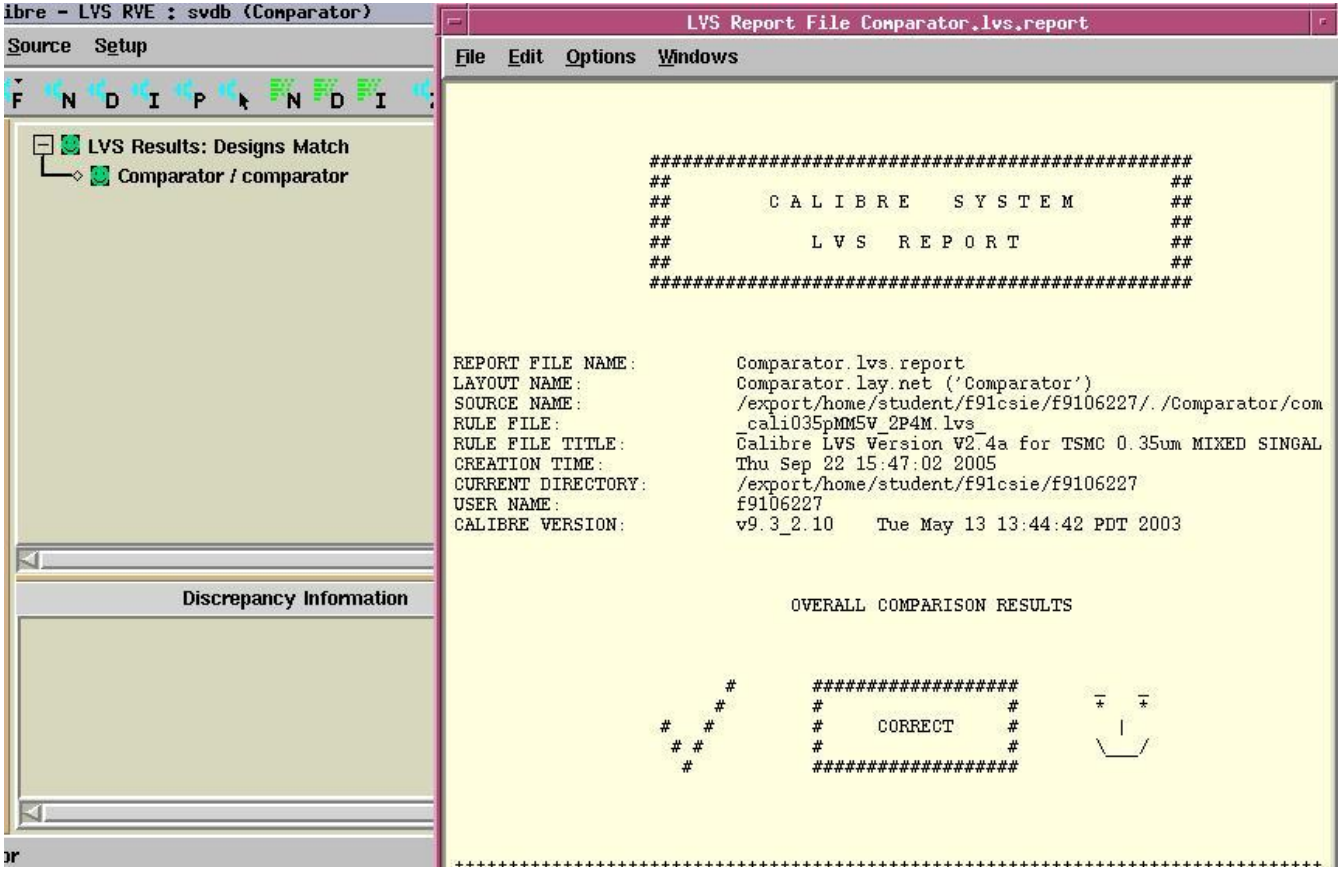
Comparator-M63~64-Layout



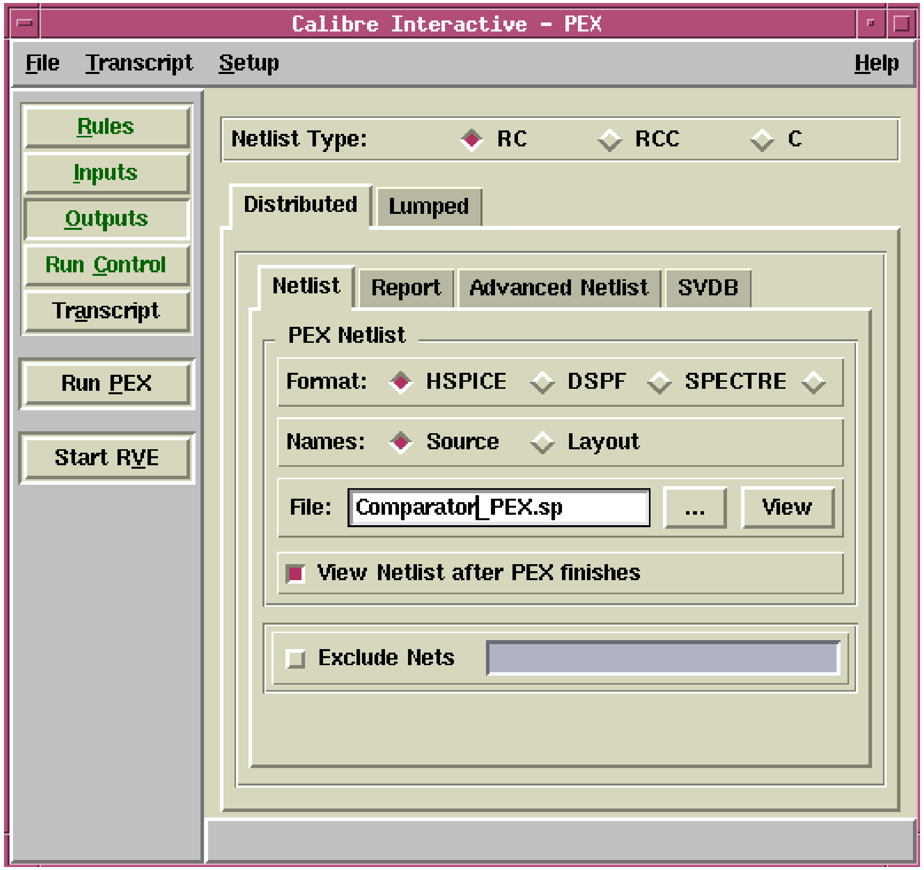
Comparator-Layout



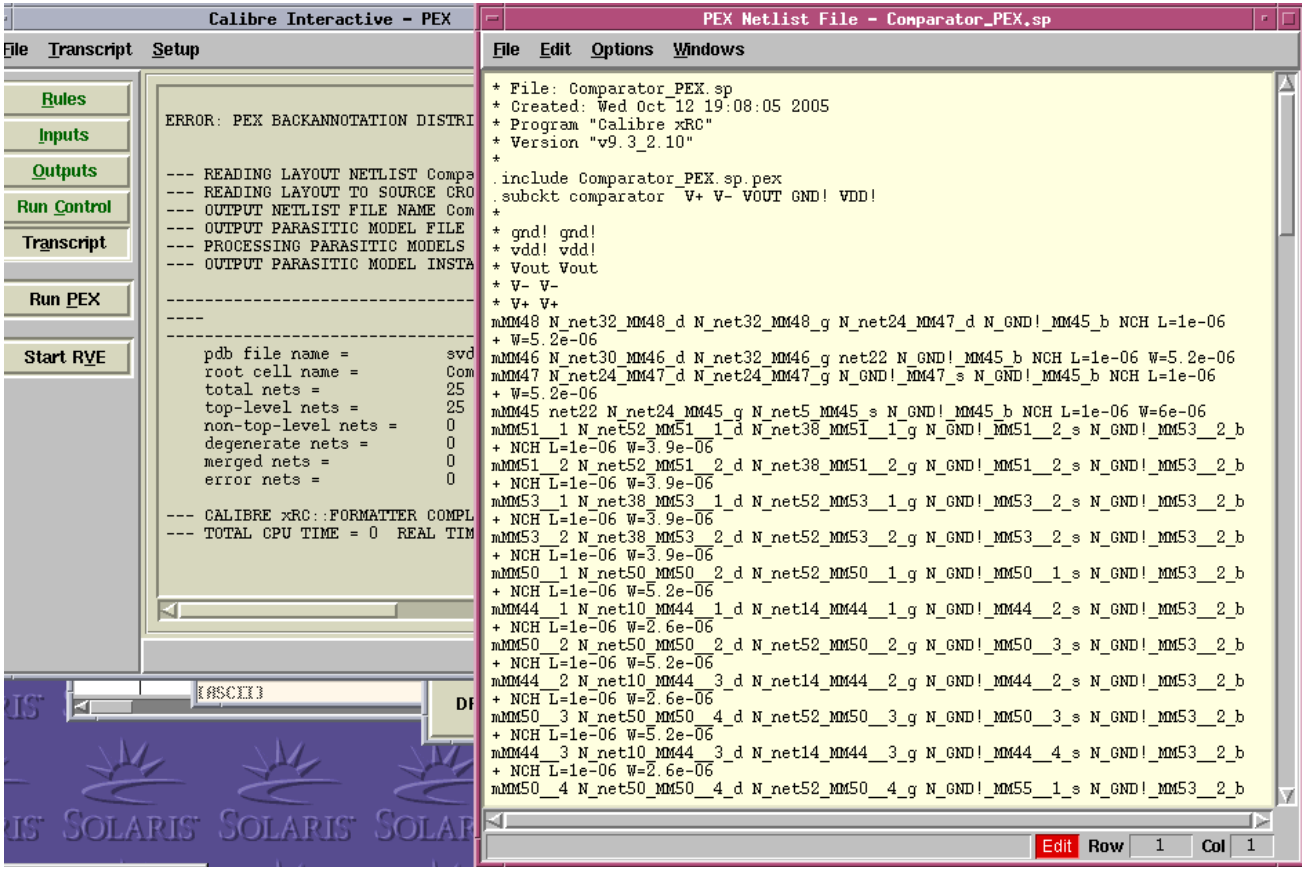
Comparator-Lvs-OK



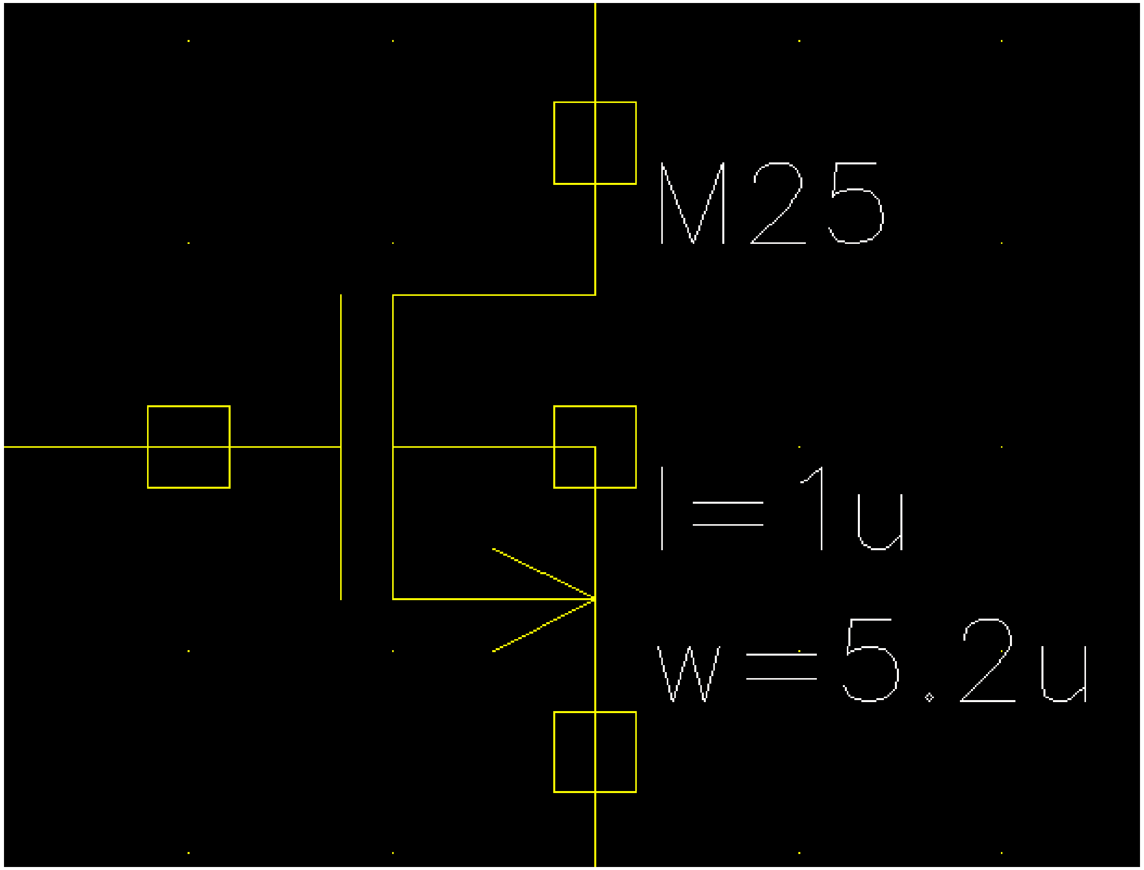
Comparator-Run PEX



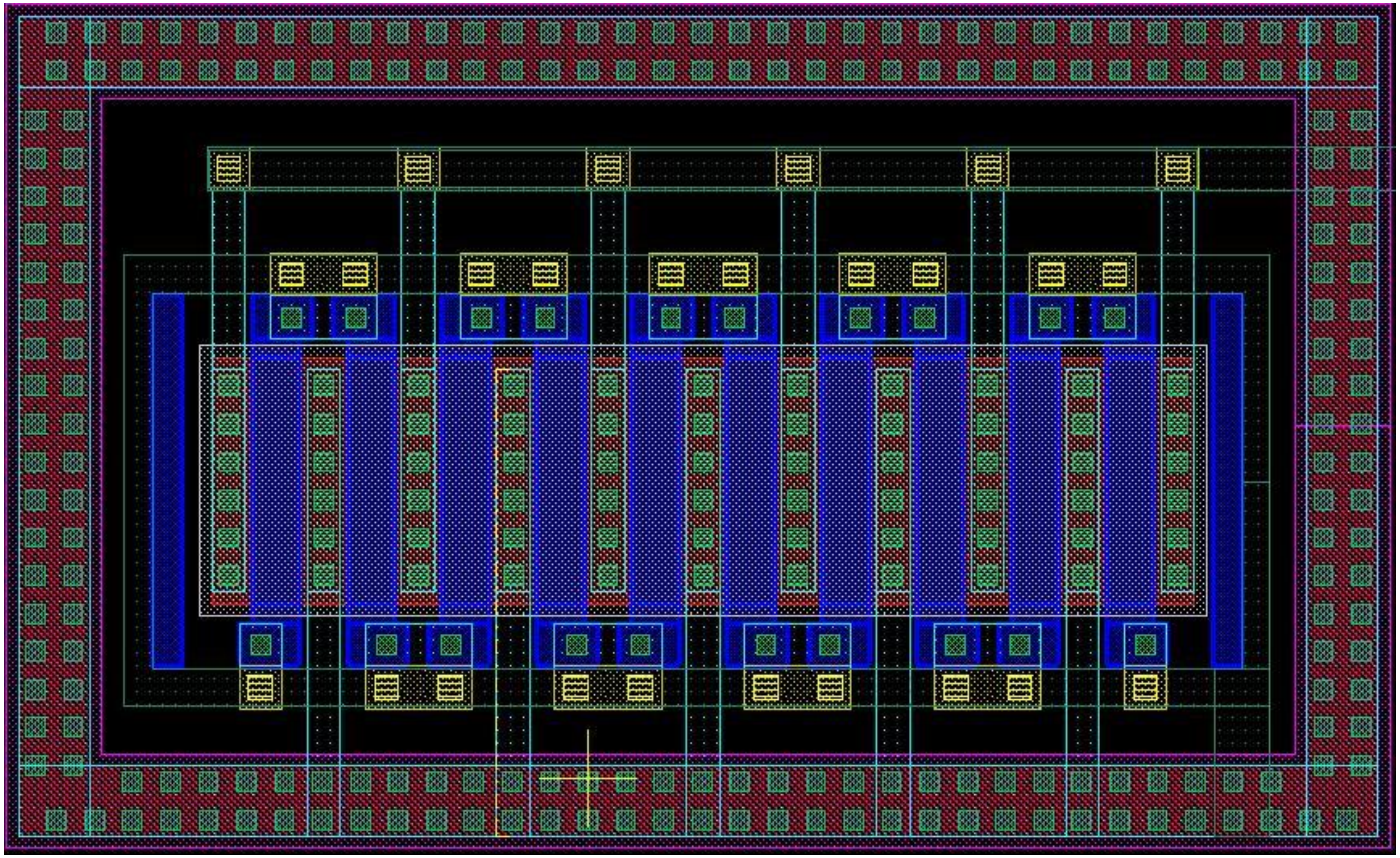
Comparator-Run PEX-Result



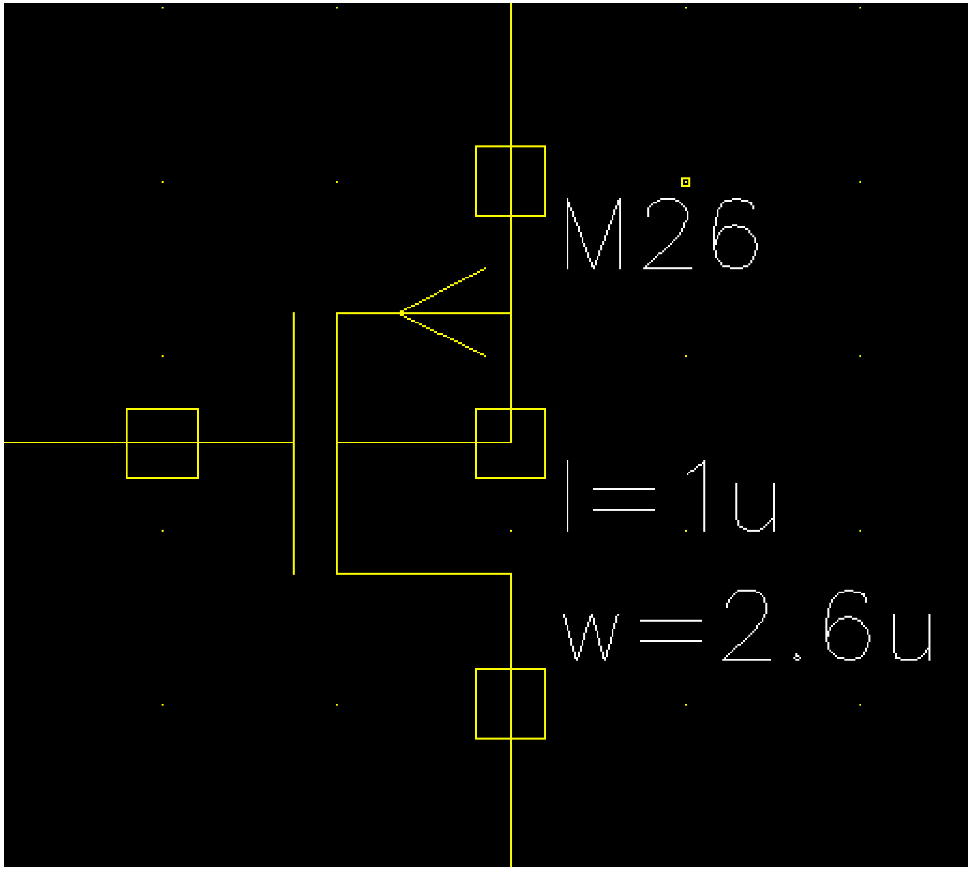
VRG-M25-Schematic



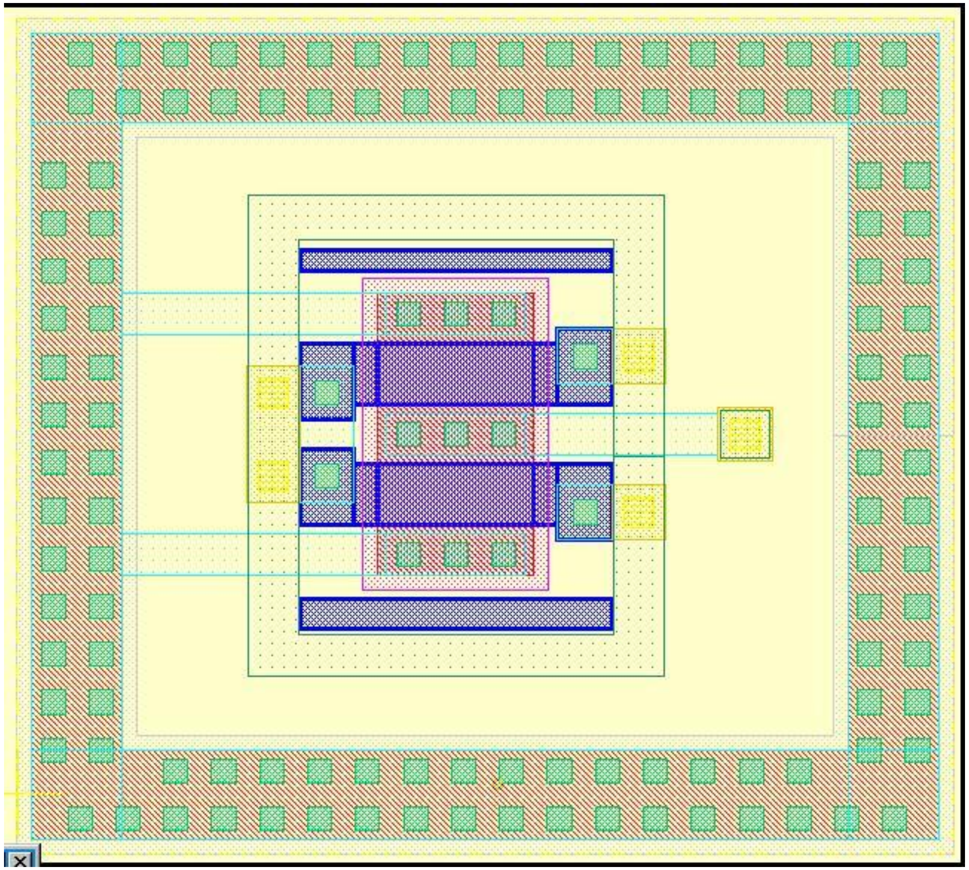
VRG-M25-Layout



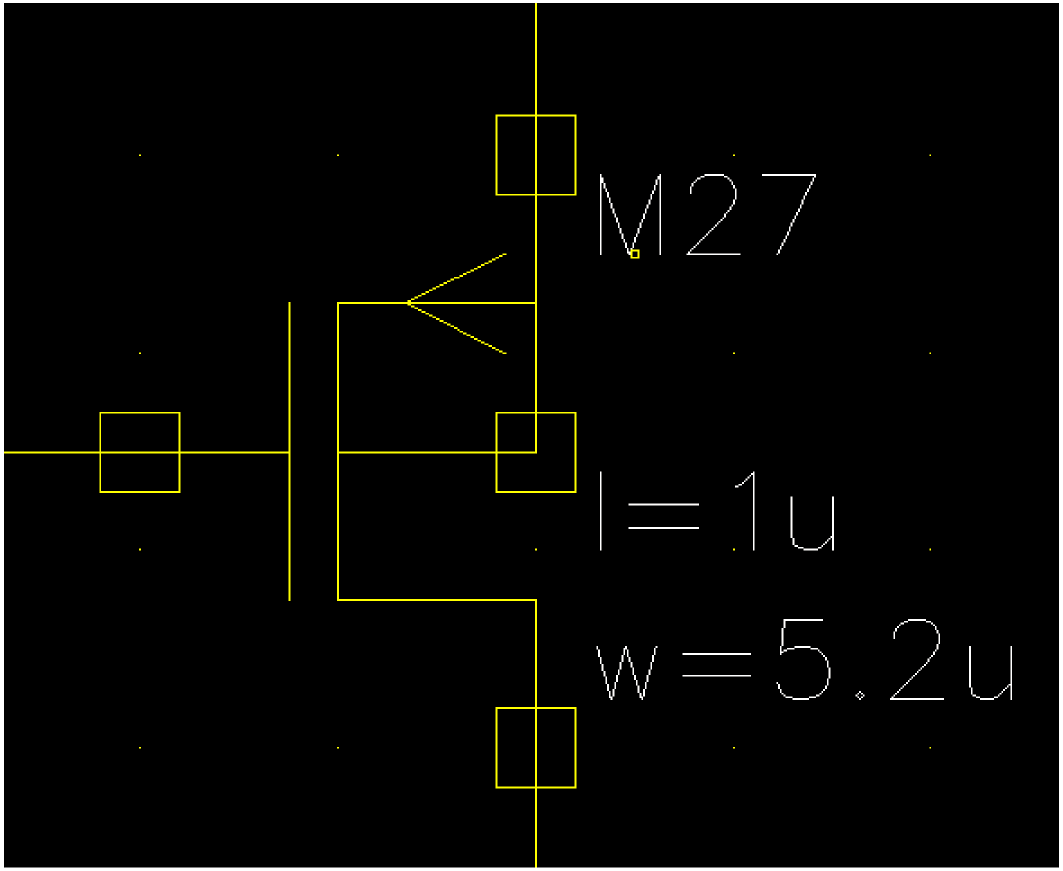
VRG-M26-Schematic



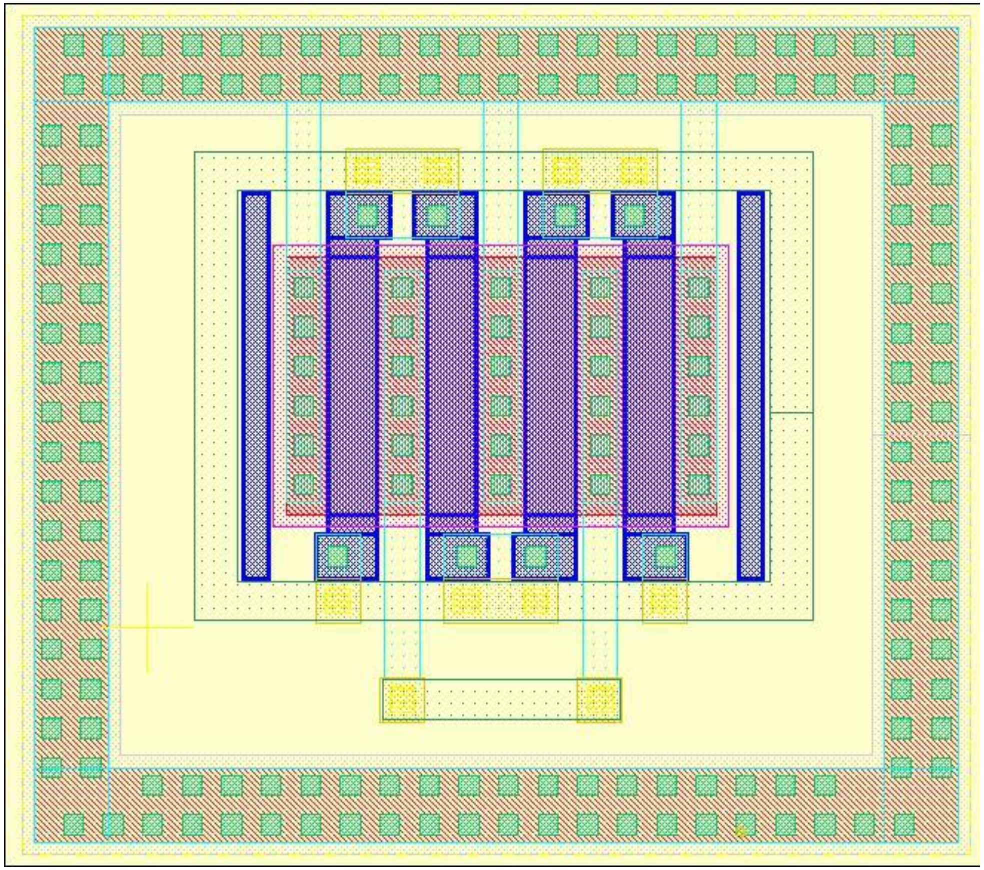
VRG-M26-Layout



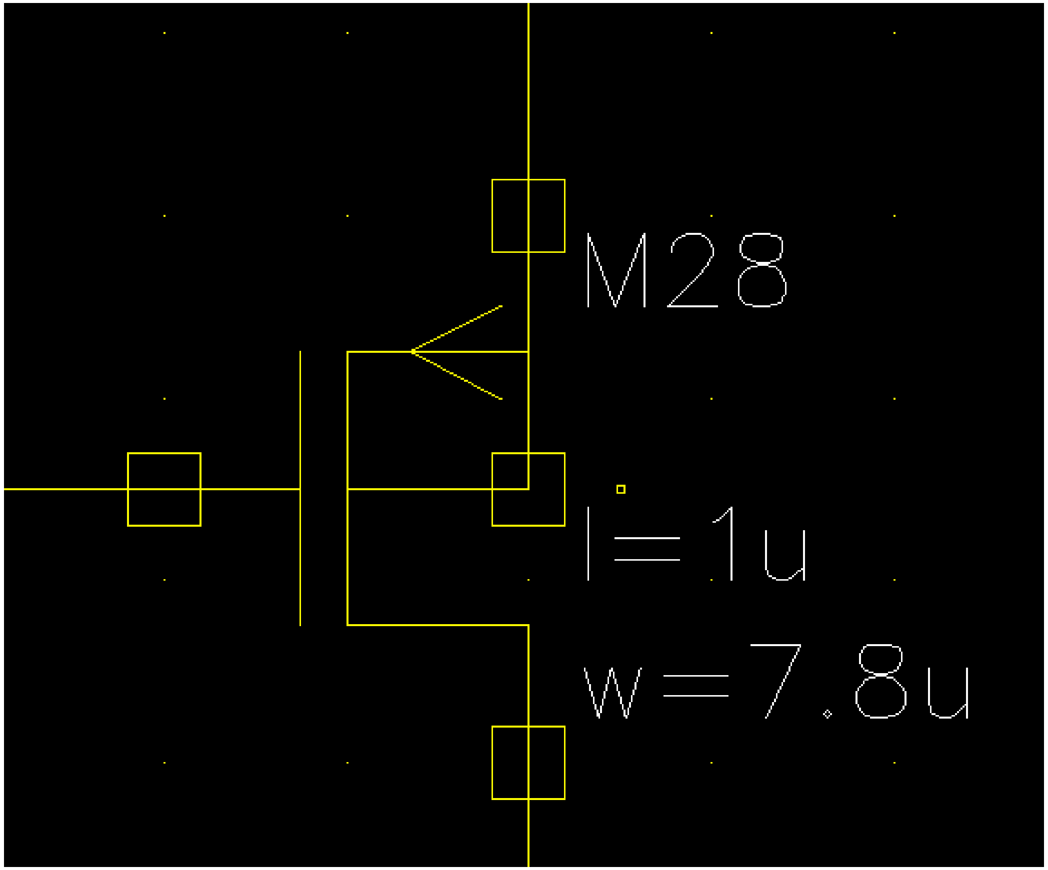
VRG-M27-Schematic



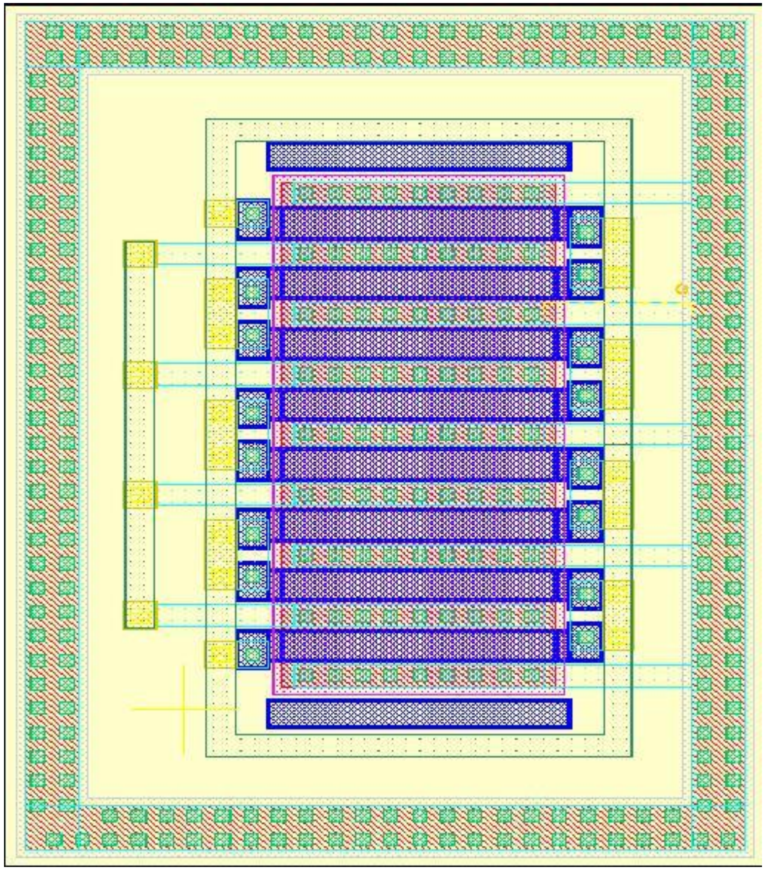
VRG-M27-Layout



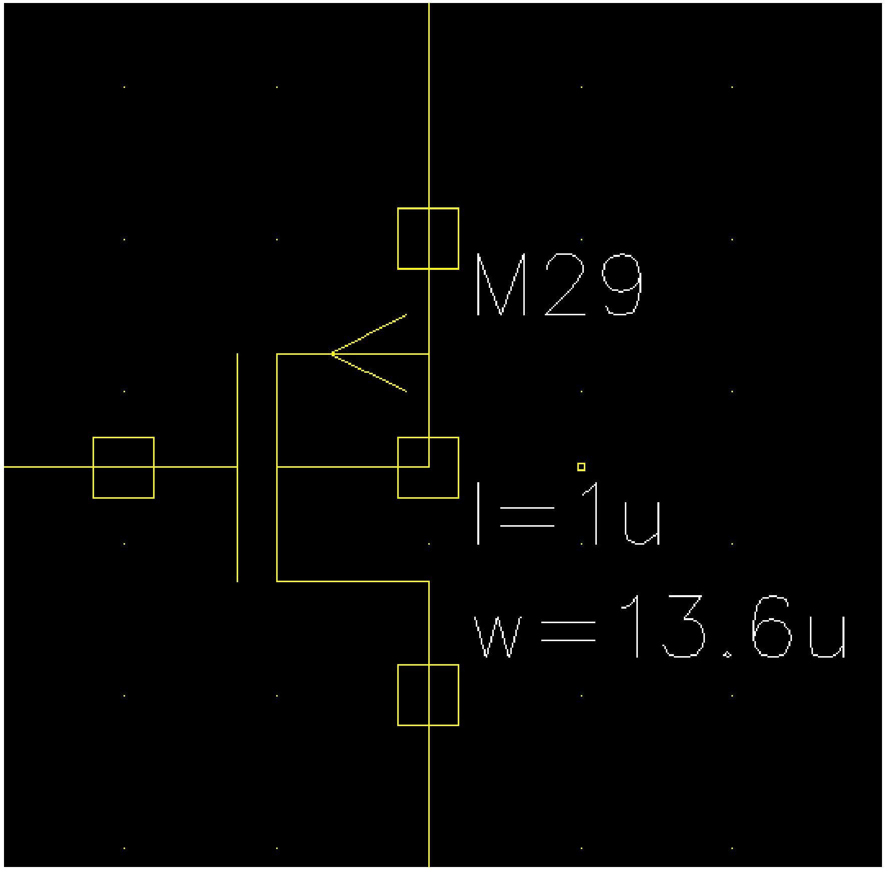
VRG-M28-Schematic



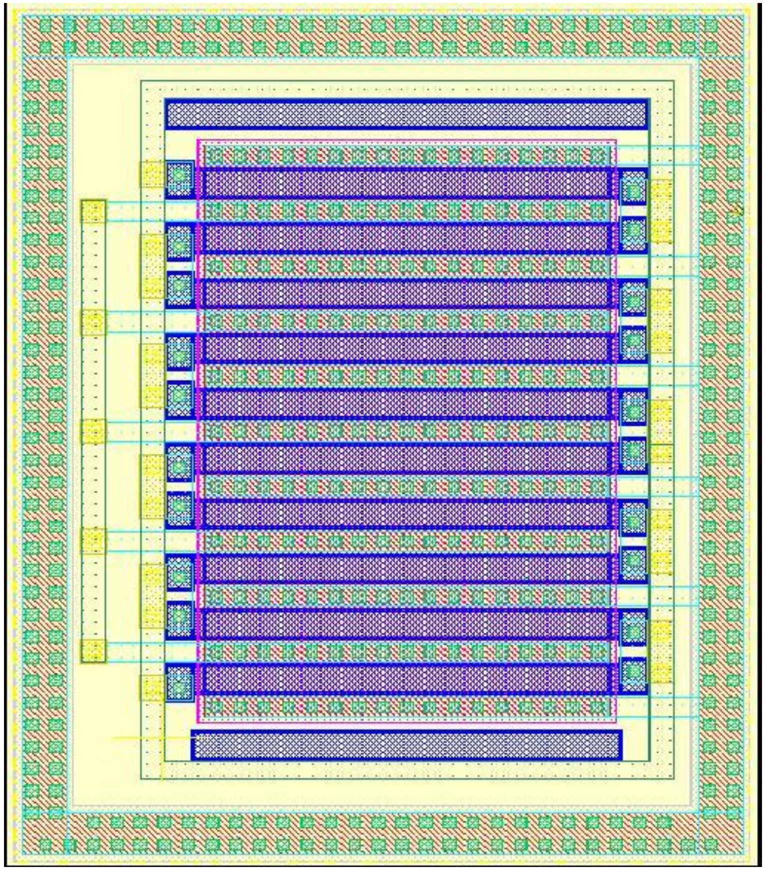
VRG-M28-Layout



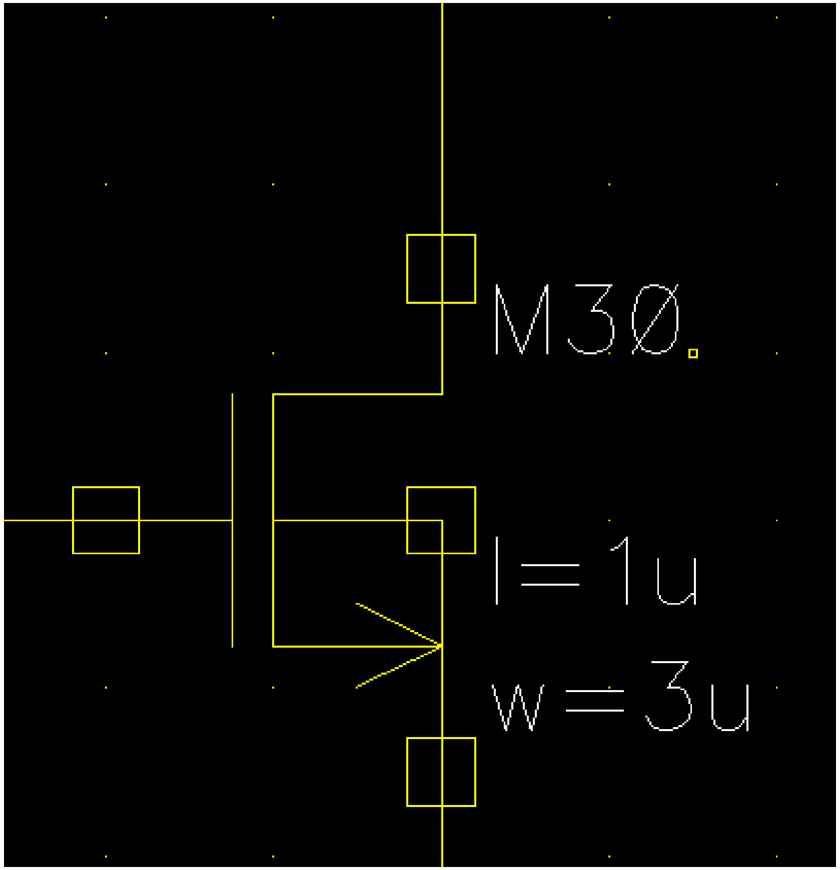
VRG-M29-Schematic



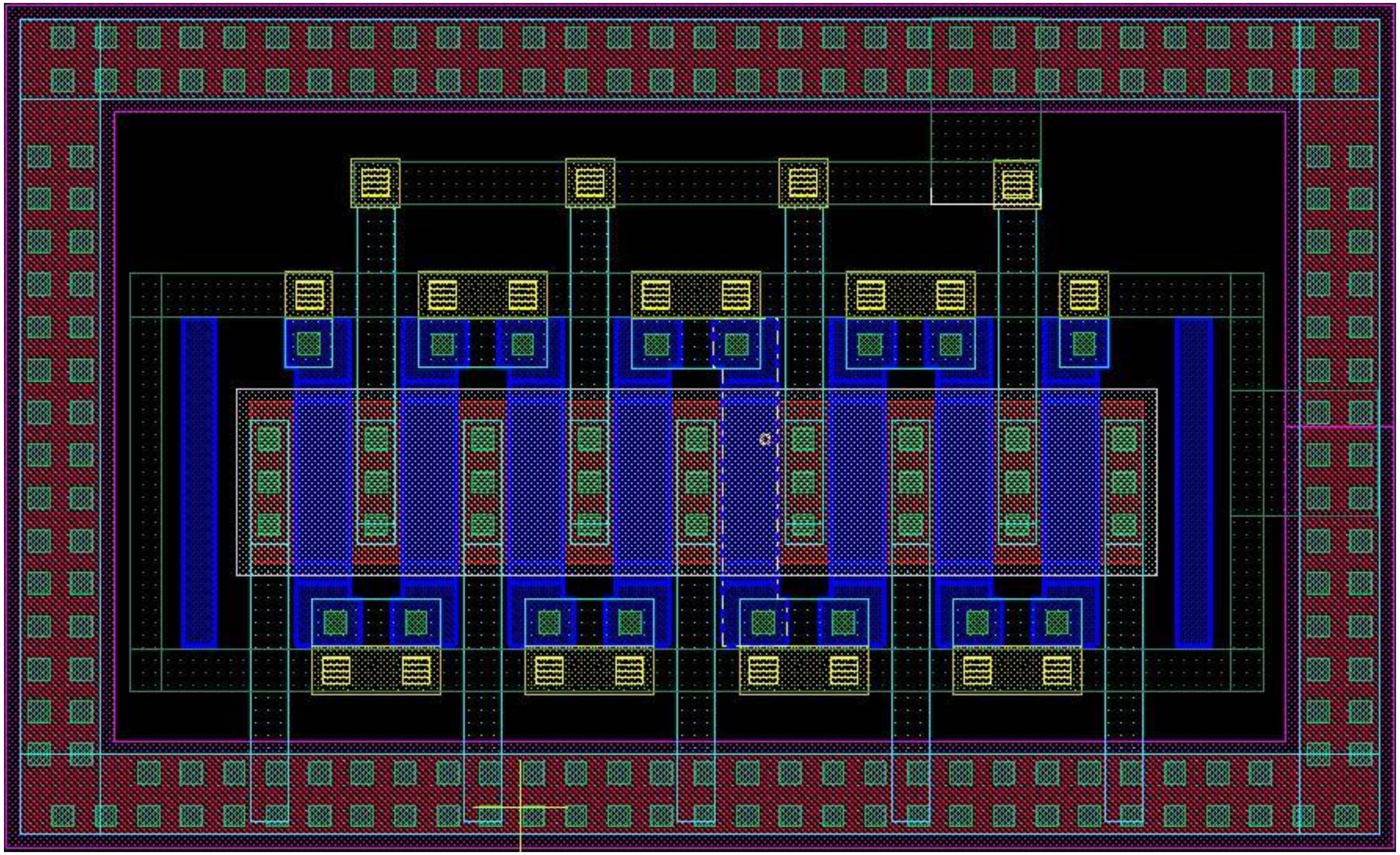
VRG-M29-Layout



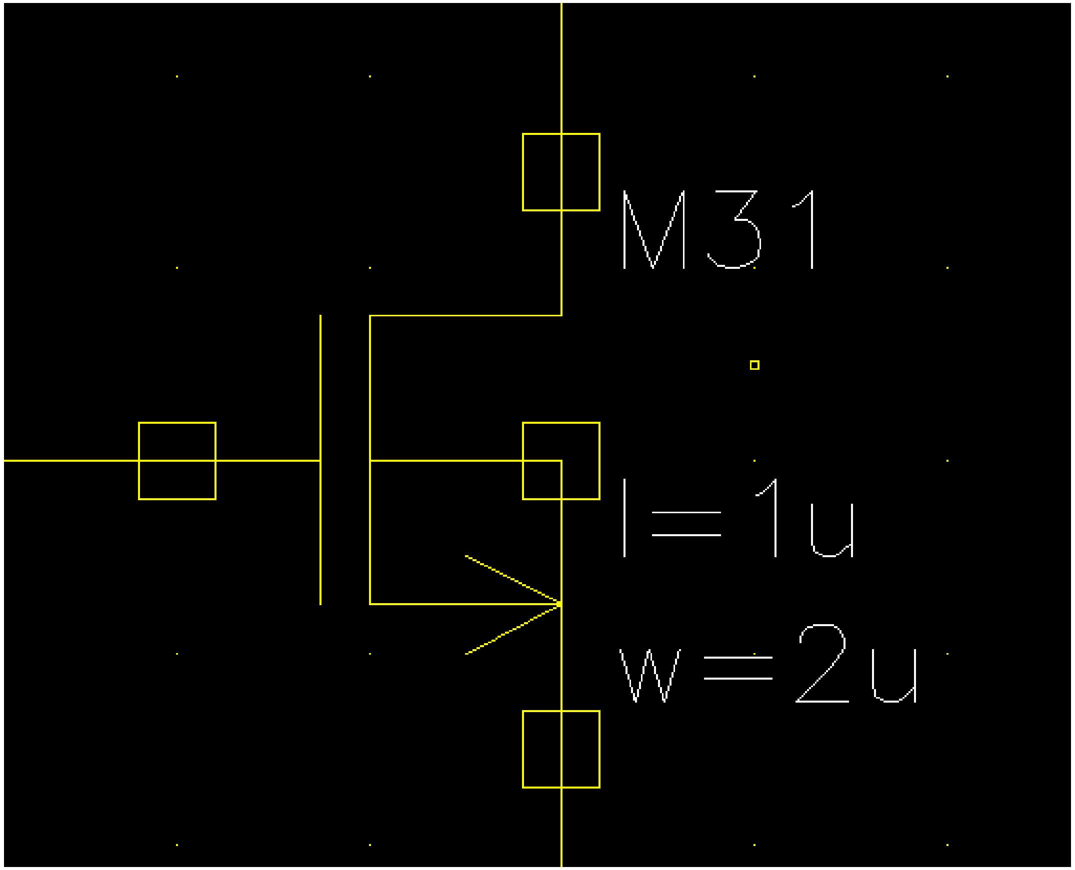
VRG-M30-Schematic



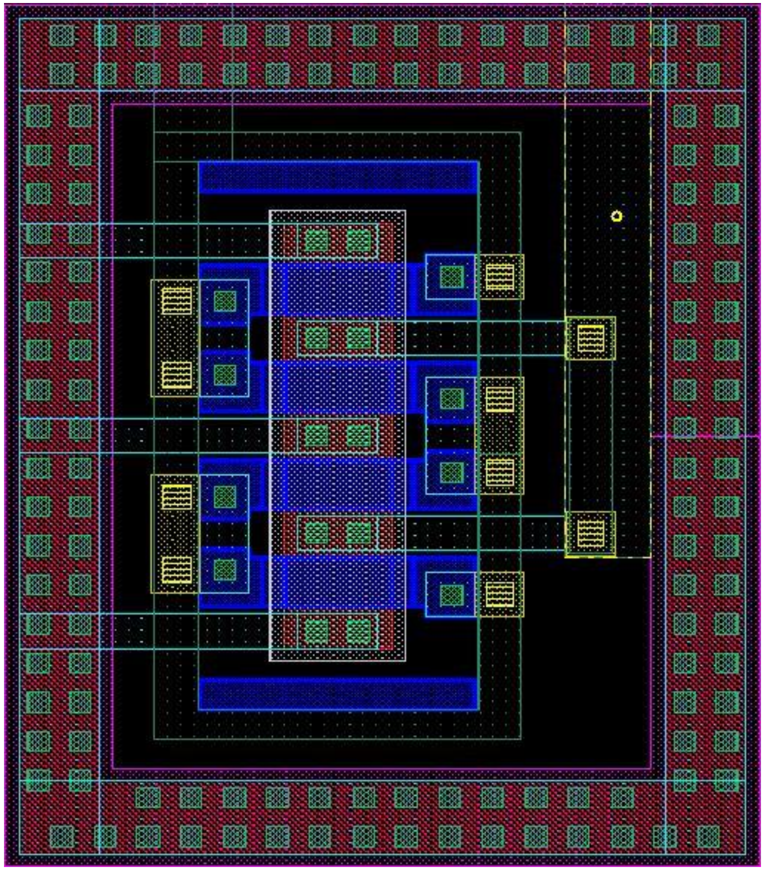
VRG-M30-Layout



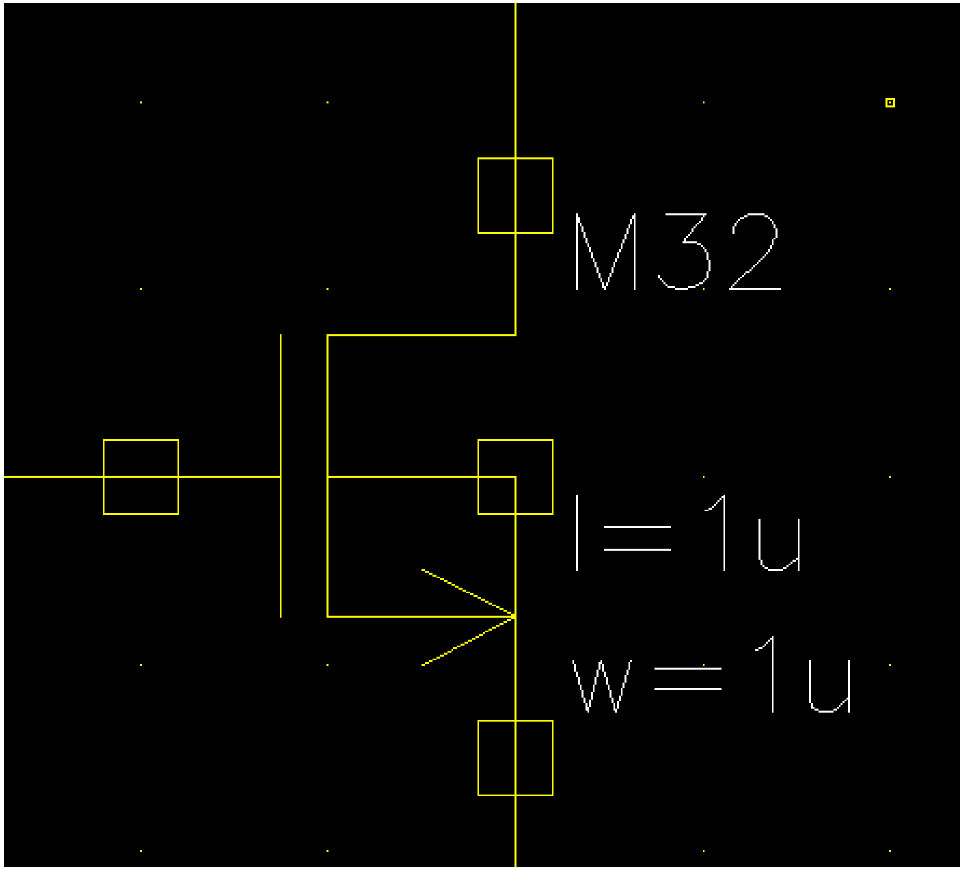
VRG-M31-Schematic



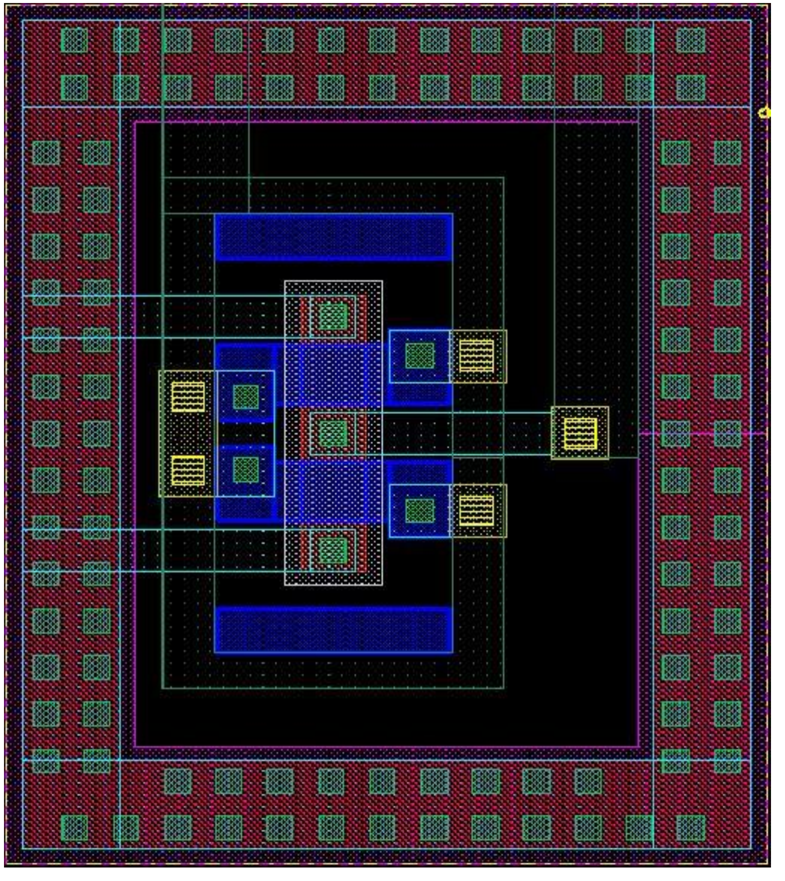
VRG-M31-Layout



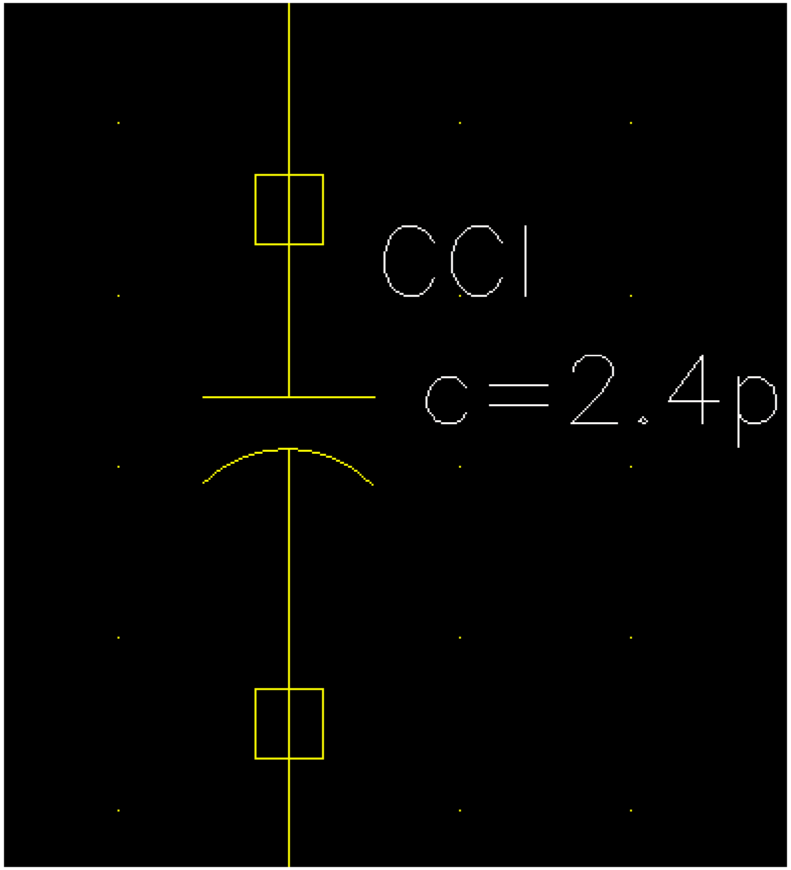
VRG-M32-Schematic



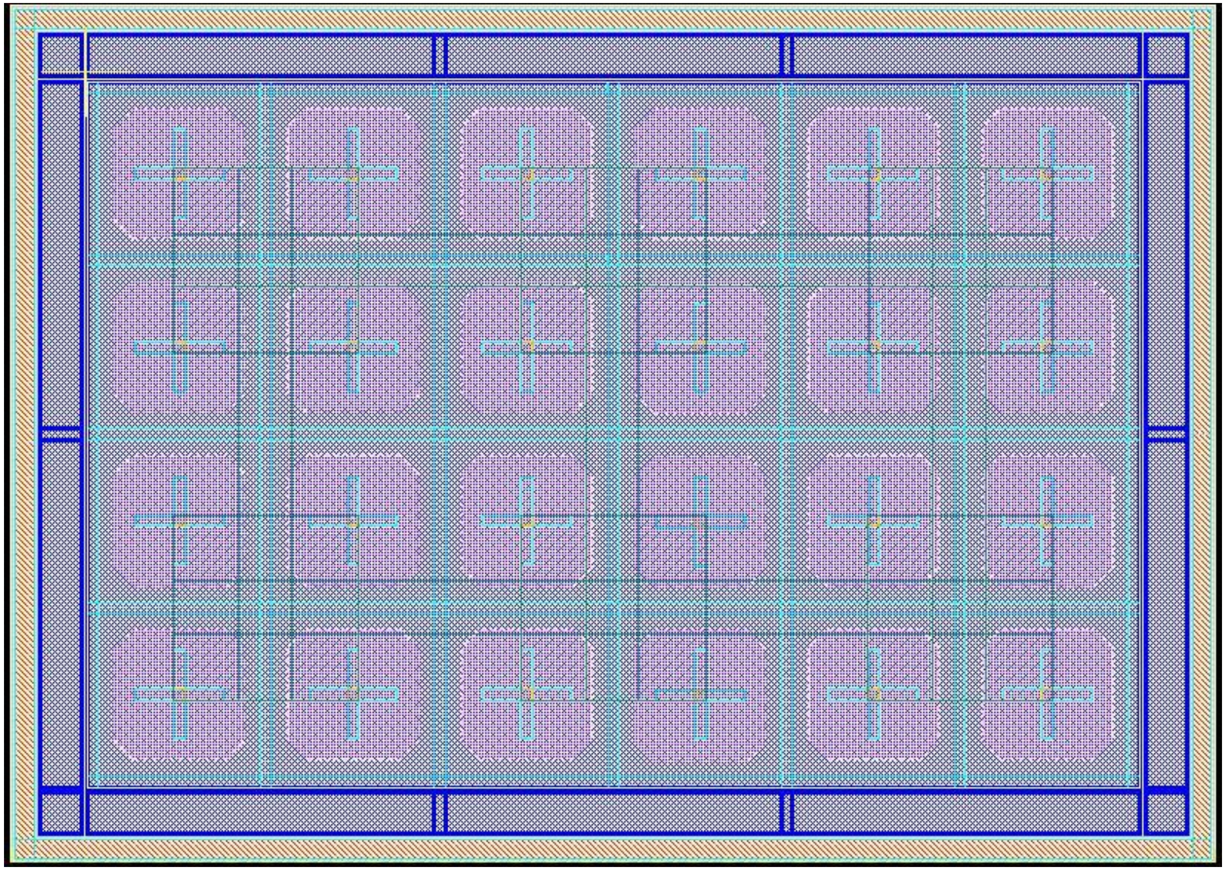
VRG-M32-Layout



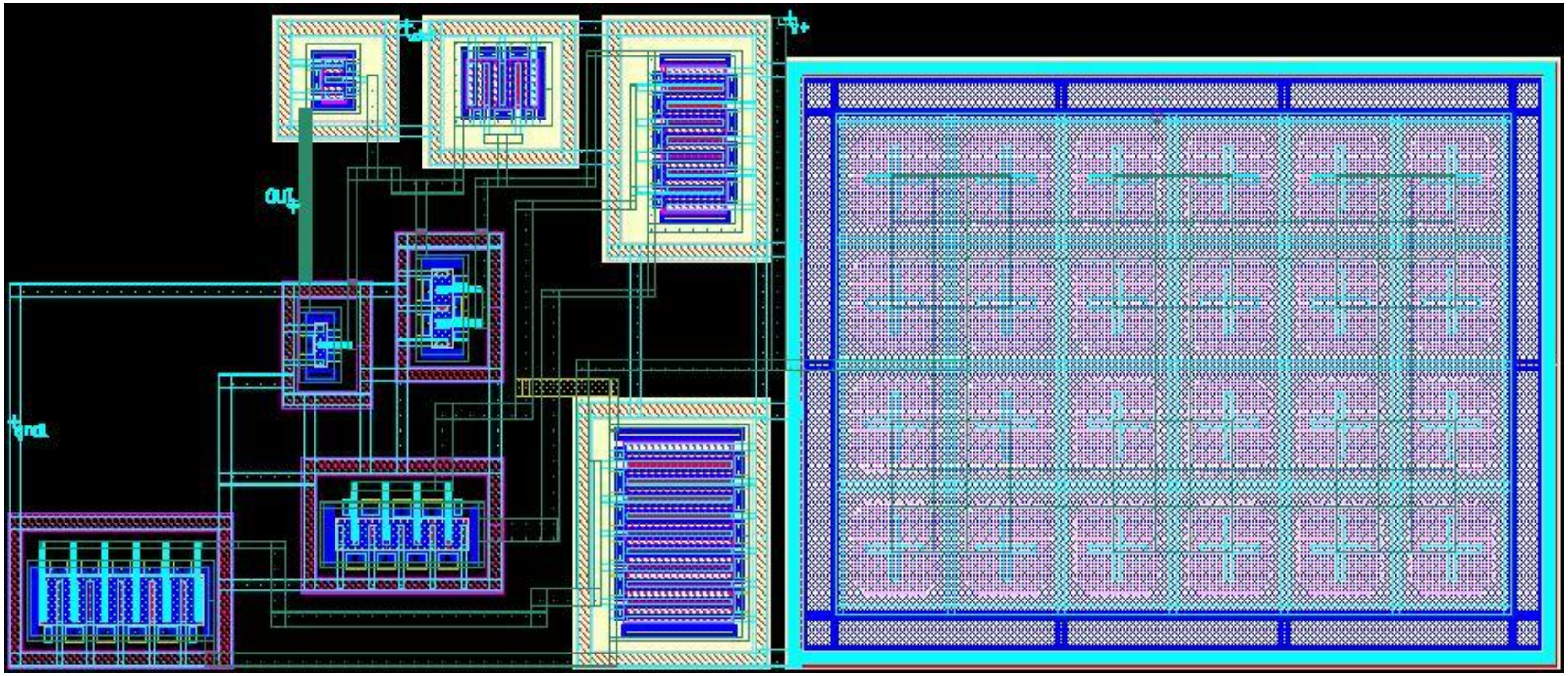
VRG-Ccl-Schematic



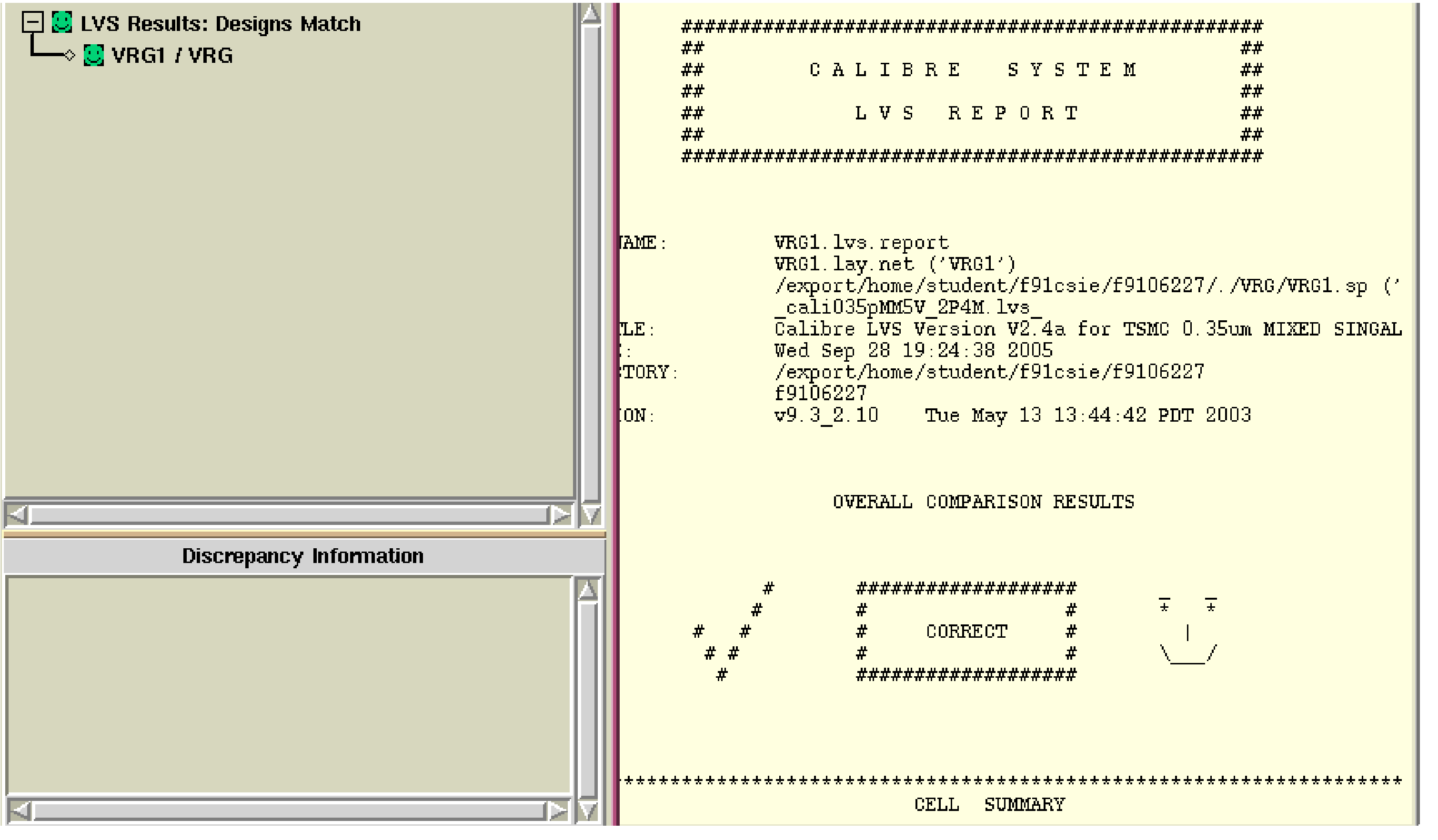
VRG-Ccl-Layout



VRG-Layout

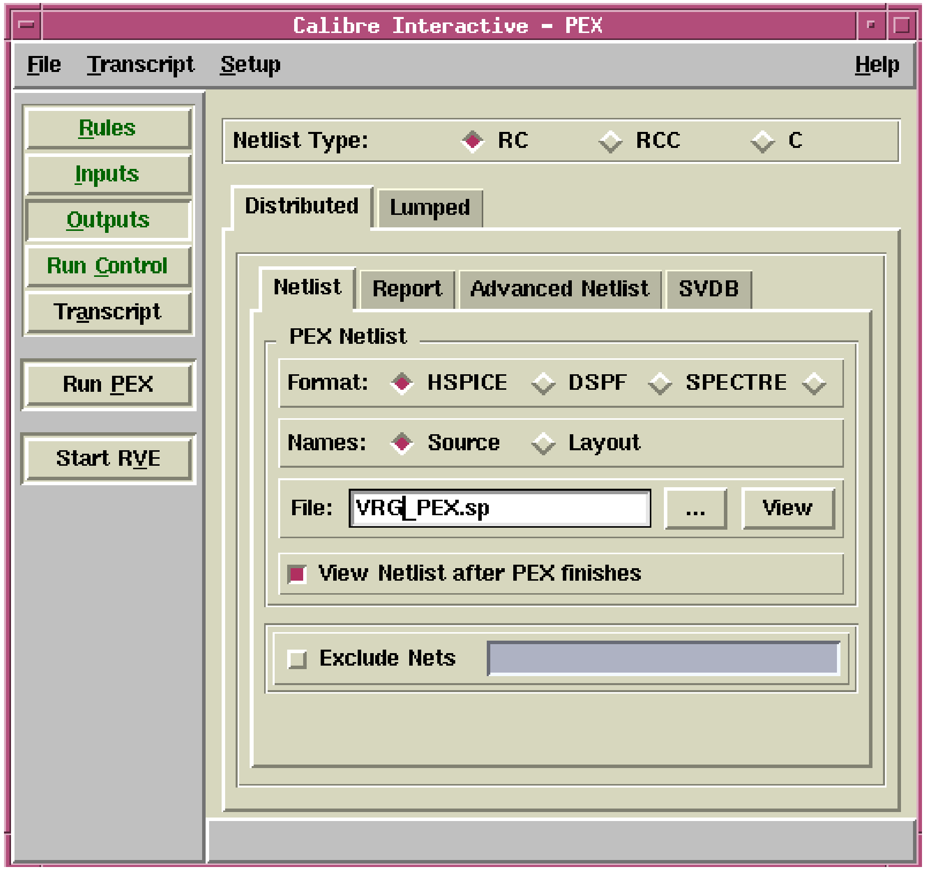


VRG-LVS-OK

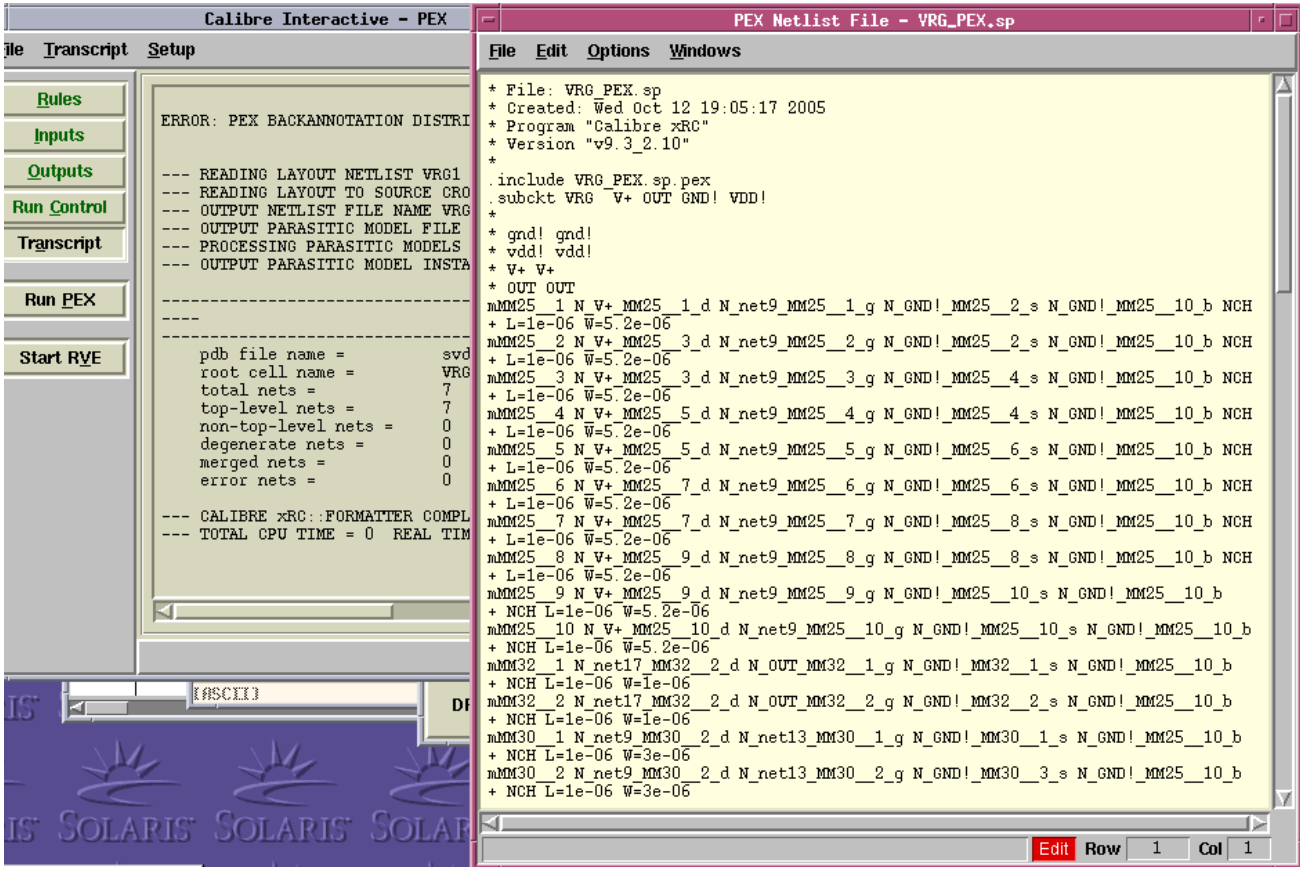


LVS Results: Designs Match
VRG1 / VRG
Discrepancy Information
C A L I B R E   S Y S T E M
L V S   R E P O R T
AME: VRG1.lvs.report
VRG1.lay.net ('VRG1')
/export/home/student/f91csie/f9106227/./VRG/VRG1.sp ('
_cali035pMM5V_2P4M.lvs_
LE: Calibre LVS Version V2.4a for TSMC 0.35um MIXED SINGAL
:   Wed Sep 28 19:24:38 2005
TORY: /export/home/student/f91csie/f9106227
f9106227
ON: v9.3_2.10   Tue May 13 13:44:42 PDT 2003
OVERALL COMPARISON RESULTS
CORRECT
CELL SUMMARY

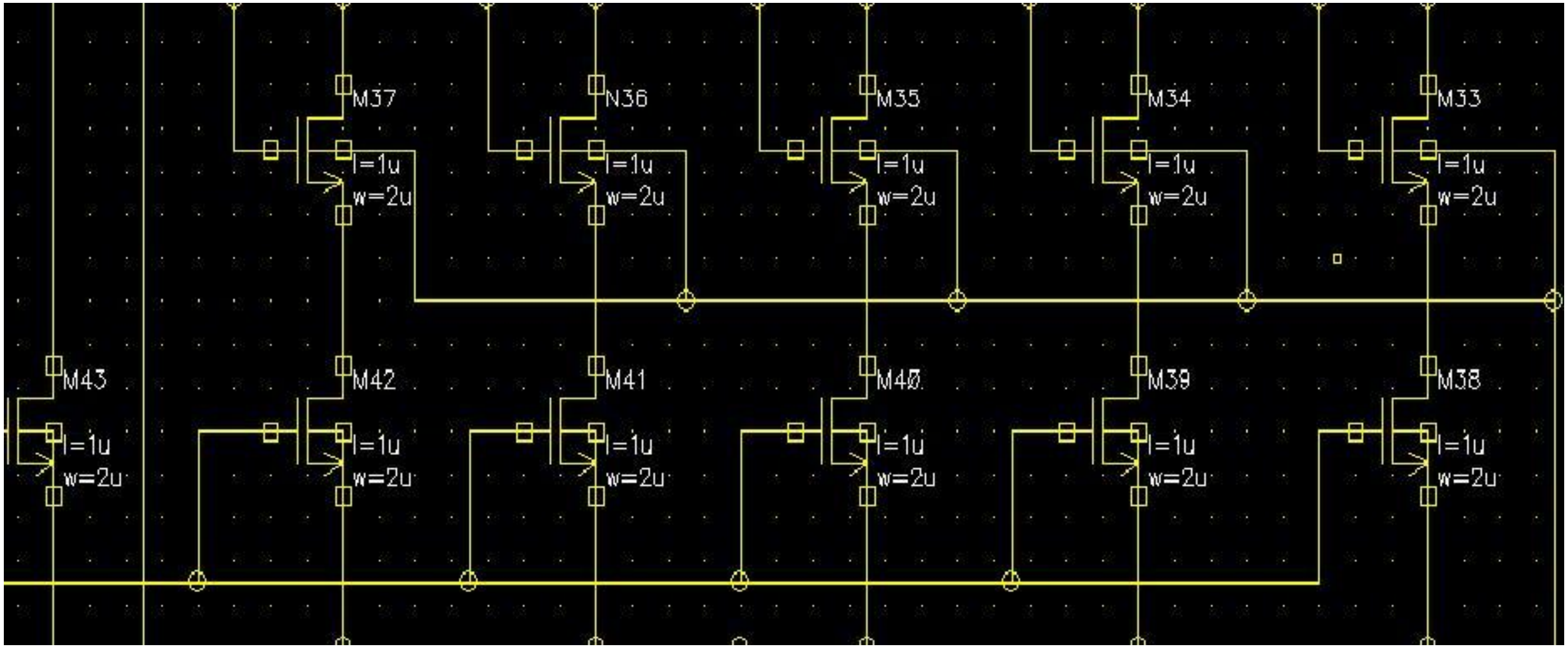
VRG-R/C Run PEX



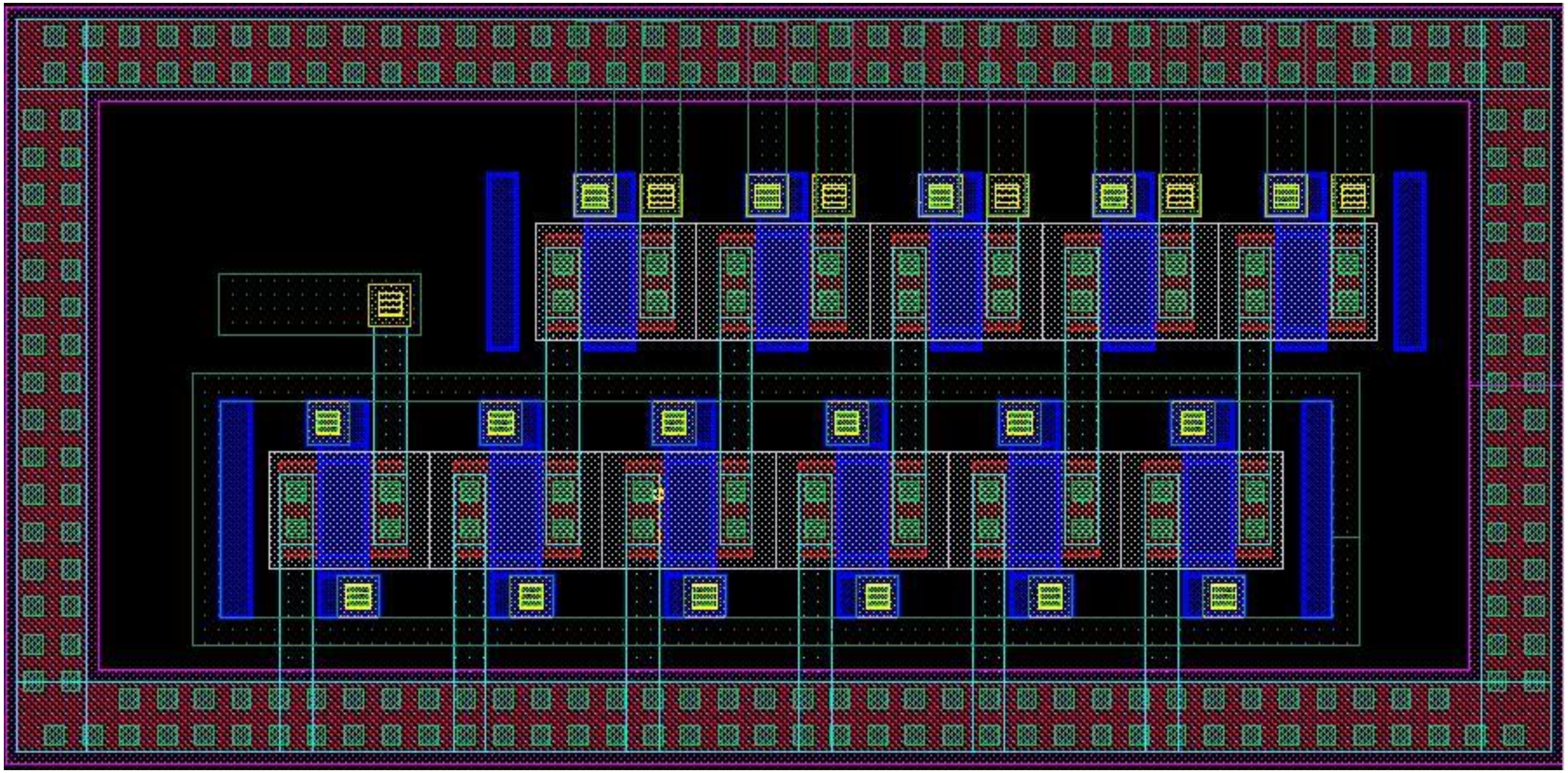
VRG-R/C Run PEX Success



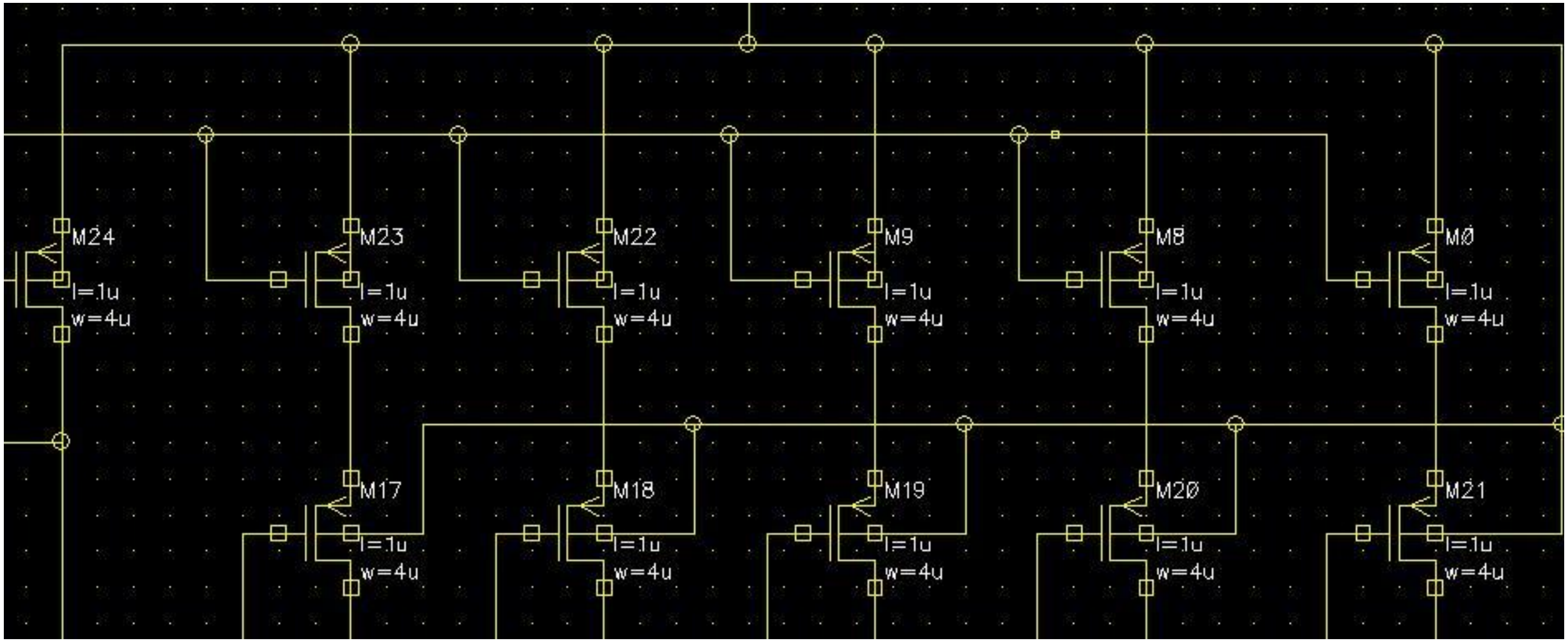
Oscillator-M33~43-Schematic



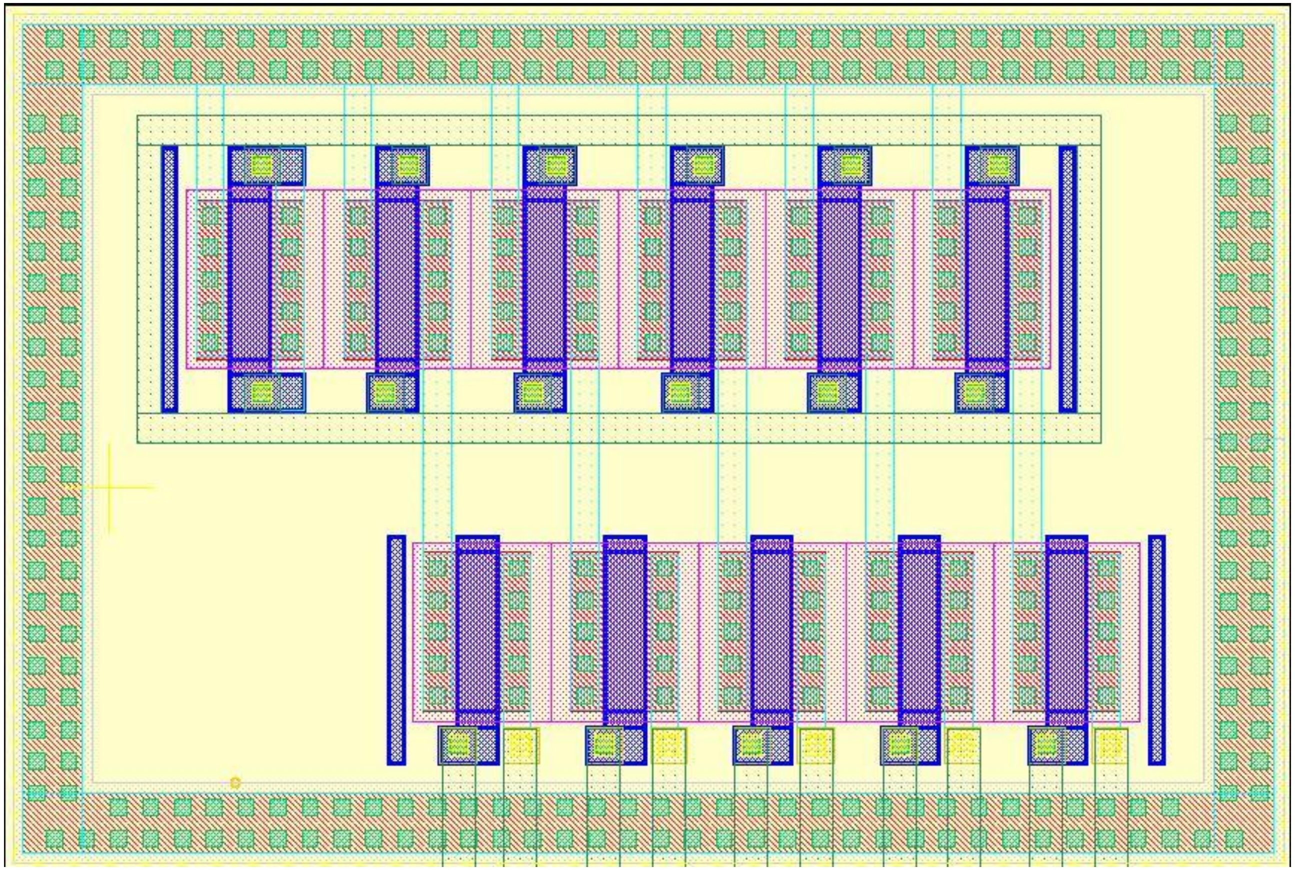
Oscillator-M33~43-Layout



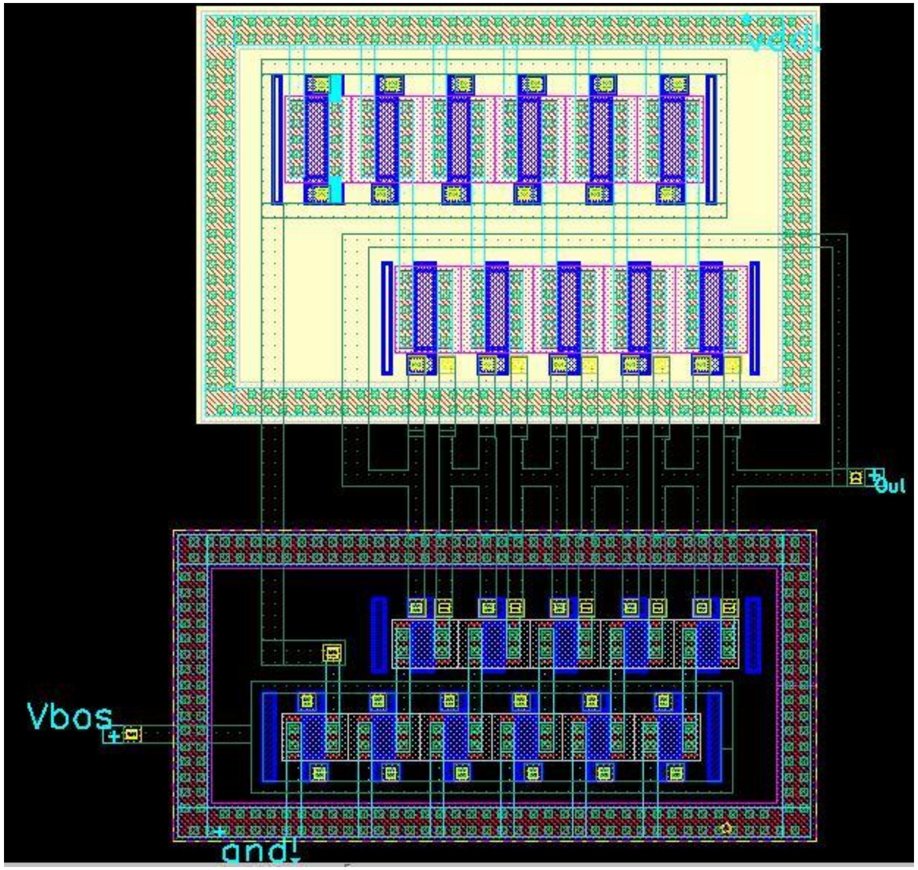
Oscillator-M0,8,9,17~24-Schematic



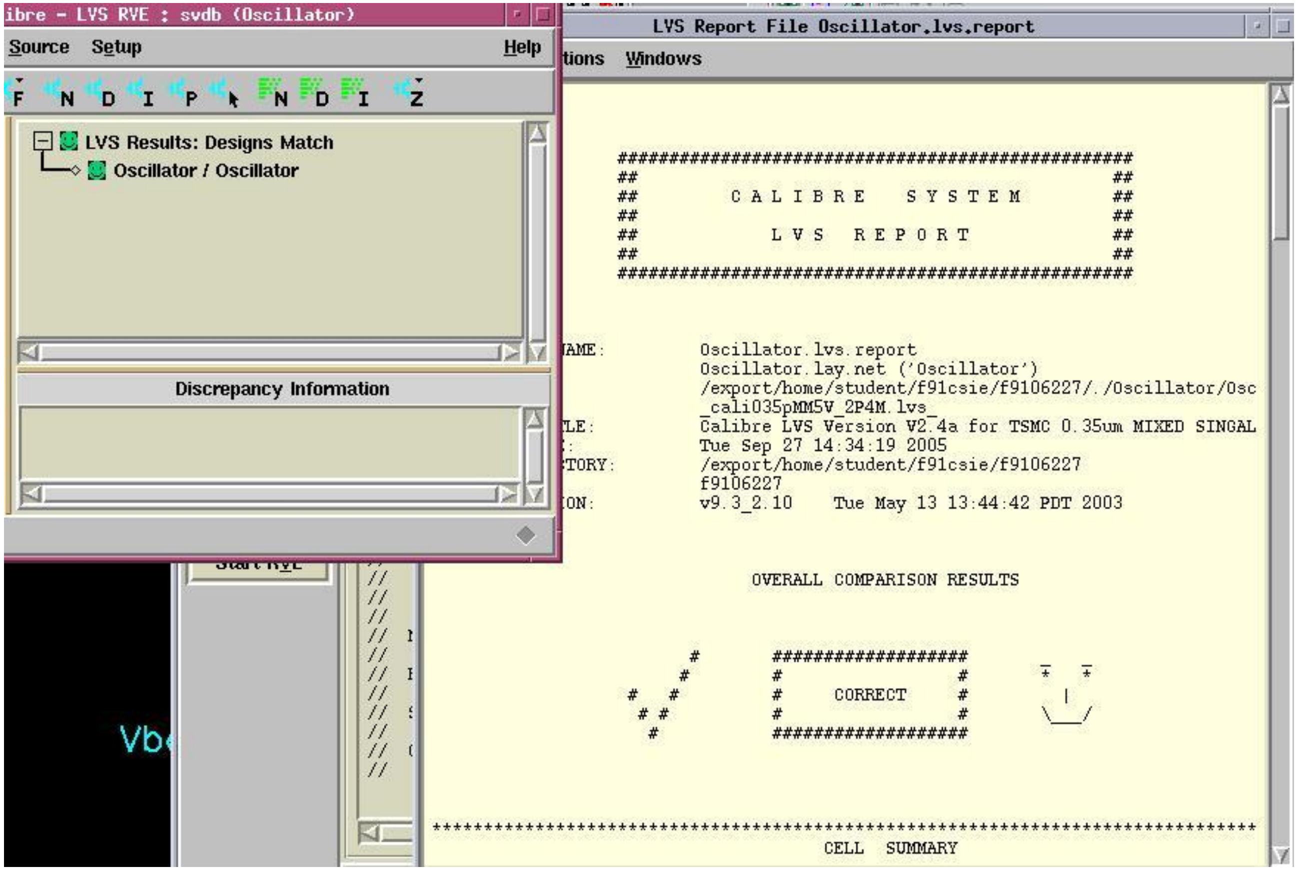
Oscillator-M0,8,9,17~24-Layout



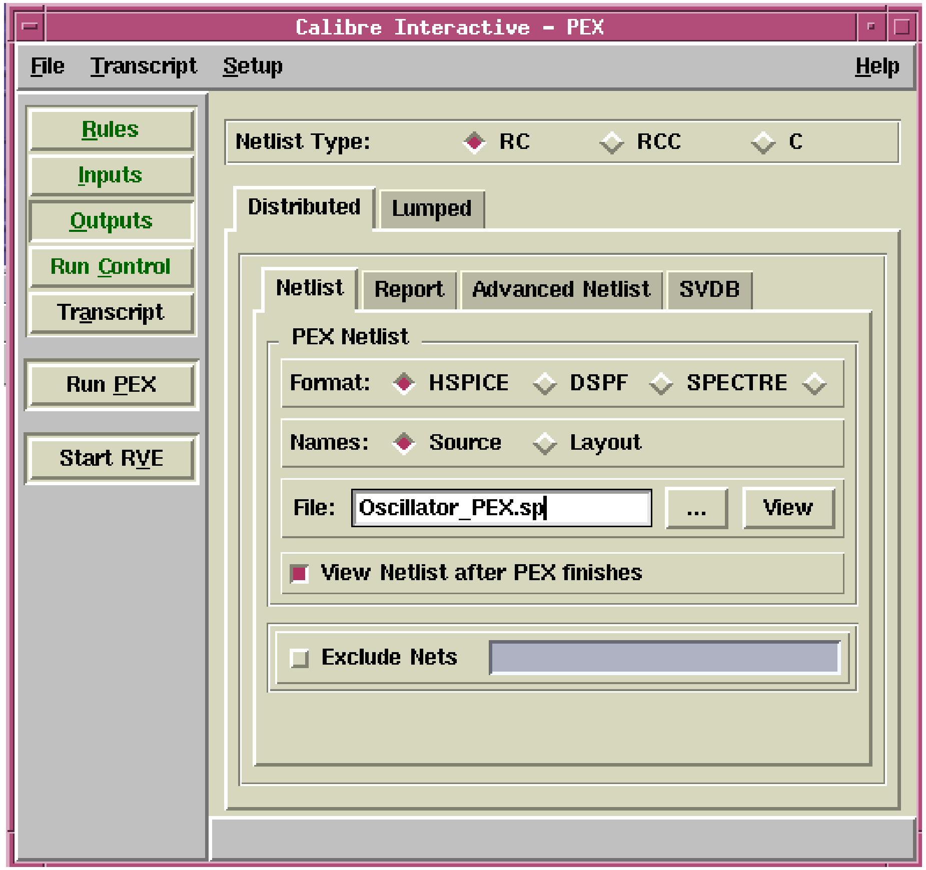
Oscillator-Layout



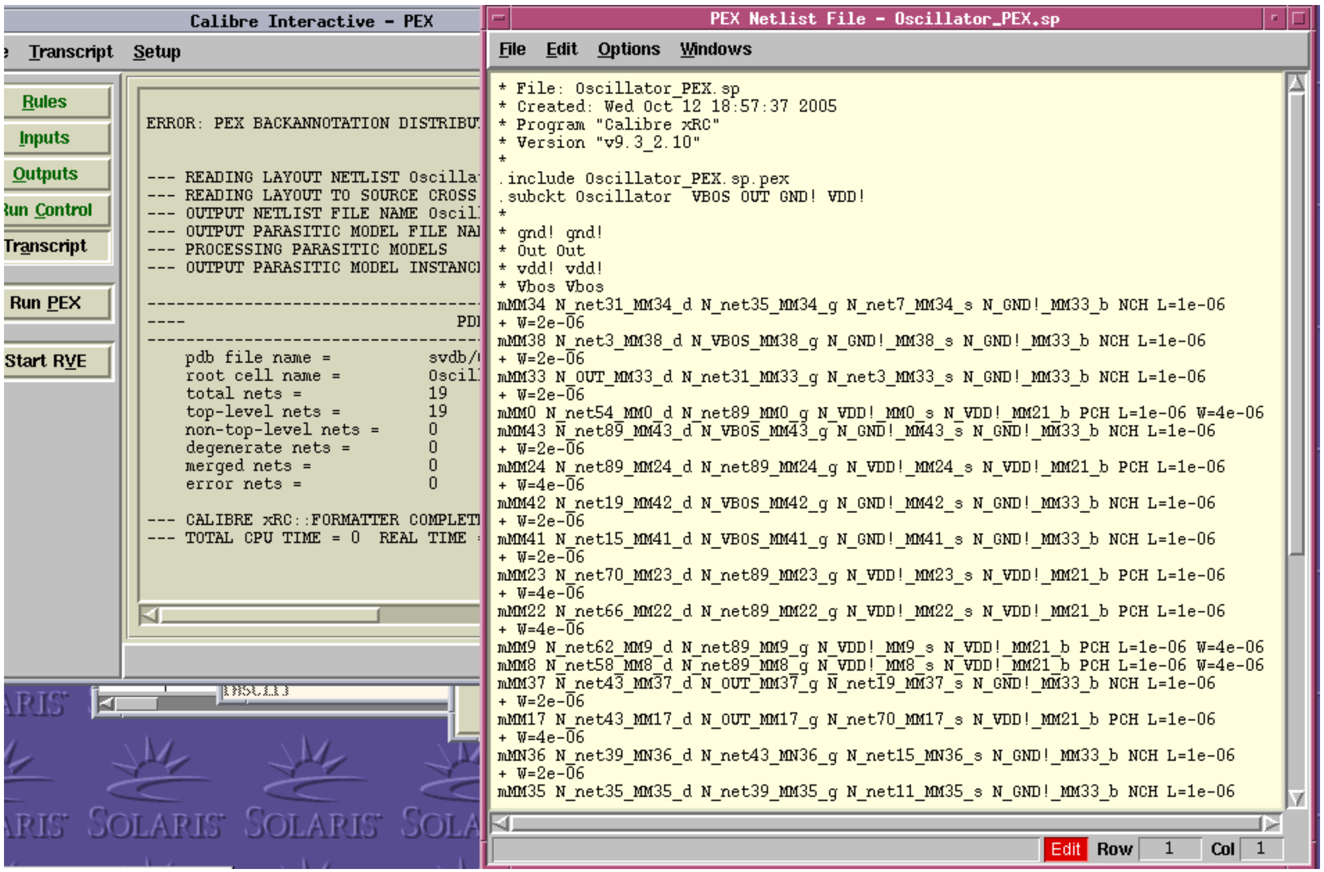
Oscillator-Lvs OK



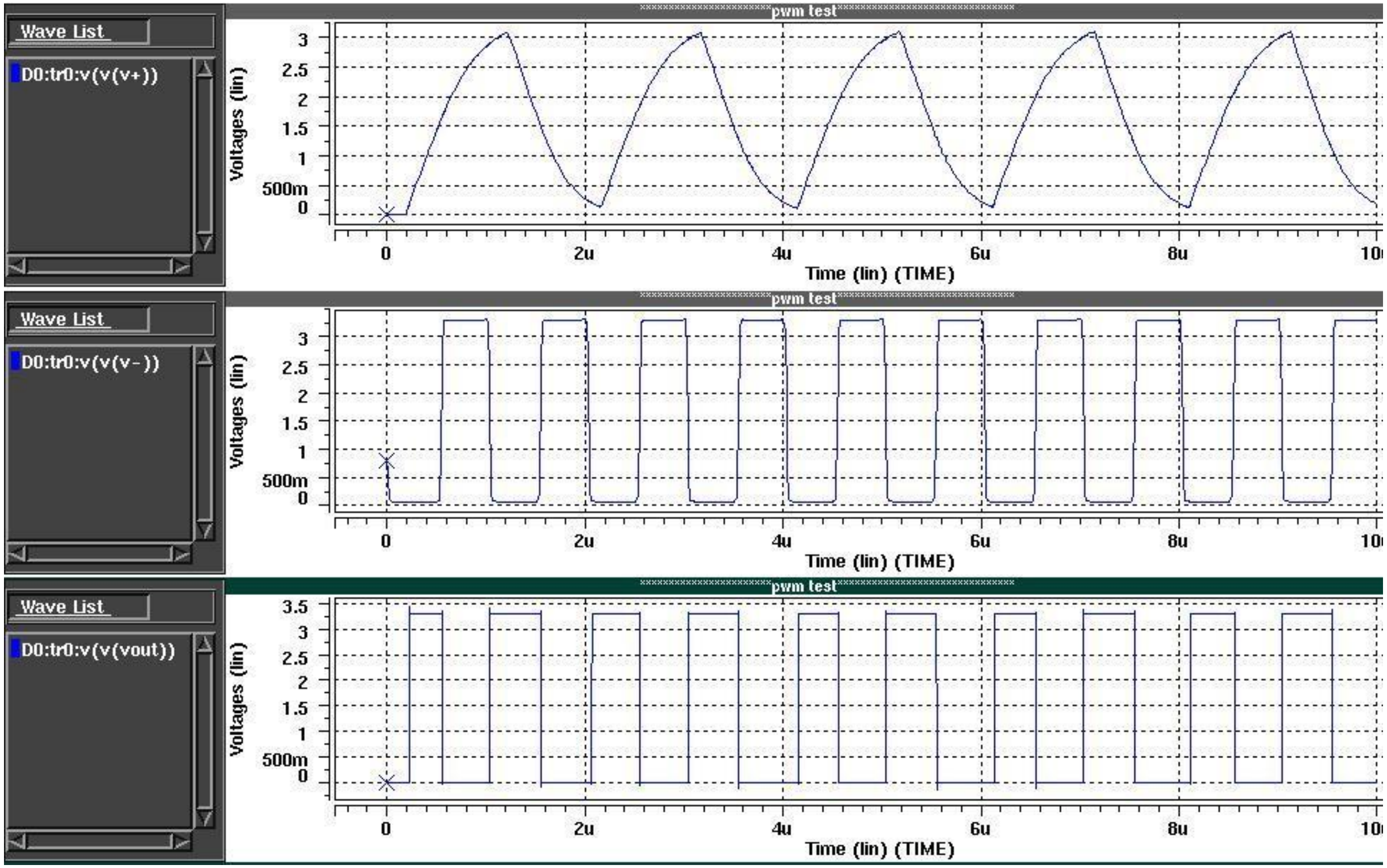
Oscillator-R/C Run PEX



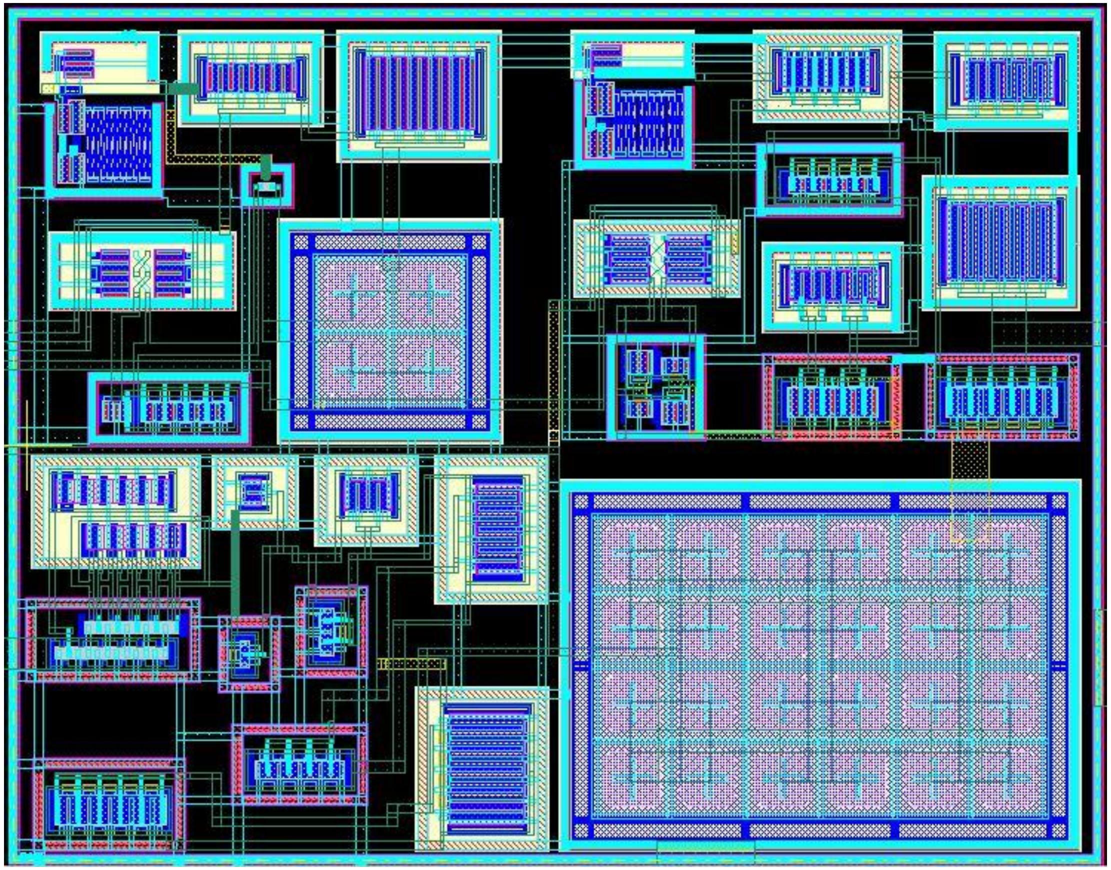
Oscillator-R/C Run PEX Success



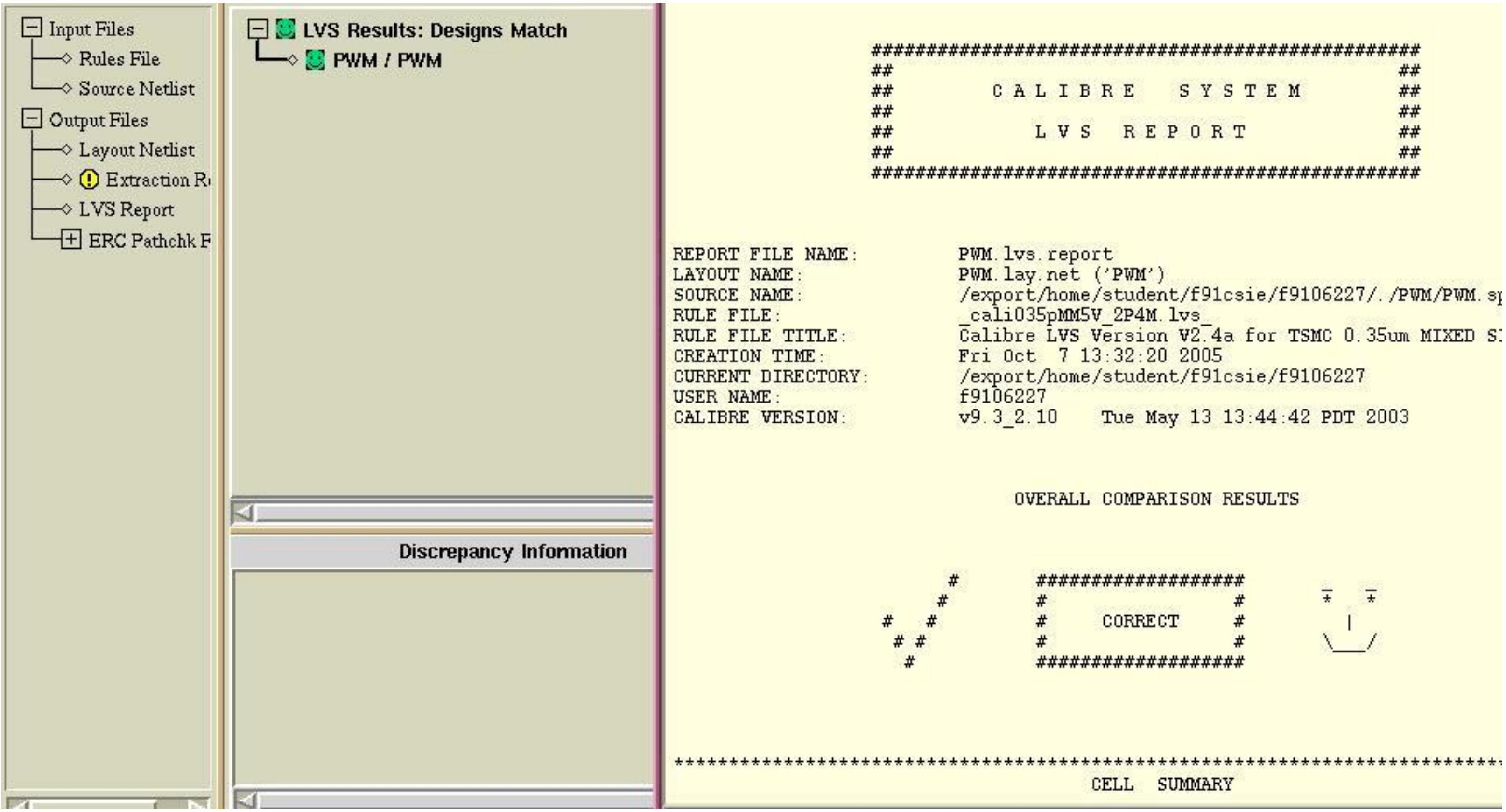
PWM-Pre-Simulation



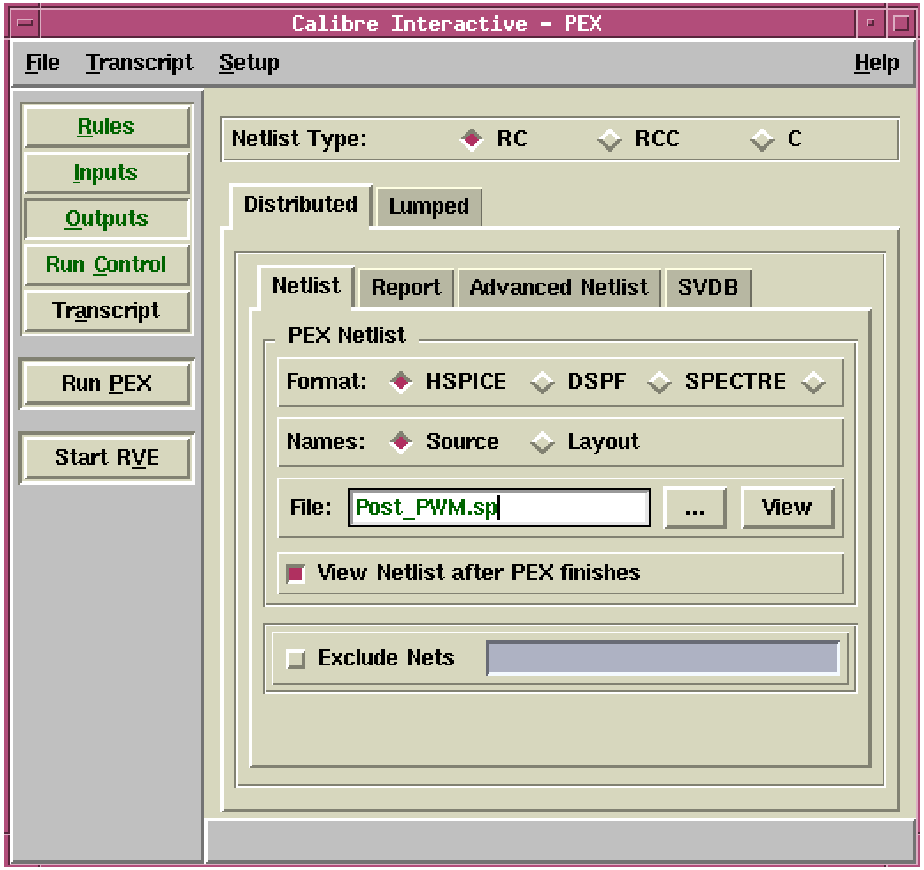
PWM-Layout



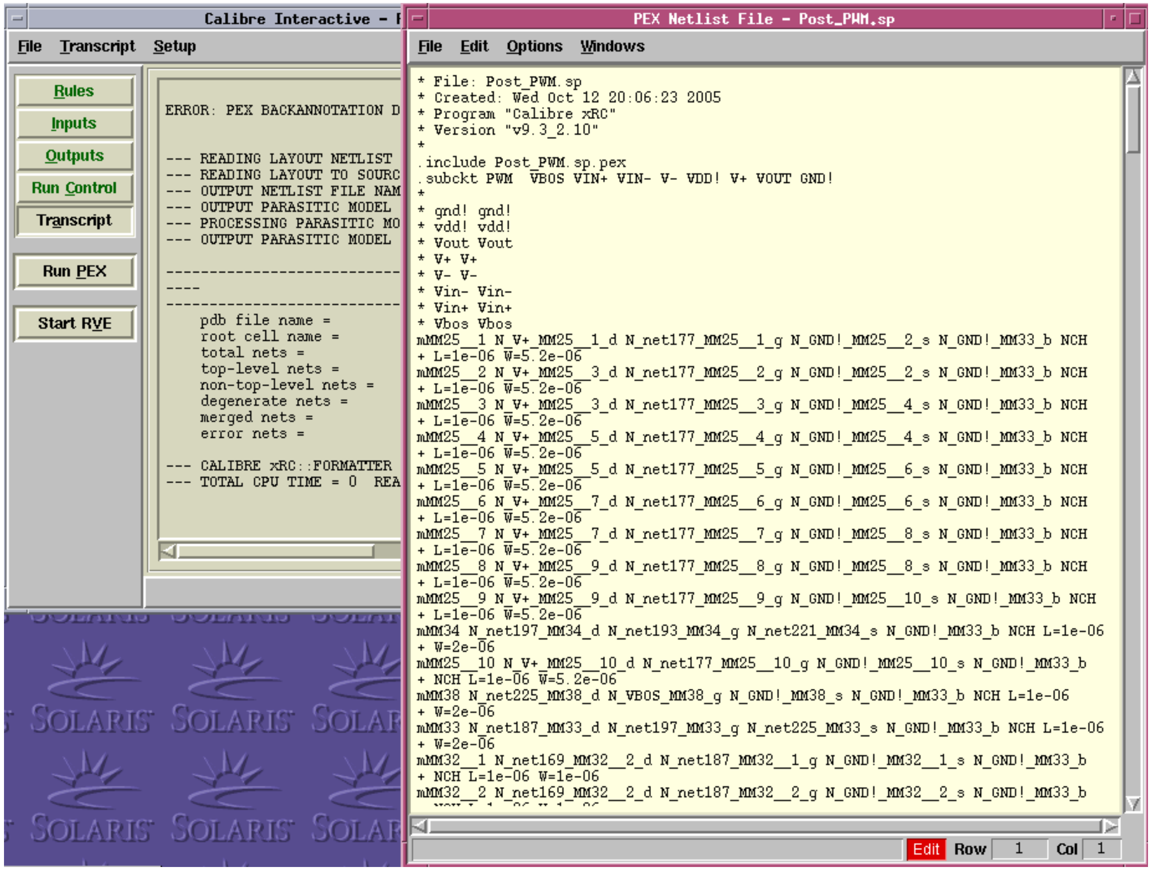
PWM-Lvs OK


Input Files
Rules File
Source Netlist
Output Files
Layout Netlist
Extraction R
LVS Report
ERC Pathchk F
LVS Results: Designs Match
PWM / PWM
Discrepancy Information
C A L I B R E   S Y S T E M
L V S   R E P O R T
REPORT FILE NAME: PWM.lvs.report
LAYOUT NAME: PWM.lay.net ('PWM')
SOURCE NAME: /export/home/student/f91csie/f9106227/./PWM/PWM.sp
RULE FILE: _cali035pMM5V_2P4M.lvs_
RULE FILE TITLE: Calibre LVS Version V2.4a for TSMC 0.35um MIXED SI
CREATION TIME: Fri Oct 7 13:32:20 2005
CURRENT DIRECTORY: /export/home/student/f91csie/f9106227
USER NAME: f9106227
CALIBRE VERSION: v9.3_2.10 Tue May 13 13:44:42 PDT 2003
OVERALL COMPARISON RESULTS
CORRECT
CELL SUMMARY

PWM-R/C Run PEX



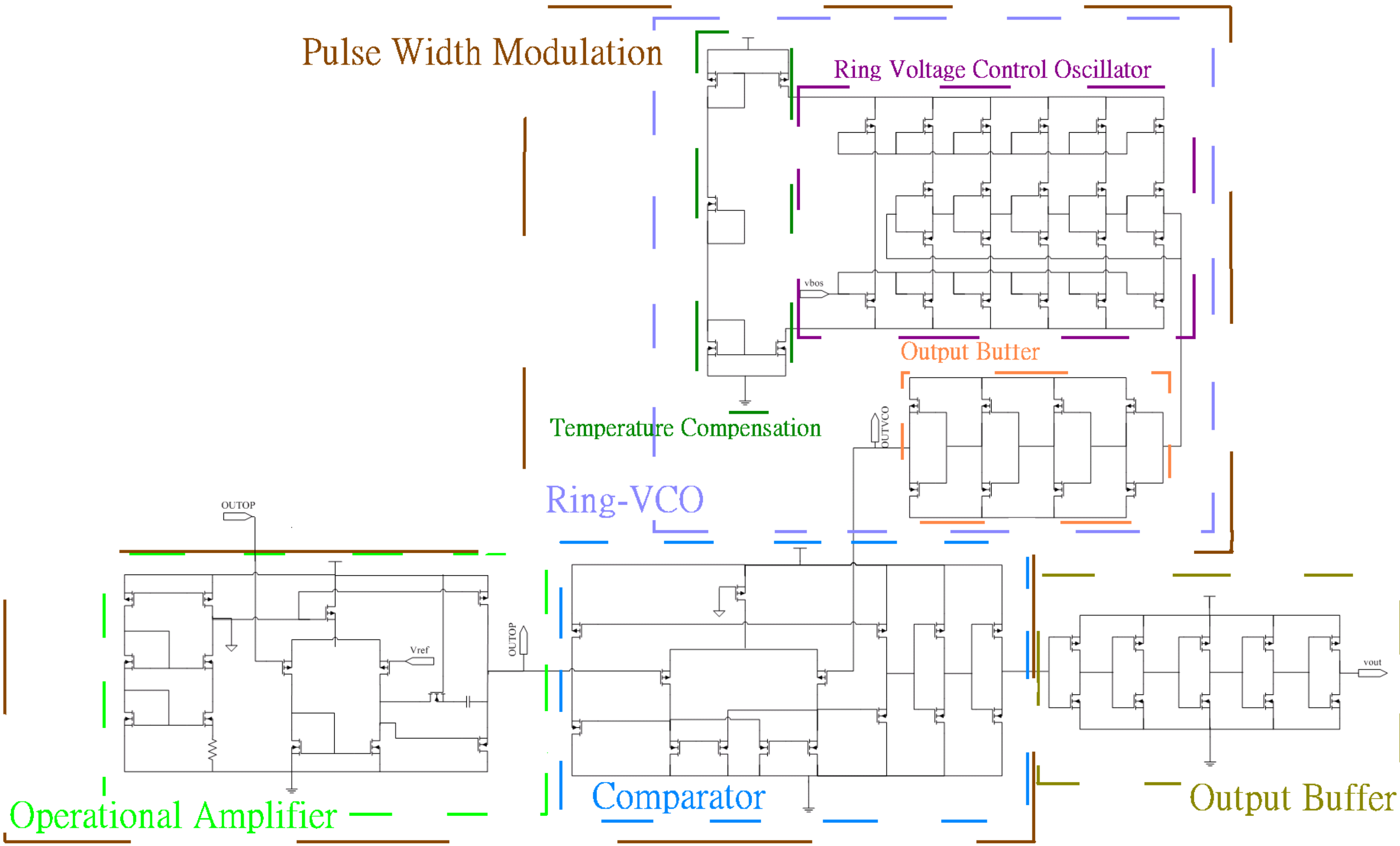
PWM-R/C Run PEX Success



Chapter 4. Final Project

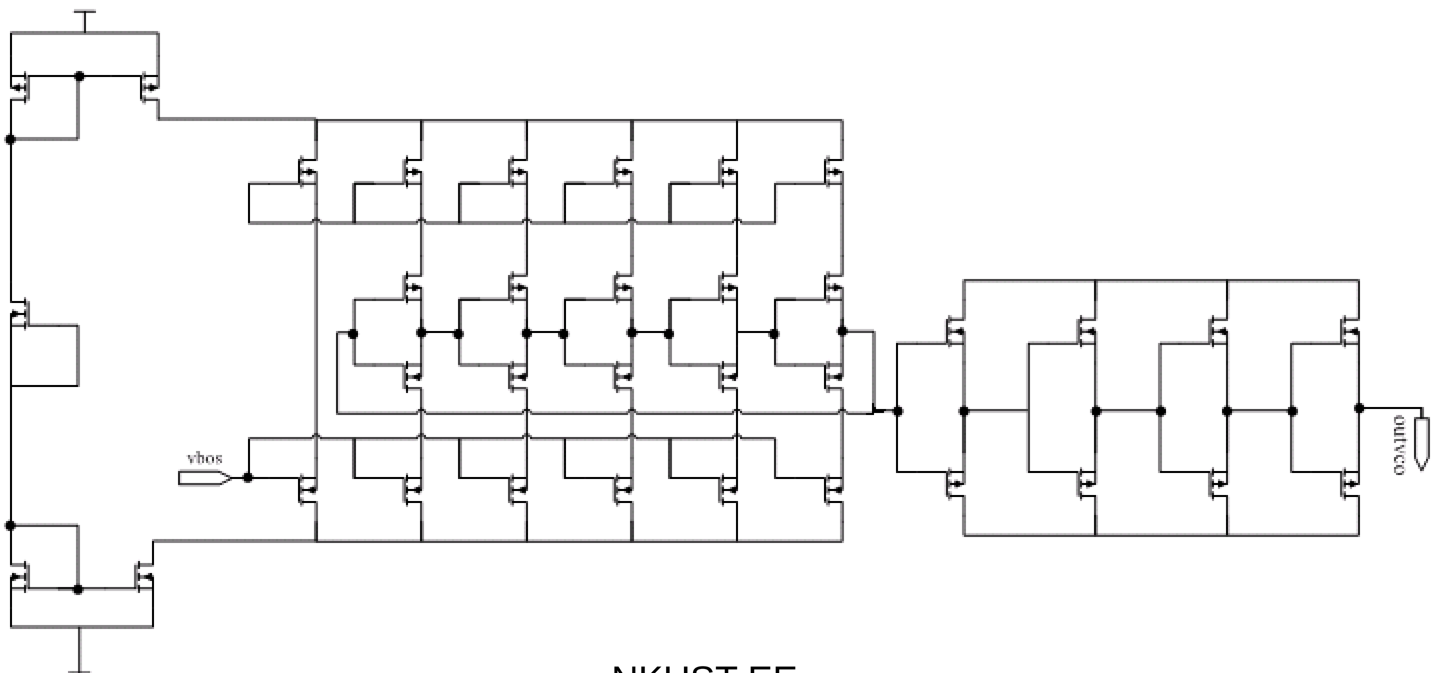
高雄第一科大電子系 郭永超副教授

PWM Design


Pulse Width Modulation
Ring Voltage Control Oscillator
vbos
Output Buffer
OUTVCO
Temperature Compensation
Ring-VCO
OUTOP
Vref
OUTOP
Operational Amplifier
Comparator
vout
Output Buffer

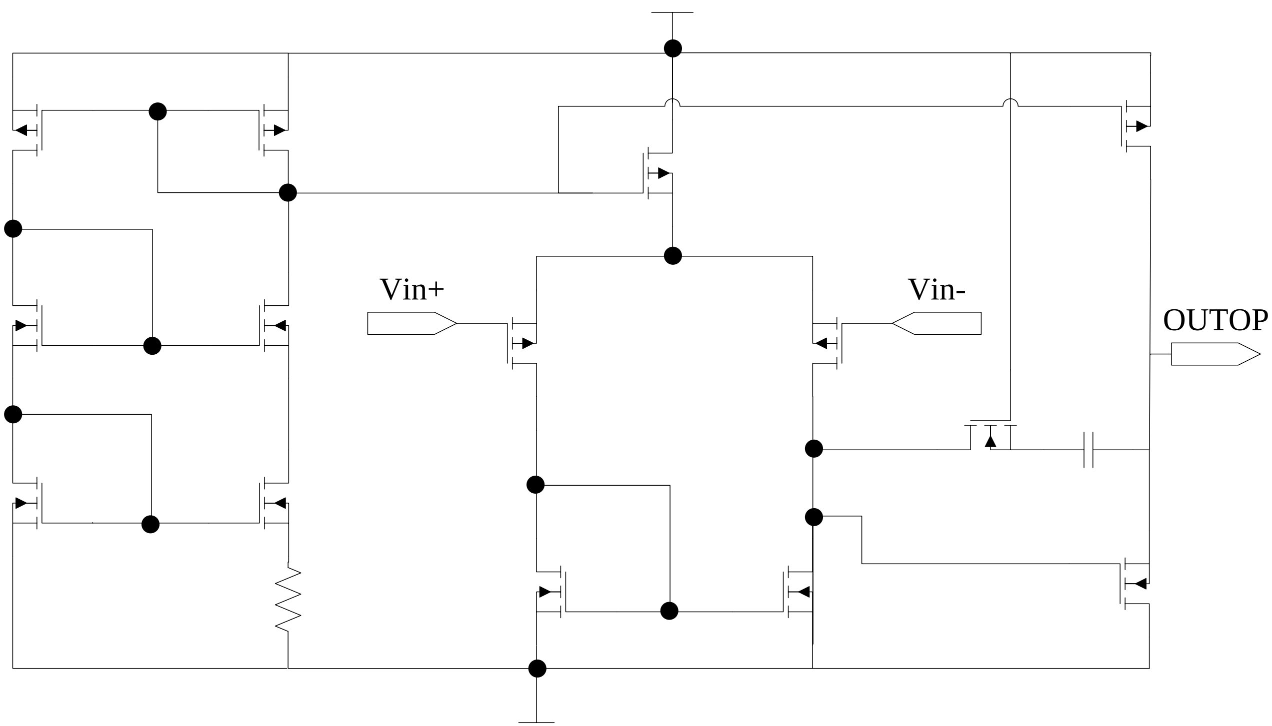
VCO

- 環型震盪器是利用奇數反向器串接，達到在輸出端產生固定頻率之方波，並且可透過vbos外接電壓訊號控制頻率，vbos電壓愈高，頻率愈高；反之vbos電壓愈低，頻率愈低。利用溫度補償的概念，將環型震盪器加以改良，使環型震盪器不易受溫度影響它的準確度。



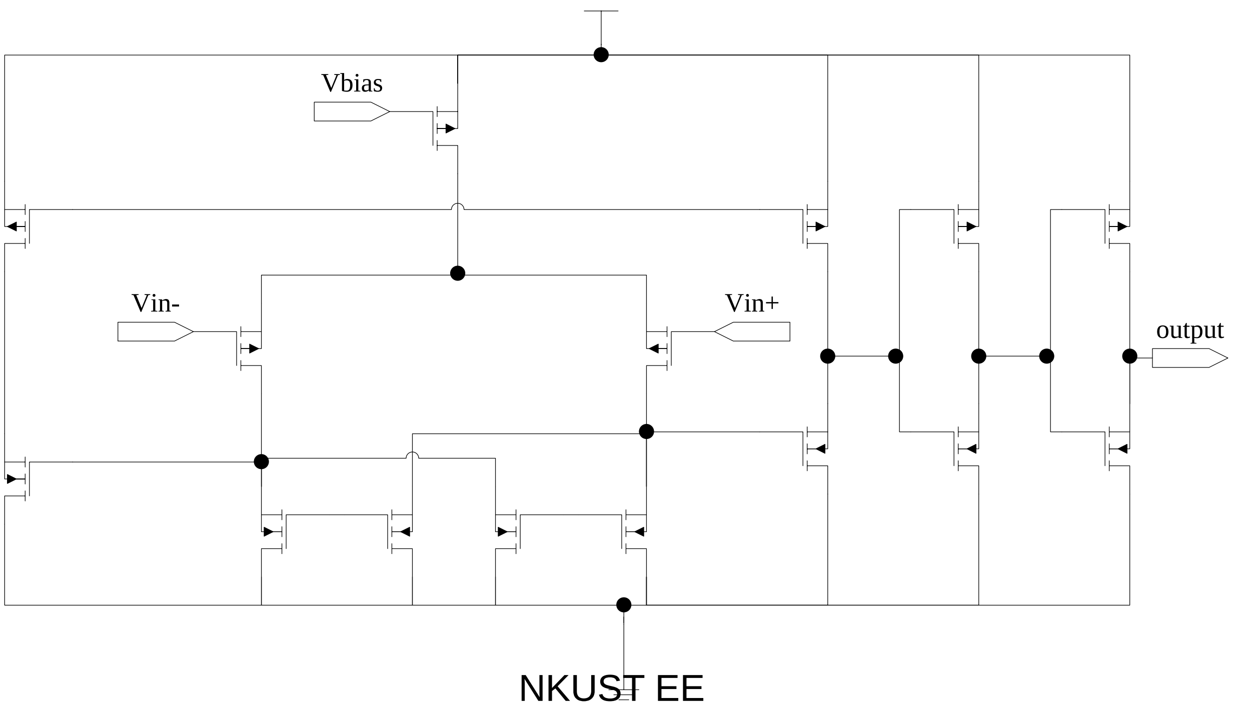
OP

- 將Converter之Sensor輸出值與參考電壓Vref作比較，並將差值放大



Comparator

- 以運算放大器輸出值與環狀電壓控制震盪器輸出值為輸入，比較兩者後輸出脈波訊號。



OP Design-Spec.

	Specification	Pre-sim	Post-sim
Av	Ao >= 70 db	70 db	72 db
Phase margin	ΦM>=45^o	48^o	53 ^{o}
Unit-gain frequency	Fo <= 200 MHz	97 MHz	71 MHz
Cc	< 4 pf	0.4 pf	0.4 pf
Slew rate	<= 250 v/μs	75 v/μs	72 v/μs
Offset	0v	0.31v	0.26v
ICMR	0v – 5v	0.31v – 4.67v	0.26v – 4.68v
PSRR	>60 db	73 db	76db
CMRR	>60 db	47 db	54 db
Power Dissipation	< 10mV	5.4768 mV	4.06 mV

OP Design (1/2)

$Let \quad SR = 250v/\mu 5 \ Cc = 0.4pF$

$\mu_p C_{ox} = 50\mu \ \ and \ \ \mu_n C_{ox} = 150\mu$

$250\mu = I_{D5}/0.4 \times 10^{-12}$

$I_{D5} = 100\mu 0$

$I_{D5} = 0.5 \times 50\mu \times (W/L)_5$

$(W/L)_5 = 16$

$I_{D3} = 0.5 \times 150\mu \times (W/L)_3$

$(W/L)_3 = (W/L)_4 = 2.7$

$f_0 = 200MHz, 2\pi \times f_0 = 1.256 \times 10^9$

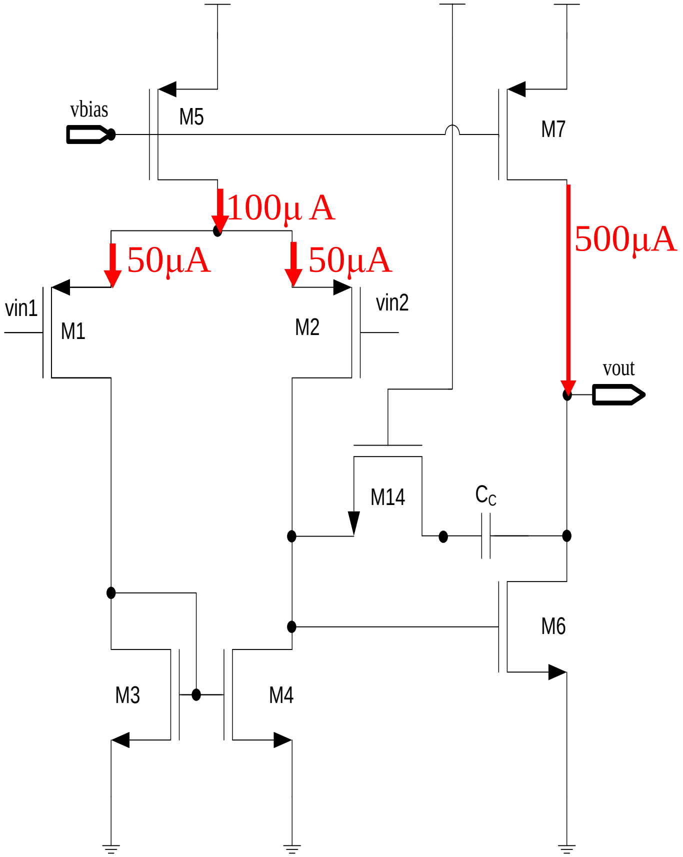
$gm1 = \omega_n C = 5.024 \times 10^{-4}$

$(gm1)^2 = 2 \times 50\mu \times (W/L)_1$

$(W/L)_1 = (W/L)_2 = 50.5$

$$(W/L)_6 = \frac{I_{D6}}{I_{D3}} \times (W/L)_3 = 27$$

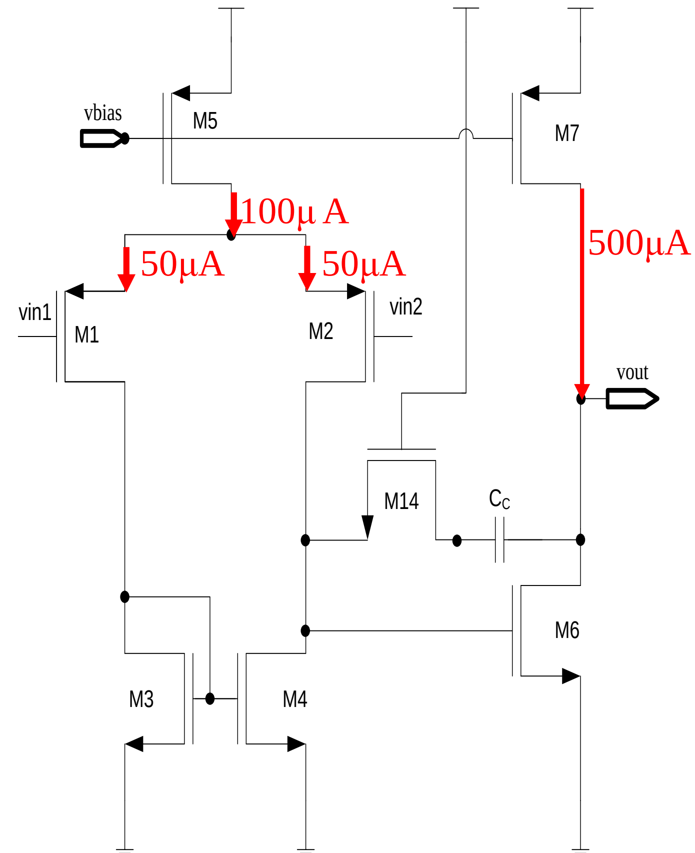
$$(W/L)_7 = \frac{500 \times 10^{-6}}{0.5 \times 50\mu \times 0.25} = 80$$



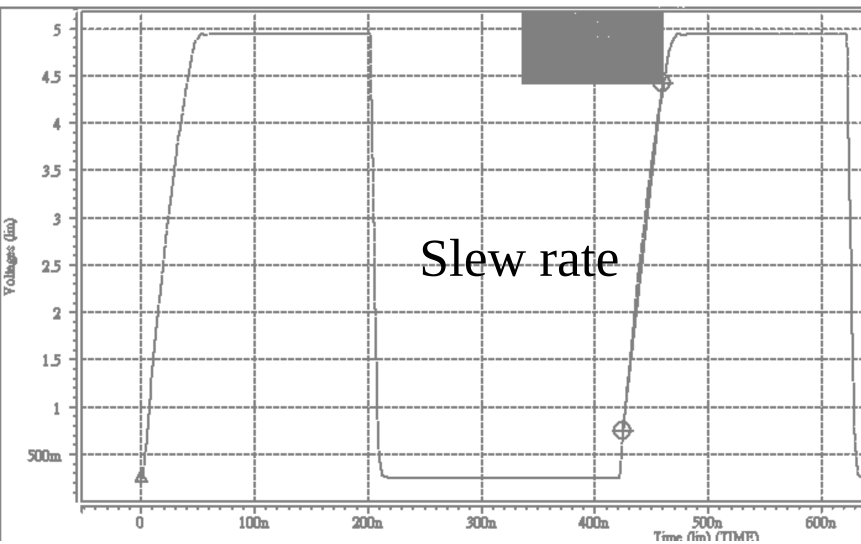
OP Design (2/2)

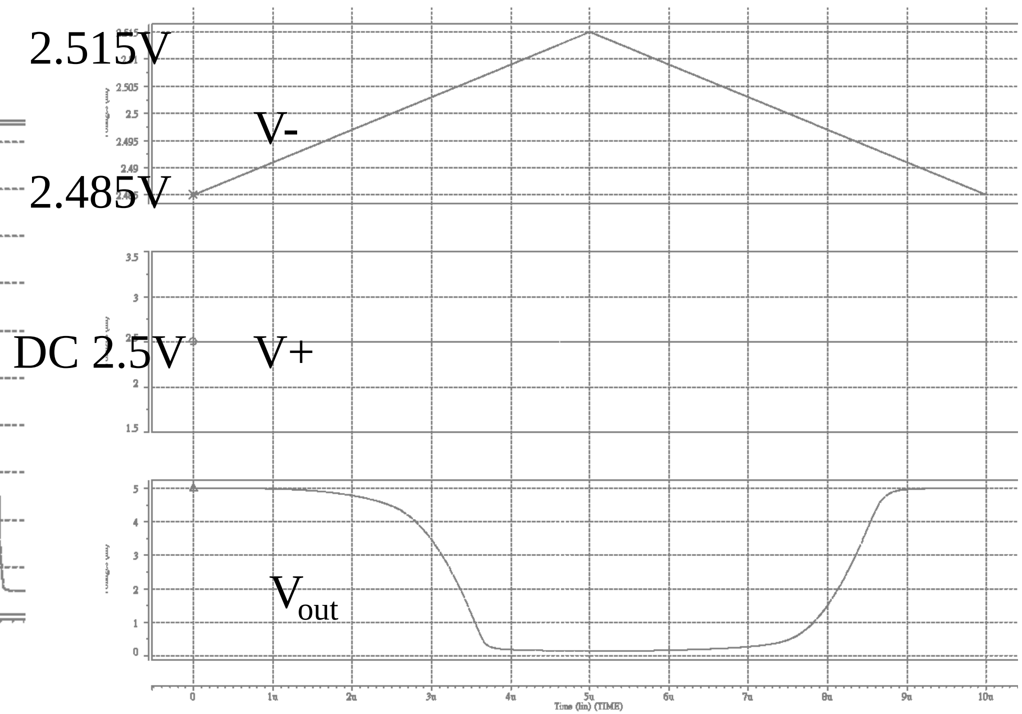
```
*Differential pair
M1    net2   vin-   net1   net1   PCH5   W=13u   L=1u   M=4
M2    net3   vin+   net1   net1   PCH5   W=13u   L=1u   M=4
M3    net2   net2   gnd!   gnd!   NCH5   W=4.5u  L=1u   M=1
M4    net3   net2   gnd!   gnd!   NCH5   W=4.5u  L=1u   M=1
M5    net1   net036 vdd!   vdd!   PCH5   W=4u    L=1u   M=4

*Single-Stage Amplifier
M6    vout   net3   gnd!   gnd!   NCH5   W=6u    L=1u   M=8
M7    vout   net036 vdd!   vdd!   PCH5   W=11u   L=1u   M=7
*Compensation
M14   net4   vdd!   net3   gnd!   NCH5   W=1.8u  L=1u   M=1
Ccc   net4   vout   0.4pF
```

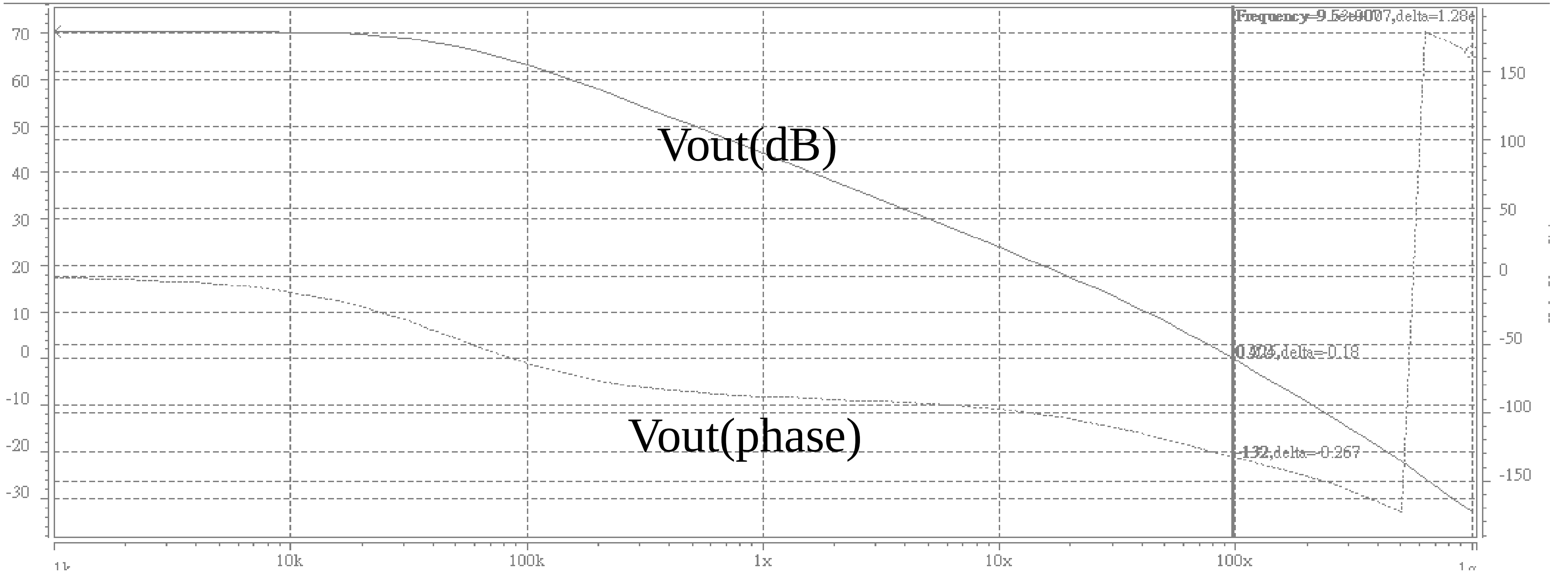


Simulation of OP (1/2)

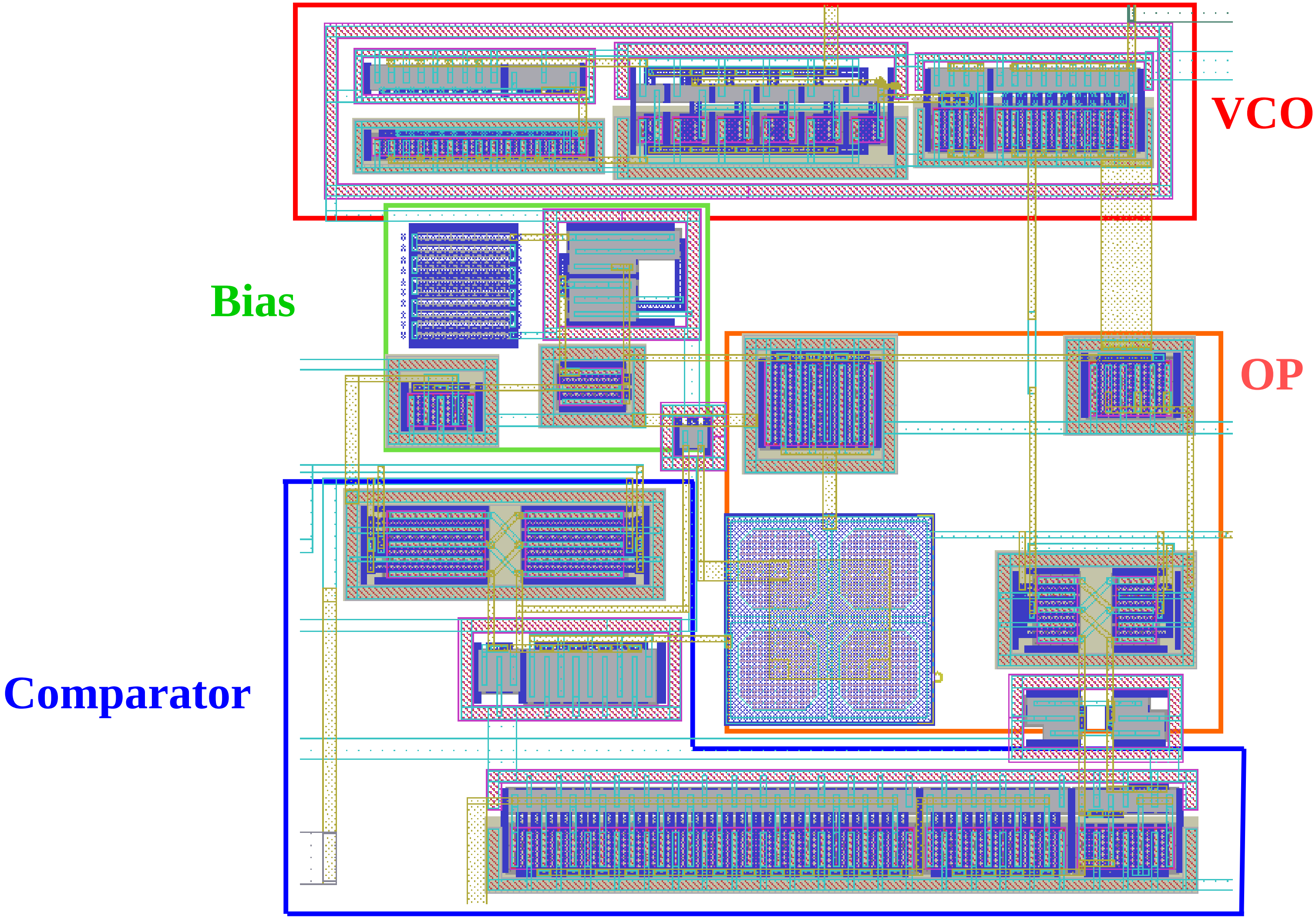




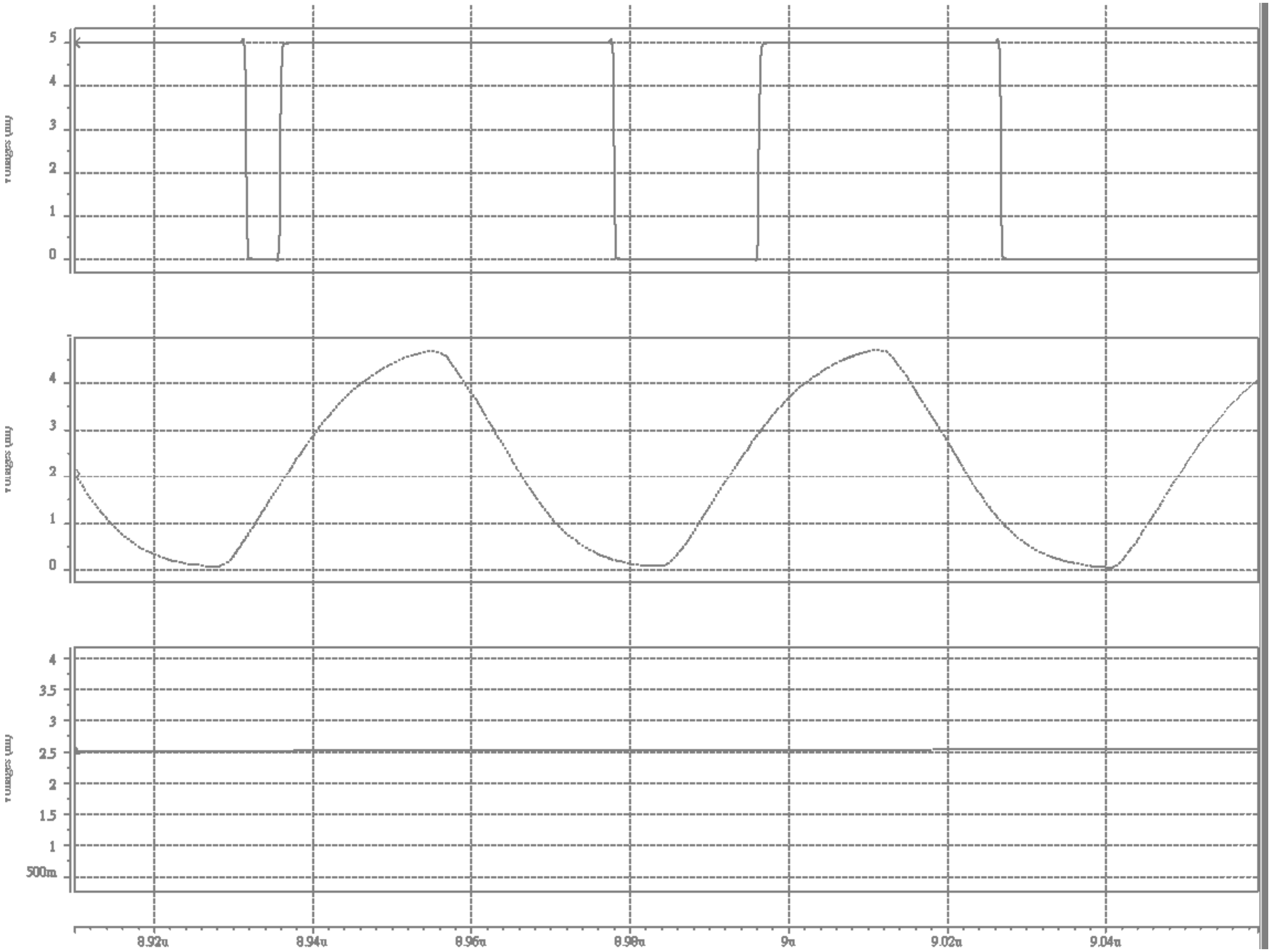
Simulation of OP (2/2)



Layout of PWM

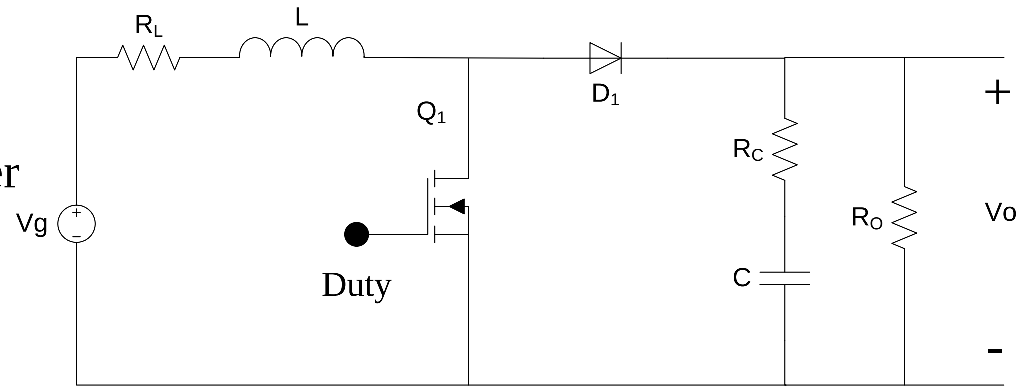


Simulation of PWM



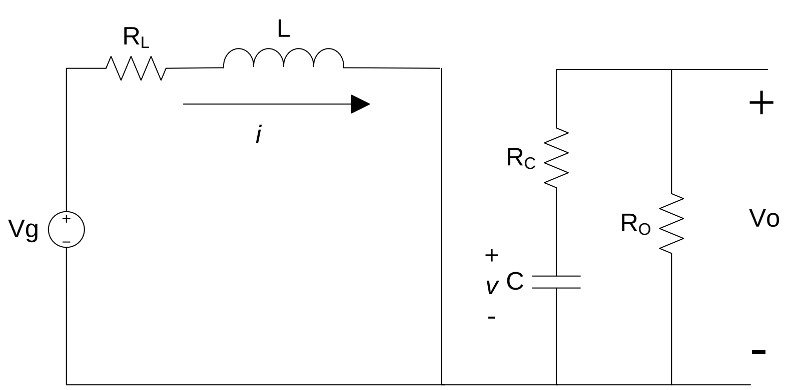
Boost’s Transfer Function in CCM Mode (1/3)

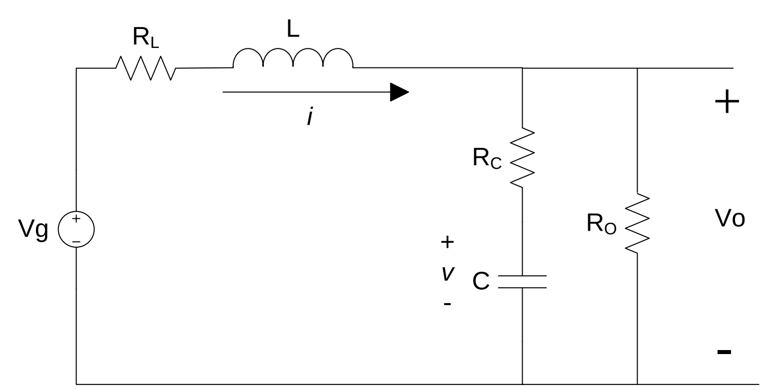
basic boost converter



switch in position 1

switch in position 2





Boost's Transfer Function in CCM Mode (2/3)

Analysis of boost converter in continuous-conduction mode (by KVL):

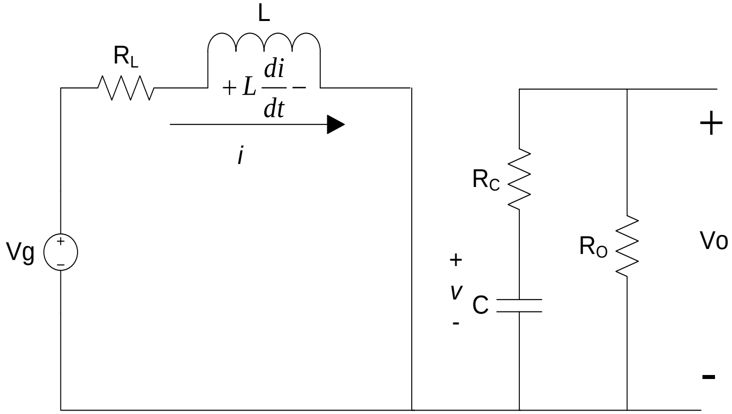
Subinterval 1: switch on

$$L\frac{di}{dt} = -(R_L)i + v_g$$

$$C\frac{dv}{dt} = \frac{v}{R_O + R_C}$$

$$V_O = \left(\frac{R_O}{R_O + R_C}\right)v$$

$$i_i = i$$



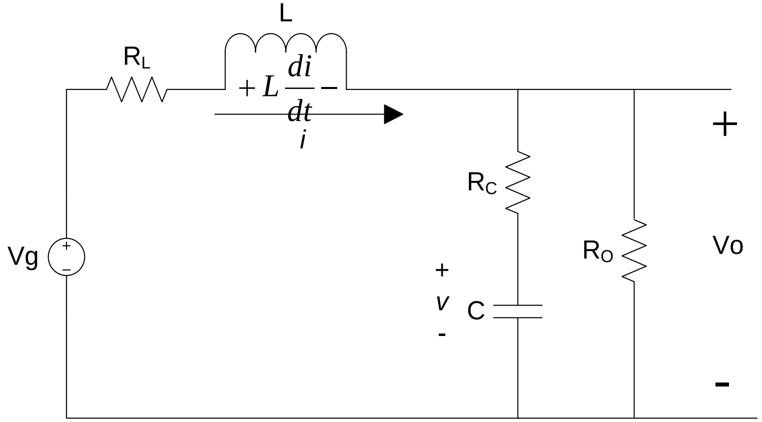
Subinterval 2: switch off

$$L\frac{di}{dt} = -\left(R_L + \frac{R_O R_C}{R_O + R_C}\right)i - \left(\frac{R_O}{R_O + R_C}\right)v + v_g$$

$$C\frac{dv}{dt} = \frac{R_O}{R_O + R_C}i - \frac{v}{R_O + R_C}$$

$$V_O = \left(\frac{R_O R_C}{R_O + R_C}\right)i + \left(\frac{R_O}{R_O + R_C}\right)v$$

$$i_i = i$$



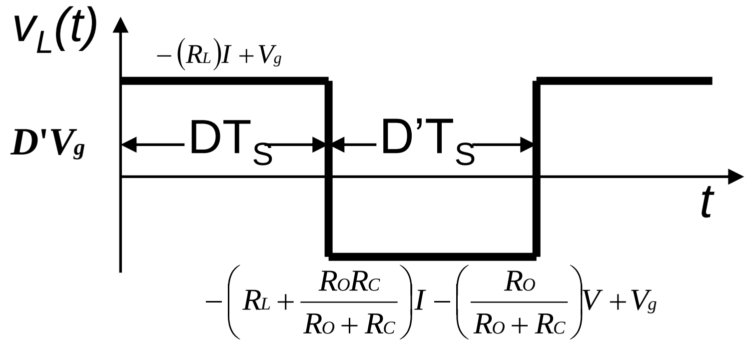
Boost's Transfer Function in CCM Mode (3/3)

Volt-second balance

$$\langle v_L \rangle = -DIR_L + DV_i - D'IR_L + D'\frac{R_O R_C}{R_O + R_C}I - D'\left(\frac{R_O}{R_O + R_C}\right)V + D'V_g$$

$$= -\left(R_L + D'\frac{R_O R_C}{R_O + R_C}\right)I - \left(\frac{D' R_O}{R_O + R_C}\right)V + V_g$$

$$V_o = D'\left(\frac{R_O R_C}{R_O + R_C}\right)I + \left(\frac{R_O}{R_O + R_C}\right)V$$

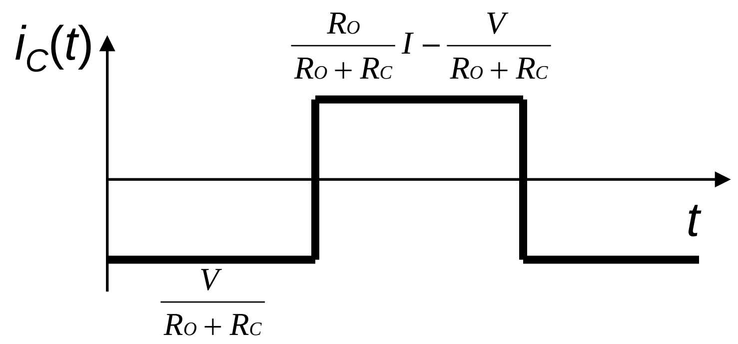


Capacitor charge balance

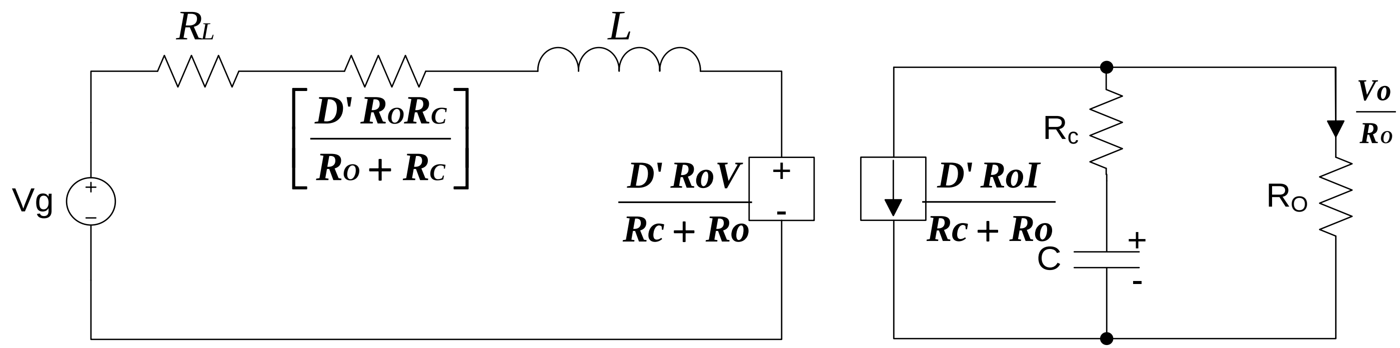
$$\langle i_c \rangle = -\frac{DV}{R_O + R_C} + \frac{D' R_O}{R_O + R_C}I - \frac{D'V}{R_O + R_C}$$

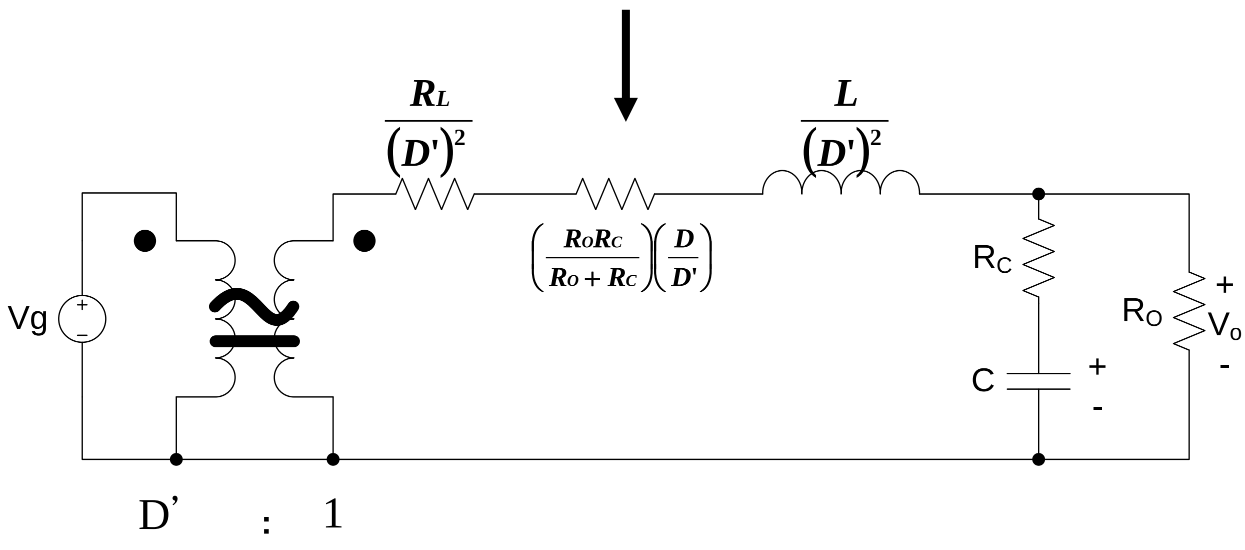
$$= \frac{D' R_O}{R_O + R_C}I - \frac{V}{R_O + R_C}$$

$$I_i = I$$



Equivalent Circuits (1/2)





Equivalent Circuits (2/2)

$$-\left(R_L + D'\frac{R_O R_C}{R_O + R_C}\right)I - \left(\frac{D' R_O}{R_O + R_C}\right)V + Vg = 0 \quad (1)$$

$$R_O D' I - V = 0; I = \frac{V}{R_O D'} \quad (2)$$

$$Vo = D'\left(\frac{R_O R_C}{R_O + R_C}\right)I + \left(\frac{R_O}{R_O + R_C}\right)V \quad (3)$$

$$Ii = I$$

$$Vo = V$$

$$Vg = \frac{R_L V}{R_O D'} + \frac{R_C V}{R_O + R_C} + \left(\frac{D' R_O}{R_O + R_C}\right)V \quad (2) \rightarrow (1)$$

$$Vg = Vo\left(\frac{R_L}{R_O D'} + \frac{R_C}{R_O + R_C} + \frac{D' R_O}{R_O + R_C}\right) \quad (2) \rightarrow (3)$$

$$\frac{Vo}{Vg} = \frac{1}{D'}\left[\frac{1}{\frac{R_L}{R_O D'^2} + \frac{R_C}{(R_O + R_C)D'} + \frac{R_O}{R_O + R_C}}\right]$$

If $Ro >> Rc$; $Ro >> R_L$ then

$$\frac{Vo}{Vg} = \frac{1}{D'}$$

Inductor and Capacitor

$$L\frac{\Delta I_L}{dt} = Vg$$

$$\Delta I_L = \frac{Vg}{L} DTs$$

Solve for peak ripple

$$2\Delta I_L = \frac{Vg}{L} DTs$$

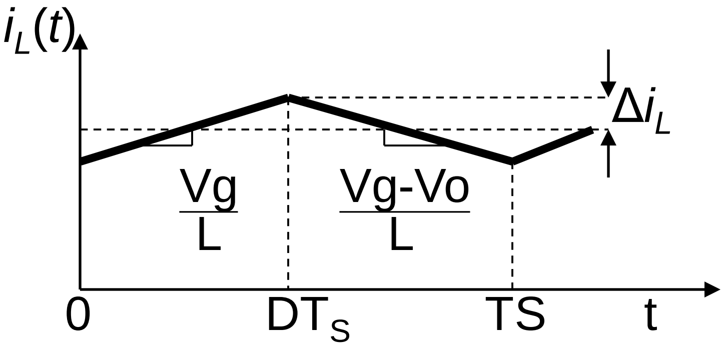
The devices L calculated as follow :

$$L = \frac{VgDTs}{2\Delta I_L}$$

$$\frac{dvc}{dt} = \frac{ic}{C} = \frac{-Vo}{RC} = \frac{I}{C} - \frac{V}{RC}$$

The devices C calculated as follow :

$$C = \frac{VDTs}{2R\Delta Vo}$$



Transient Steady

- Perturbation and linearization
- State-space averaging
- Simplified analysis of PWM converters using model of PWM switch

Perturbation and Linearization (1/4)

The averaged converter equations

$$L\frac{di}{dt} = -\left(R_L + d'\frac{R_O R_C}{R_O + R_C}\right)i - \left(\frac{d' R_O}{R_O + R_C}\right)v + v_g$$

$$C\frac{dv}{dt} = \frac{d' R_O}{R_O + R_C}i - \frac{v}{R_O + R_C}$$

$$v_o = d'\left(\frac{R_O R_C}{R_O + R_C}\right)i + \left(\frac{R_O}{R_O + R_C}\right)v$$

$$i_i = i$$

Next step: perturbation and linearization.

$$\langle Vg(t)\rangle_{Ts} = \mathrm{Vg} + \hat{\mathrm{v}}_{\mathrm{g}}$$

$$\mathrm{d(t)} = \mathrm{D} + \hat{\mathrm{d}}$$

$$\langle \mathrm{i(t)}\rangle_{\mathrm{Ts}} = \mathrm{I} + \hat{\mathrm{i}}$$

$$\langle \mathrm{v(t)}\rangle_{\mathrm{Ts}} = \mathrm{V} + \hat{\mathrm{v}}$$

$$\langle i_i(\mathrm{t})\rangle_{\mathrm{Ts}} = \mathrm{Ii} + \hat{\mathrm{i}}_{\mathrm{i}}$$

$$\mathrm{d'} = 1 - \left(\mathrm{D} + \hat{\mathrm{d}}\right) = \mathrm{D'} - \hat{\mathrm{d}}$$

Substitution to the averaged converter equations

Perturbation and Linearization (2/4)

Perturbation of the averaged inductor equation

$$L\frac{d\left(I+\hat{i}\right)}{dt}=-\left(R_L+\left(D'-\hat{d}\right)\left(\frac{R_O R_C}{R_O+R_C}\right)\right)\left(I+\hat{i}\right)-\left(\frac{\left(D'-\hat{d}\right)R_O}{R_O+R_C}\right)\left(V+\hat{v}\right)+V_g+\hat{v}_g$$

$$=-IR_L-\hat{i}R_L-D'\left(\frac{R_O R_C}{R_O+R_C}\right)I+\hat{d}\left(\frac{R_O R_C}{R_O+R_C}\right)I-D'\left(\frac{R_O R_C}{R_O+R_C}\right)\hat{i}+\hat{d}\left(\frac{R_O R_C}{R_O+R_C}\right)\hat{i}-\left(\frac{D'R_O}{R_O+R_C}\right)V+\left(\frac{\hat{d}(t)R_O}{R_O+R_C}\right)V-\left(\frac{D'R_O}{R_O+R_C}\right)\hat{v}+\left(\frac{\hat{d}R_O}{R_O+R_C}\right)\hat{v}+V_g+\hat{v}_g$$

$$=\underbrace{\left(V_g-IR_L-D'\left(\frac{R_O R_C}{R_O+R_C}\right)I-\left(\frac{D'R_O}{R_O+R_C}\right)V\right)}_{Dc\ terms}+\underbrace{\left(\hat{d}\left(\frac{R_O R_C}{R_O+R_C}\right)I-D'\left(\frac{R_O R_C}{R_O+R_C}\right)\hat{i}+\left(\frac{\hat{d}R_O}{R_O+R_C}\right)V-\left(\frac{D'R_O}{R_O+R_C}\right)\hat{v}+\hat{v}_g-\hat{i}R_L\right)}_{\substack{1st\ order\ ac\ terms \\ (linear)}}+\underbrace{\left(\hat{d}\left(\frac{R_O R_C}{R_O+R_C}\right)\hat{i}+\left(\frac{\hat{d}R_O}{R_O+R_C}\right)\hat{v}\right)}_{\substack{2nd\ order\ ac\ terms \\ (nonlinear)}}$$

Upon discarding second-order terms, and removing dc terms (which add to zero), we are left with

$$L\frac{d\hat{i}}{dt}=\left(\frac{\hat{d}R_O R_C}{R_O+R_C}\right)I+\left(\frac{\hat{d}R_O}{R_O+R_C}\right)V-\left(R_L+\frac{D'R_O R_C}{R_O+R_C}\right)\hat{i}-\left(\frac{D'R_O}{R_O+R_C}\right)\hat{v}+\hat{v}_g$$

$$L\frac{d\hat{i}}{dt}=-\left[R_L+\frac{D'R_O R_C}{R_O+R_C}\right]\hat{i}-\left[\frac{D'R_O}{R_O+R_C}\right]\hat{v}+\left[\left(\frac{R_O R_C}{R_O+R_C}\right)I+\left(\frac{R_O}{R_O+R_C}\right)V\right]\hat{d}+\hat{v}_g$$

Perturbation and Linearization (3/4)

Perturbation of the averaged capacitor equation

$$C\frac{d(V+\hat{v})}{dt}=\left(\frac{(D'-\hat{d})R_O}{R_O+R_C}\right)(I+\hat{i})-\frac{(V+\hat{v})}{R_O+R_C}$$

$$=\underbrace{\left(\frac{D'R_O}{R_O+R_C}I-\frac{V}{R_O+R_C}\right)}_{DC\ terms}+\underbrace{\left(\frac{D'R_O}{R_O+R_C}\right)\hat{i}-\frac{\hat{d}R_O}{R_O+R_C}I-\frac{\hat{v}}{R_O+R_C}}_{\substack{1st\ order\ ac\ terms\\ (linear)}}-\underbrace{\left(\frac{\hat{d}R_O}{R_O+R_C}\hat{i}\right)}_{\substack{2nd\ order\ ac\ terms\\ (nonlinear)}}$$

Upon discarding second-order terms, and removing dc terms (which add to zero), we are left with

$$C\frac{d\hat{v}}{dt}=\left(\frac{D'R_O}{R_O+R_C}\right)\hat{i}-\left(\frac{1}{R_O+R_C}\right)\hat{v}-\left(\frac{IR_O}{(R_O+R_C)}\right)\hat{d}$$

Perturbation and Linearization (4/4)

Perturbation of the average input current and output voltage

$$V_O + \hat{v}_o = \left(D' - \hat{d}\right)\left(\frac{R_O R_C}{R_O + R_C}\right)\left(I + \hat{i}\right) + \left(\frac{R_O}{R_O + R_C}\right)\left(V + \hat{v}\right)$$

$$= \underbrace{\left(\frac{D'\left(R_O R_C\right)}{R_O + R_C} I + \frac{R_O V}{R_O + R_C}\right)}_{DC \quad terms} + \underbrace{\frac{D'\left(R_O R_C\right)}{R_O + R_C}\hat{i} - \frac{\hat{d}\left(R_O R_C\right)}{R_O + R_C} I + \frac{R_O \hat{v}}{R_O + R_C}}_{\substack{1nd \quad order \quad ac \quad terms \\ (linear)}} - \underbrace{\left(\frac{\hat{d}\left(R_O R_C\right)}{R_O + R_C}\hat{i}\right)}_{\substack{2nd \quad order \quad ac \quad terms \\ (nonlinear)}}$$

Upon discarding second-order terms, and removing dc terms (which add to zero), we are left with

$$\hat{v}_o = \left[\frac{D'\left(R_O R_C\right)}{R_O + R_C}\right]\hat{i} + \left[\frac{R_O}{R_O + R_C}\right]\hat{v} - \left[\frac{I R_O R_C}{\left(R_O + R_C\right)}\right]\hat{d}$$

$$I_i + \hat{i}_i = I + \hat{i}$$

removing dc terms

$$\hat{i}_i = \hat{i}$$

Small-Signal AC Equations

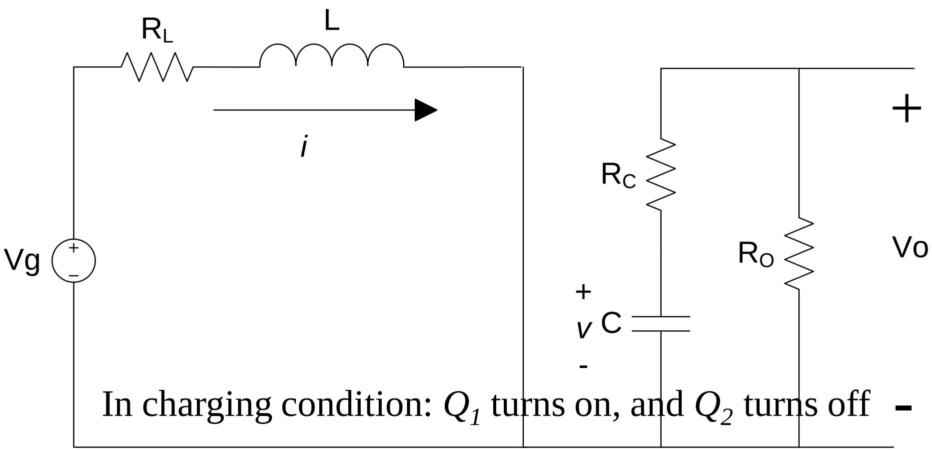
$$L\frac{d\hat{i}}{dt} = -\left[R_L + \frac{D'R_O R_C}{R_O + R_C}\right]\hat{i} - \left[\frac{D'R_O}{R_O + R_C}\right]\hat{v} + \left[\left(\frac{R_O R_C}{R_O + R_C}\right)I + \left(\frac{R_O}{R_O + R_C}\right)V\right]\hat{d} + \hat{v}_g$$

$$C\frac{d\hat{v}}{dt} = \left[\frac{D'R_O}{R_O + R_C}\right]\hat{i} - \left[\frac{1}{R_O + R_C}\right]\hat{v} - \left[\frac{IR_O}{(R_O + R_C)}\right]\hat{d}$$

$$\hat{v}_o = \left[\frac{D'(R_O R_C)}{R_O + R_C}\right]\hat{i} + \left[\frac{R_O}{R_O + R_C}\right]\hat{v} - \left[\frac{IR_O R_C}{(R_O + R_C)}\right]\hat{d}$$

$$\hat{i}_i = \hat{i}$$

State-Space Averaging (1/13)



In charging condition: Q_1 turns on, and Q_2 turns off

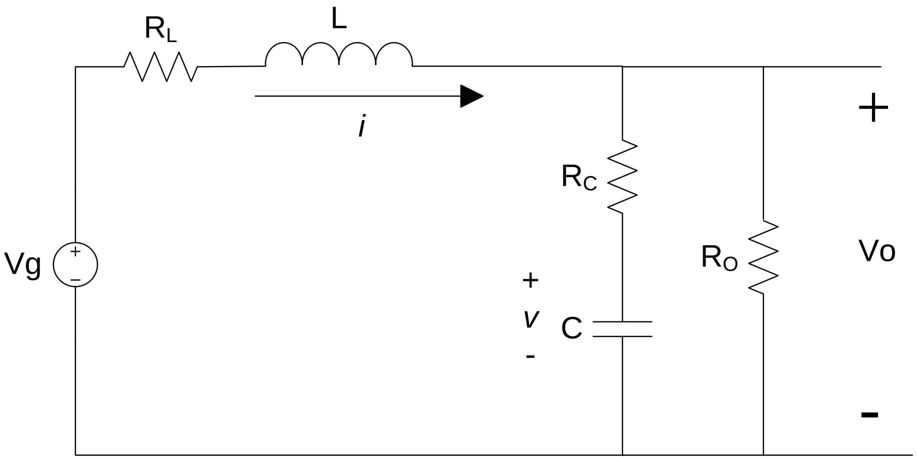
$$L\frac{di}{dt} = Vg - iR_L$$

$$C\frac{dv}{dt} = -\left(\frac{v}{R_C + R_O}\right)$$

$$\frac{di}{dt} = \frac{Vg}{L} - \frac{iR_L}{L} \qquad (1)$$

$$\frac{dv}{dt} = -\left(\frac{v}{(R_C + R_O)C}\right) \qquad (2)$$

$$V_O = \left(\frac{R_O}{R_C + R_O}\right)v \qquad (3)$$



In discharging condition: Q_1 turns off, and Q_2 turns on

$$L\frac{di}{dt} = \left(R_L + \frac{R_O R_C}{R_O + R_C}\right)i - \left(\frac{R_O}{R_O + R_C}\right)v + Vg$$

$$C\frac{dv}{dt} = \left(\frac{R_O}{R_O + R_C}\right)i - \frac{v}{R_O + R_C}$$

$$\frac{di}{dt} = \left(\frac{R_L + \frac{R_O R_C}{R_O + R_C}}{L}\right)i - \frac{1}{L}\left(\frac{R_O}{R_O + R_C}\right)v + \frac{Vg}{L} \qquad (4)$$

$$\frac{dv}{dt} = \left[\frac{R_O}{(R_O + R_C)C}\right]i - \frac{v}{(R_O + R_C)C} \qquad (5)$$

$$Vo = \left(\frac{R_O R_C}{R_O + R_C}\right)i - \left(\frac{R_O}{R_O + R_C}\right)v \qquad (6)$$

State-Space Averaging (2/13)

We change equ.(1), (2) and (3) into state space functions:

$$\dot{x} = A_1 x + B_1 u$$
$$y = C_1 x + E_1 u$$

$x^T = [i \quad v]$ state vector $\quad u^T = [V_g]$ input vector $\quad y^T = [V_o]$ output vector

$\text{A} \longrightarrow$ System Matrix $\quad \text{B} \longrightarrow$ Input Matrix

$\text{C} \longrightarrow$ Output Matrix $\quad \text{E} \longrightarrow$ Feedforward Matrix

D is replace of E, so we dont confuse about duty and feedforward Matrix

The state equation is as follows

$$\dot{x} = \begin{bmatrix} \dot{i} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -\frac{R_{L1}}{L} & 0 \\ 0 & -\left[\frac{1}{(R_C + R_O)C}\right] \end{bmatrix} \cdot \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cdot [V_g] = A_1 x + B_1 u$$

$$y = [V_o] = \begin{bmatrix} 0 & \frac{R_O}{R_C + R_O} \end{bmatrix} \cdot \begin{bmatrix} i \\ v \end{bmatrix} + [0] \cdot [V_g] = C_1 x$$

the constant vectors are

$$A_1 = \begin{bmatrix} -\frac{R_L}{L} & 0 \\ 0 & -\left[\frac{1}{(R_C + R_O)C}\right] \end{bmatrix} \quad B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \quad C_1 = \begin{bmatrix} 0 & \frac{R_O}{R_C + R_O} \end{bmatrix}$$

State-Space Averaging (3/13)

In the same way, We change equ.(4), (5) and (6) into state space functions:

$$\dot{x} = A_2 x + B_2 u$$

$$y = C_2 x$$

The state equation is as follows

$$\dot{x} = \begin{bmatrix} \dot{i} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_L + \dfrac{R_O R_C}{R_C + R_O}}{L} & -\dfrac{R_O}{(R_C + R_O)L} \\ \dfrac{R_O}{(R_C + R_O)C} & -\left[\dfrac{1}{(R_C + R_O)C}\right] \end{bmatrix} \cdot \begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \cdot [V_g] = A_1 x + B_1 u$$

$$y = [V_o] = \begin{bmatrix} \dfrac{R_O R_C}{R_C + R_O} & \dfrac{R_O}{R_C + R_O} \end{bmatrix} \cdot \begin{bmatrix} i \\ v \end{bmatrix} + [0] \cdot [V_g] = C_2 x$$

the constant vectors are

$$A_2 = \begin{bmatrix} -\dfrac{R_L + \dfrac{R_O R_C}{R_C + R_O}}{L} & \dfrac{R_O}{(R_C + R_O)L} \\ \dfrac{R_O}{(R_C + R_O)C} & -\left[\dfrac{1}{(R_C + R_O)C}\right] \end{bmatrix} \quad B_2 = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \quad C_2 = \begin{bmatrix} \dfrac{R_O R_C}{R_C + R_O} & \dfrac{R_O}{R_C + R_O} \end{bmatrix}$$

State-Space Averaging (4/13)

Evaluate averaged matrices

$$A' = DA_1 + D'A_2$$

$$= D \cdot \begin{bmatrix} -\frac{R_L}{L} & 0 \\ 0 & -\left[\frac{1}{(R_C+R_O)C}\right] \end{bmatrix} + D' \cdot \begin{bmatrix} -\left(\frac{R_L + \frac{R_O R_C}{R_C+R_O}}{L}\right) & -\frac{R_O}{(R_C+R_O)L} \\ \frac{R_O}{(R_C+R_O)C} & -\left[\frac{1}{(R_C+R_O)C}\right] \end{bmatrix}$$

$$= \begin{bmatrix} -\left(\frac{R_L + D'\frac{R_O R_C}{R_C+R_O}}{L}\right) & -\left(\frac{R_O}{(R_C+R_O)L}\right)D' \\ \left(\frac{R_O}{(R_C+R_O)C}\right)D' & -\left[\frac{1}{(R_C+R_O)C}\right] \end{bmatrix}$$

$$C' = DC_1 + D'C_2$$

$$= D\begin{bmatrix} 0 & \frac{R_O}{R_C+R_O} \end{bmatrix} + D'\begin{bmatrix} \frac{R_O R_C}{R_C+R_O} & \frac{R_O}{R_C+R_O} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{D' R_O R_C}{R_C+R_O} & \frac{R_O}{R_C+R_O} \end{bmatrix}$$

$$B' = DB_1 + D'B_2$$

$$= D \cdot \begin{bmatrix} 1/L \\ 0 \end{bmatrix} + D' \cdot \begin{bmatrix} 1/L \\ 0 \end{bmatrix} = \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$$

State-Space Averaging (5/13)

The averaged (nonlinear) state equations:

$$\dot{x} = \left(\mathrm{d(t)}\,A_1 + \mathrm{d(t)'}\,A_2\right)\langle x(t)\rangle_{\mathrm{Ts}} + \left(\mathrm{d(t)}\,B_1 + \mathrm{d(t)'}\,B_2\right)\langle \mathrm{u(t)}\rangle_{\mathrm{Ts}}$$

$$\langle \mathrm{y(t)}\rangle_{\mathrm{Ts}} = \left(\mathrm{d(t)}\,C_1 + \mathrm{d(t)'}\,C_2\right)x + \left(\mathrm{d(t)}\,E_1 + \mathrm{d(t)'}\,E_2\right)\langle \mathrm{u(t)}\rangle_{\mathrm{Ts}}$$

Let

$$\langle x(t)\rangle_{\mathrm{Ts}} = \mathrm{X} + \hat{\mathrm{x}}$$

$$\langle \mathrm{y(t)}\rangle_{\mathrm{Ts}} = \mathrm{Y} + \hat{\mathrm{y}}$$

$$\langle \mathrm{u(t)}\rangle_{\mathrm{Ts}} = \mathrm{U} + \hat{\mathrm{u}}$$

$$\mathrm{d(t)} = \mathrm{D} + \hat{\mathrm{d}}$$

Dc terms drop out of equations. Second-order (nonlinear) terms are small when the small-signal assumption is satisfied. We are left with:

$$\mathrm{K}\cdot\dot{\mathrm{x}} = A\cdot x + B\cdot u + \left[\left(A_1 - A_2\right)x + \left(B_1 - B_2\right)u\right]\hat{d}\left(t\right)$$

$$y = C\cdot x + E\cdot u + \left[\left(C_1 - C_2\right)x + \left(E_1 - E_2\right)u\right]\hat{d}\left(t\right)$$

This is the desired result.

State-Space Averaging (6/13)

Evaluate matrices in small-signal model:

$$E = (A_1 - A_2)x + (B_1 - B_2)u$$

$$= \begin{bmatrix} -\frac{R_L}{L} + \frac{R_L + \frac{R_O R_C}{R_C + R_O}}{L} & 0 + \frac{R_O}{(R_C + R_O)L} \\ 0 - \frac{R_O}{(R_C + R_O)C} & -\left[\frac{1}{(R_C + R_O)C}\right] + \left[\frac{1}{(R_C + R_O)C}\right] \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix} + \begin{bmatrix} \frac{1}{L} - \frac{1}{L} \\ 0 \end{bmatrix} V_g$$

$$= \begin{bmatrix} \left(\frac{\frac{R_O R_C}{R_C + R_O}}{L}\right) I + \left(\frac{R_O}{(R_C + R_O)L}\right) V \\ \left(-\frac{R_O}{(R_C + R_O)C}\right) I \end{bmatrix} = \begin{bmatrix} \left(\frac{(R_C + D' R_O)V}{D'(R_C + R_O)L}\right) \\ \left(-\frac{V}{D'(R_C + R_O)C}\right) \end{bmatrix} \qquad (V=(D')R_O I)$$

$$(C_1 - C_2)x + (E_1 - E_2)u$$

$$= \begin{bmatrix} 0 - \frac{R_O R_C}{R_C + R_O} & \frac{R_O}{R_C + R_O} - \frac{R_O}{R_C + R_O} \end{bmatrix} \begin{bmatrix} I \\ V \end{bmatrix}$$

$$= \left(-\frac{R_O R_C}{R_C + R_O}\right) I$$

State-Space Averaging (7/13)

Small-signal ac state equations:

$$\begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}\begin{bmatrix} \dot{i} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -\left(\dfrac{R_L + D'\dfrac{R_O R_C}{R_C + R_O}}{L}\right) & \left(\dfrac{R_O}{(R_C + R_O)L}\right)D' \\ \left(\dfrac{R_O}{(R_C + R_O)C}\right)D' & -\left[\dfrac{1}{(R_C + R_O)C}\right] \end{bmatrix}\begin{bmatrix} i \\ v \end{bmatrix} + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} V_g$$

$$[V_O] = \begin{bmatrix} \dfrac{D' R_O R_C}{R_C + R_O} & \dfrac{R_O}{R_C + R_O} \end{bmatrix}\begin{bmatrix} i \\ v \end{bmatrix}$$

Construction of ac equivalent circuit

Small-signal ac equations, in scalar form:

$$\frac{di}{dt} = -\left(\frac{R_L + D'\dfrac{R_O R_C}{R_C + R_O}}{L}\right) i - \left(\frac{D' R_O}{(R_C + R_O)L}\right) v + \frac{1}{L} V_g + \left(\frac{(R_C + D' R_O)V}{D'(R_C + R_O)L}\right) d$$

$$\frac{dv}{dt} = \left[\frac{D' R_O}{(R_C + R_O)C}\right] i - \left[\frac{1}{(R_C + R_O)C}\right] v - \left[\frac{I R_O}{(R_C + R_O)C}\right] d$$

$$V_O = \left(\frac{D' R_O R_C}{R_C + R_O}\right) i + \left(\frac{R_O}{R_C + R_O}\right) v - \left(\frac{I R_O R_C}{R_C + R_O}\right) d$$

State-Space Averaging (8/13)

Next, we transformac equivalent circuit small - signal ac equations transforminto laplace

inductor equation

$$sL\hat{i}(s) = -\left(R_L + \frac{D'R_O R_C}{R_C + R_O}\right)\hat{i}(s) - \left(\frac{D'R_O}{R_C + R_O}\right)\hat{v}(s) + \left(\frac{V(D'R_O + R_C)}{D'(R_C + R_O)}\right)\hat{d}(s) + v_g(s)$$

$$\left(sL + R_L + \frac{D'R_O R_C}{R_C + R_O}\right)\hat{i}(s) + \left(\frac{D'R_O}{R_C + R_O}\right)\hat{v}(s) = \left(\frac{V(D'R_O + R_C)}{D'(R_C + R_O)}\right)\hat{d}(s) + v_g(s)$$

capacitor eqnation

$$sC\hat{v}(s) = \left(\frac{D'R_O}{R_C + R_O}\right)\hat{i}(s) - \frac{\hat{v}(s)}{R_C + R_O} - \left(\frac{IR_O}{R_C + R_O}\right)\hat{d}(s)$$

output eqnation

$$\hat{V}_O(s) = \left(\frac{D'R_O R_C}{R_C + R_O}\right)\hat{i}(s) + \left(\frac{R_O}{R_C + R_O}\right)\hat{v}(s) - \left(\frac{IR_O R_C}{R_C + R_O}\right)\hat{d}(s)$$

State-Space Averaging (9/13)

$$G_{vg}(s)=\left.\frac{\hat{V}_o(s)}{\hat{V}_g(s)}\right|_{\substack{\hat{d}(s)=0\\ \hat{i}_{load}=0}}=\left.\frac{\hat{y}(s)}{\hat{u}(s)}\right|_{\substack{\hat{d}(s)=0\\ \hat{i}_{load}=0}}=C'(SI-A')^{-1}B'$$

$$=\begin{bmatrix}\frac{D'R_OR_C}{R_C+R_O} & \frac{R_O}{R_C+R_O}\end{bmatrix}\left(\begin{bmatrix}S & 0\\ 0 & S\end{bmatrix}-\begin{bmatrix}-\left(\frac{R_L}{L}+\frac{D'R_OR_C}{(R_C+R_O)L}\right) & -\left(\frac{R_O}{(R_C+R_O)L}\right)D'\\ \left(\frac{R_O}{(R_C+R_O)C}\right)D' & -\left[\frac{1}{(R_C+R_O)C}\right]\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}1/L\\ 0\end{bmatrix}$$

$$=\begin{bmatrix}\frac{D'R_OR_C}{R_C+R_O} & \frac{R_O}{R_C+R_O}\end{bmatrix}\cdot\begin{bmatrix}S+\left(\frac{R_L}{L}+\frac{D'R_OR_C}{(R_C+R_O)L}\right) & \left(\frac{R_O}{(R_C+R_O)L}\right)D'\\ -\left(\frac{R_O}{(R_C+R_O)C}\right)D' & S+\left[\frac{1}{(R_C+R_O)C}\right]\end{bmatrix}^{-1}\cdot\begin{bmatrix}1/L\\ 0\end{bmatrix}$$

$$\text{ps}:\left(K=\begin{bmatrix}a & b\\ c & d\end{bmatrix},\ K^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}\right)$$

State-Space Averaging (10/13)

$$G_{vg}(s)=\left.\frac{\hat{V}_o(s)}{\hat{V}_g(s)}\right|_{\substack{\hat{d}(s)=0\\ \hat{i}_{load}=0}}=\left.\frac{\hat{y}(s)}{\hat{u}(s)}\right|_{\substack{\hat{d}(s)=0\\ \hat{i}_{load}=0}}=C'(SI-A')^{-1}B'$$

$$=\frac{\dfrac{R_O}{R_C+R_O}}{S^2+\dfrac{S\left[\left(\dfrac{R_L}{R_C+R_O}+D'R_OR_C\right)C+L\right]+R_L+D'\dfrac{R_OR_C}{R_C+R_O}}{LC(R_C+R_O)}+\dfrac{\left(\dfrac{D'R_O}{R_C+R_O}\right)^2}{LC}}\cdot\begin{bmatrix}D'R_C & 1\end{bmatrix}\cdot\begin{bmatrix}S+\left[\dfrac{1}{(R_C+R_O)C}\right] & \left(\dfrac{R_O}{(R_C+R_O)L}\right)D' \\ -\left(\dfrac{R_O}{(R_C+R_O)C}\right)D' & S+\left(\dfrac{R_L}{L}+\dfrac{D'R_OR_C}{(R_C+R_O)L}\right)\end{bmatrix}\cdot\begin{bmatrix}1/L\\ 0\end{bmatrix}$$

$$Let\ \ R'=R_L+D'\frac{R_OR_C}{R_C+R_O}+\left(\frac{D'^2R_O{}^2}{R_C+R_O}\right)$$

$$=\frac{\dfrac{R_O}{R_C+R_O}}{S^2+\dfrac{S\left[\left(\dfrac{R_L}{R_C+R_O}+D'R_OR_C\right)C+L\right]+R'}{LC(R_C+R_O)}}\cdot\begin{bmatrix}\dfrac{D'}{C}(SR_CC+1) & \left(\dfrac{(D')^2R_CR_O}{(R_C+R_O)L}\right)-S-\left(\dfrac{R_L}{L}+\dfrac{D'R_OR_C}{(R_C+R_O)L}\right)\end{bmatrix}\cdot\begin{bmatrix}1/L\\ 0\end{bmatrix}$$

$$=\frac{\dfrac{R_O}{R_C+R_O}\left[D'(SR_CC+1)\right]\dfrac{1}{L}}{S^2+\dfrac{S\left[\left(\dfrac{R_L}{R_C+R_O}+D'R_OR_C\right)C+L\right]+R'}{LC(R_C+R_O)}}=\frac{D'R_O[R_CSC+1]}{(R_C+R_O)LCS^2+S\left[\left((R_C+R_O)R_L+D'R_OR_C\right)C+L\right]+R'}$$

State-Space Averaging (11/13)

$$G_{vd}(s)=\left.\frac{\hat{V}_o(s)}{\hat{d}(s)}\right|_{\substack{\hat{V}_g(s)=0\\ \hat{i}_{load}=0}}=\left.\frac{\hat{y}(s)}{\hat{d}(s)}\right|_{\substack{\hat{V}_g(s)=0\\ \hat{i}_{load}=0}}=C'(SI-A')^{-1}E$$

$$=\begin{bmatrix}\frac{D'R_OR_C}{R_C+R_O} & \frac{R_O}{R_C+R_O}\end{bmatrix}\left(\begin{bmatrix}S & 0\\ 0 & S\end{bmatrix}-\begin{bmatrix}-\left(\frac{R_L}{L}+\frac{D'R_OR_C}{(R_C+R_O)L}\right) & -\left(\frac{R_O}{(R_C+R_O)L}\right)D'\\ \left(\frac{R_O}{(R_C+R_O)C}\right)D' & -\left[\frac{1}{(R_C+R_O)C}\right]\end{bmatrix}\right)^{-1}\cdot\begin{bmatrix}\left(\frac{(R_C+D'R_O)V}{D'(R_C+R_O)L}\right)\\ \left(-\frac{V}{D'(R_C+R_O)C}\right)\end{bmatrix}$$

$$=\frac{LCR_O}{LC(R_C+R_O)S^2+S\left[\left(\frac{R_L}{R_C+R_O}+D'R_OR_C\right)C+L\right]+R'}\begin{bmatrix}\frac{D'}{C}(SR_CC+1) & \left(\frac{(D')^2R_CR_O}{(R_C+R_O)L}\right)-S-\left(\frac{R_L}{L}+\frac{D'R_OR_C}{(R_C+R_O)L}\right)\end{bmatrix}\cdot\begin{bmatrix}\left(\frac{(R_C+D'R_O)V}{D'(R_C+R_O)L}\right)\\ \left(-\frac{V}{D'(R_C+R_O)C}\right)\end{bmatrix}$$

$$=\frac{\left(\frac{V}{D'}\right)(SR_CC+1)\left[\left(\frac{R_O}{R_C+R_O}\right)\left[\left(\frac{D'R_OR_C+(D'R_O)^2}{R_C+R_O}\right)-\frac{D'R_OR_C-(D')^2R_CR_O}{R_C+R_O}\right]-SL-R_L\right]}{LC(R_C+R_O)S^2+S\left[\left(\frac{R_L}{R_C+R_O}+D'R_OR_C\right)C+L\right]+R'}$$

$$=\frac{\left(\frac{V}{D'}\right)\left[\frac{(D')^2R_O^2}{R_C+R_O}-R_L\right](1+SR_CC)\left[1-\frac{SL}{\frac{(D')^2R_O^2}{R_C+R_O}-R_L}\right]}{LC(R_C+R_O)S^2+S\left[\left(\frac{R_L}{R_C+R_O}+D'R_OR_C\right)C+L\right]+R'}$$

State-Space Averaging (12/13)

The output impedance Zout(s) is

$$Zout(s) = \frac{\hat{V}_{o1}(s)}{\hat{i}_{o1}(s)} = \frac{\hat{V}_{o1}(s)}{\hat{V}_g(s)} \Bigg/ \frac{\hat{i}_{o1}(s)}{\hat{V}_g(s)}$$

the output current is expressed

$$i_o = \frac{i_L}{D'}$$

$$\hat{y}_1(s) = i_o(s) = \begin{bmatrix} \frac{1}{D'} & 0 \end{bmatrix} \begin{bmatrix} i_L(s) \\ V_C(s) \end{bmatrix} = C''\hat{x}(s)$$

we substituted equ.(31) for the above equation,

$$\hat{y}_1(s) = C''^T(SI - A')^{-1} B'\hat{u}(s) + C''^T(SI - A')^{-1} E\hat{d}(s)$$

therefore,

$$Zout(s) = \left.\frac{\hat{V}_i(s)}{\hat{i}_{load}(s)}\right|_{\substack{\hat{d}(s)=0 \\ \hat{V}_g=0}} = \left.\frac{\hat{y}_2(s)}{\hat{u}(s)}\right|_{\substack{\hat{d}(s)=0 \\ \hat{V}_g=0}}$$

$$= C''(SI - A')^{-1} B'$$

State-Space Averaging (13/13)

$$\frac{\hat{i}_{o1}(s)}{\hat{V}_g(s)} = \begin{bmatrix} \frac{1}{D'} & 0 \end{bmatrix} \left(\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} -\left(\frac{R_L}{L} + \frac{D' R_O R_C}{(R_C + R_O)L} \right) & -\left(\frac{R_O}{(R_C + R_O)L} \right) D' \\ \left(\frac{R_O}{(R_C + R_O)C} \right) D' & -\left[\frac{1}{(R_C + R_O)C} \right] \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1/L \\ 0 \end{bmatrix}$$

$$= \frac{\left(SC(R_C + R_O) + 1\right) / D'}{(R_C + R_O)LCS^2 + S\left[\left((RC + RO)R_L + D' R_O R_C\right)C + L\right] + R'}$$

$$Z_{out}(s) = \frac{\hat{V}_{o1}(s)}{\hat{i}_{o1}(s)} = \frac{\hat{V}_{o1}(s)}{\hat{V}_g(s)} \bigg/ \frac{\hat{i}_{o1}(s)}{\hat{V}_g(s)}$$

$$= \frac{\left[\frac{D' R_O [R_C SC + 1]}{(R_C + R_O)LCS^2 + S\left[\left((R_C + R_O)R_L + D' R_O R_C\right)C + L\right] + R'} \right]}{\left[\frac{\left(SC(R_C + R_O) + 1\right) / D'}{(R_C + R_O)LCS^2 + S\left[\left((R_C + R_O)R_L + D' R_O R_C\right)C + L\right] + R'} \right]}$$

$$= \frac{(D')^2 R_O [R_C SC + 1]}{SC(R_C + R_O) + 1}$$

Equivalent Circuits

inductor equation

$$\left(sL+R_L+\frac{D'R_OR_C}{R_C+R_O}\right)\hat{i}(s)+\left(\frac{D'R_O}{R_C+R_O}\right)\hat{v}(s)$$

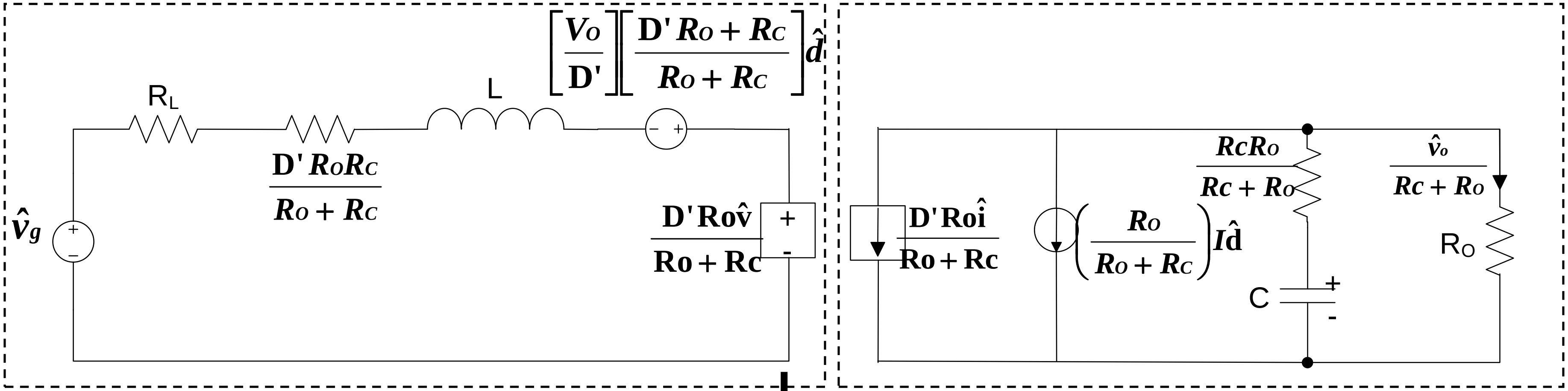
$$=\left(\frac{V\left(D'R_O+R_C\right)}{D'\left(R_C+R_O\right)}\right)\hat{d}(s)+v_g(s)$$

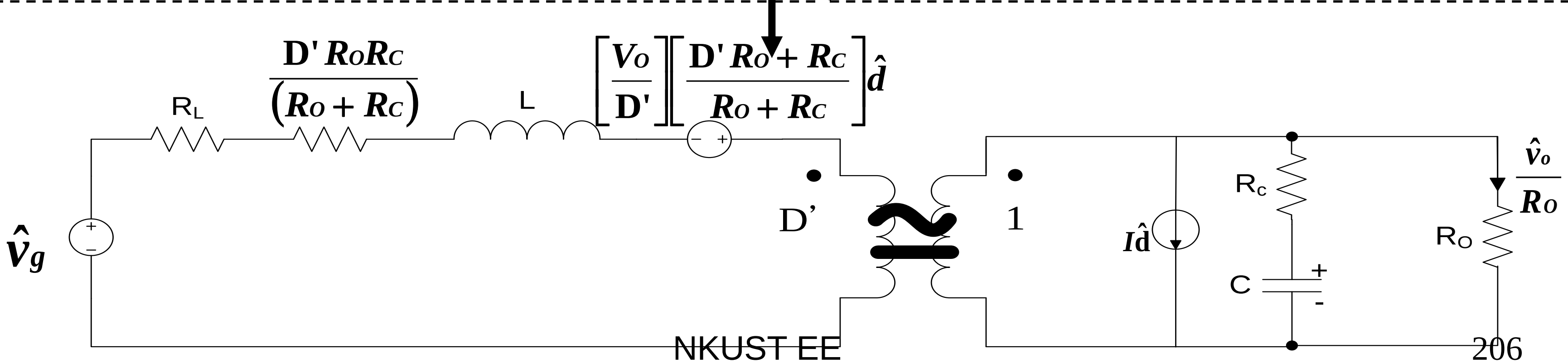
capacitor eqnation

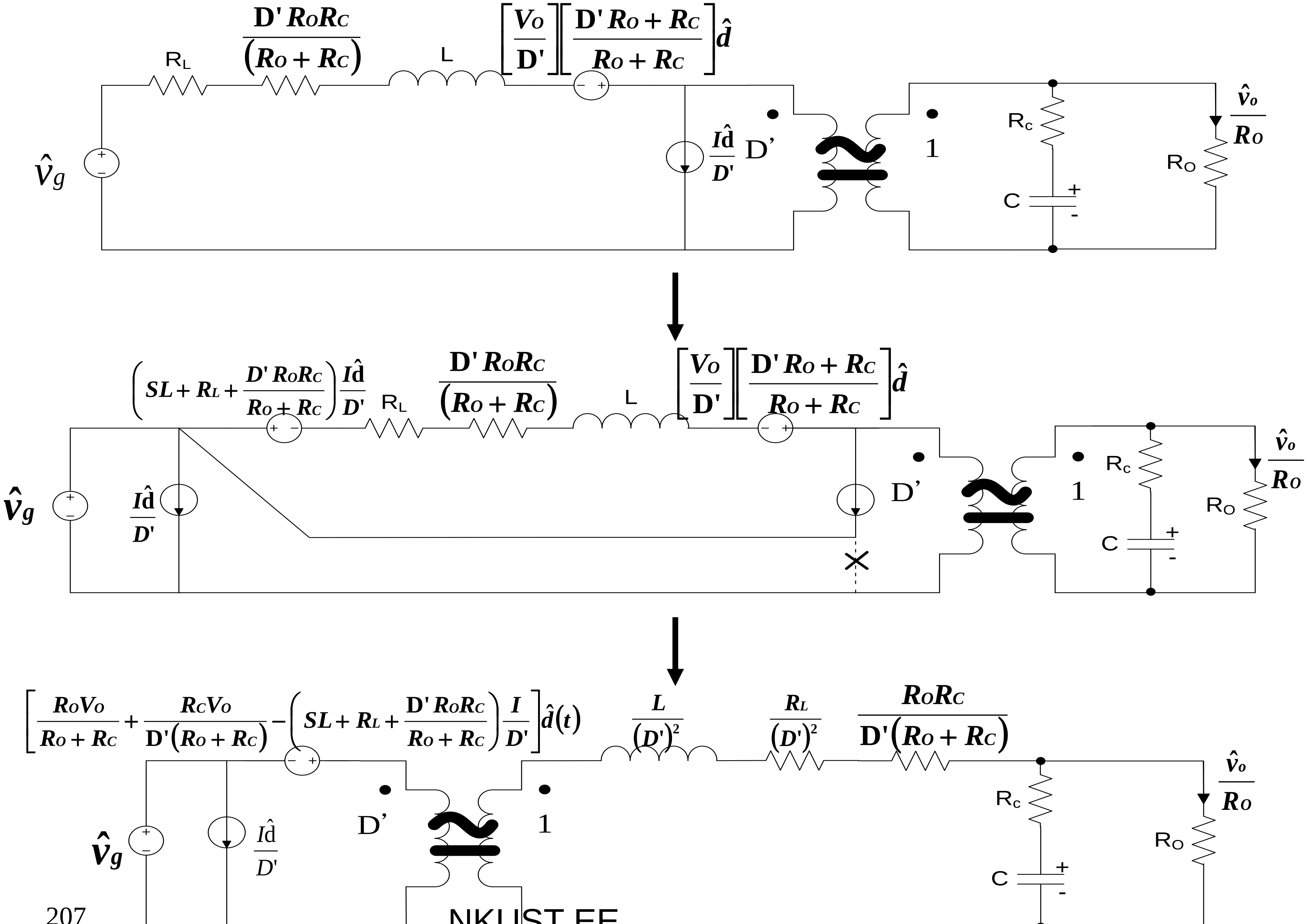
$$sC\hat{v}(s)=\left(\frac{D'R_O}{R_C+R_O}\right)\hat{i}(s)-\frac{\hat{v}(s)}{R_C+R_O}-\left(\frac{IR_O}{R_C+R_O}\right)\hat{d}(s)$$

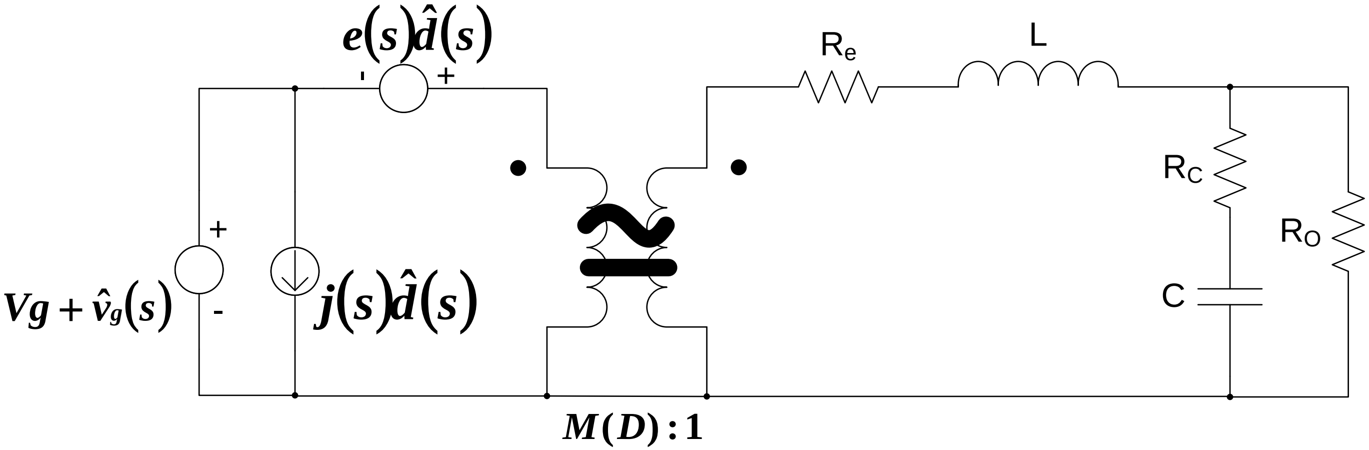
output eqnation

$$\hat{V}_O(s)=\left(\frac{D'R_OR_C}{R_C+R_O}\right)\hat{i}(s)+\left(\frac{R_O}{R_C+R_O}\right)\hat{v}(s)-\left(\frac{IR_OR_C}{R_C+R_O}\right)\hat{d}(s)$$






RL
D' RoRc / (Ro + Rc)
L
[Vo / D'][D' Ro + Rc / Ro + Rc] d̂
v̂g
Id̂ / D'
D'
1
Rc
C
Ro
v̂o / Ro
(SL + RL + D' RoRc / Ro + Rc) Id̂ / D'
[RoVo / Ro + Rc + RcVo / D'(Ro + Rc) − (SL + RL + D' RoRc / Ro + Rc) I / D'] d̂(t)
L / (D')²
RL / (D')²
RoRc / D'(Ro + Rc)



Model parameters

$$e(s) = V_O\left[\frac{R_O}{R_O + R_C} - \frac{R_L}{(D')^2 R_O}\right]\left[1 - \left(\frac{sL}{\frac{(D' R_O)^2}{R_O + R_C} - R_L}\right)\right]$$

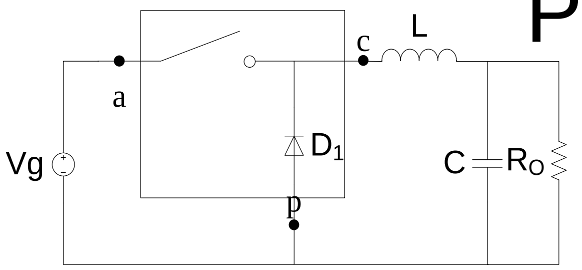
$$j(s) = \frac{V_O}{(D')^2 R_O}$$

$$Le = \frac{L}{(D')^2}$$

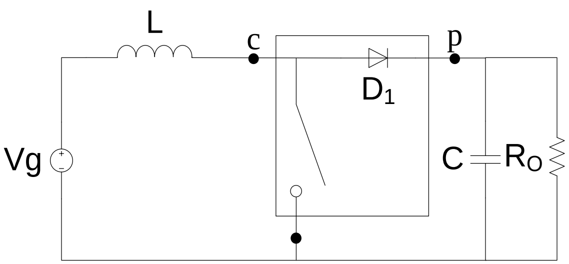
$$R = \frac{R_L + (R_O // R_C)DD'}{(D')^2}$$

$$M(D) = D'$$

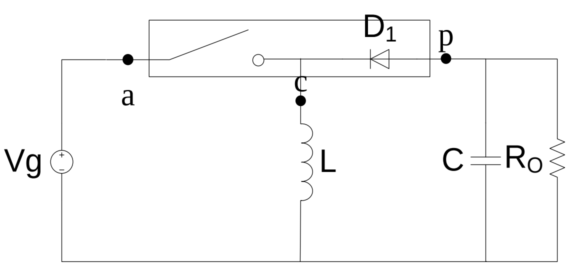
PWM Modeling



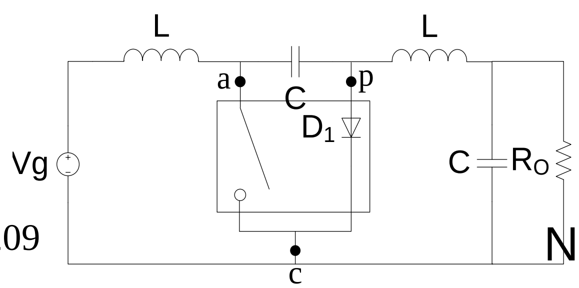
buck



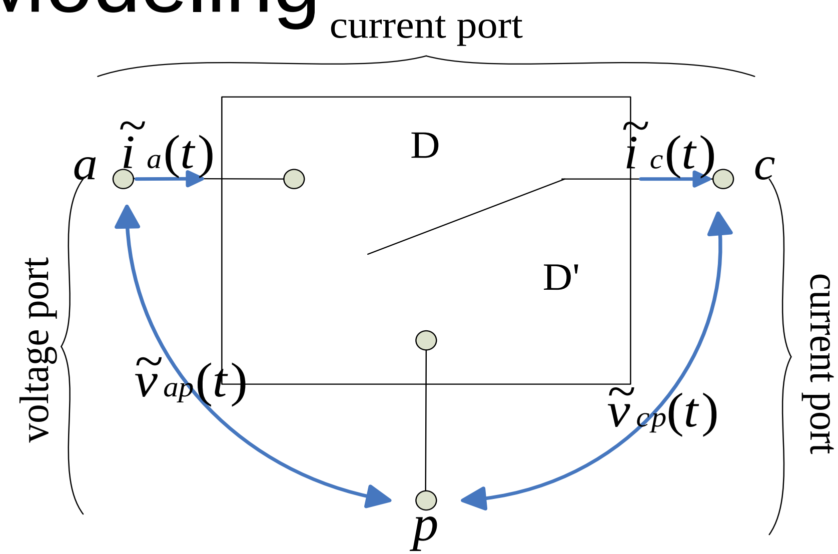
boost



Buck-boost



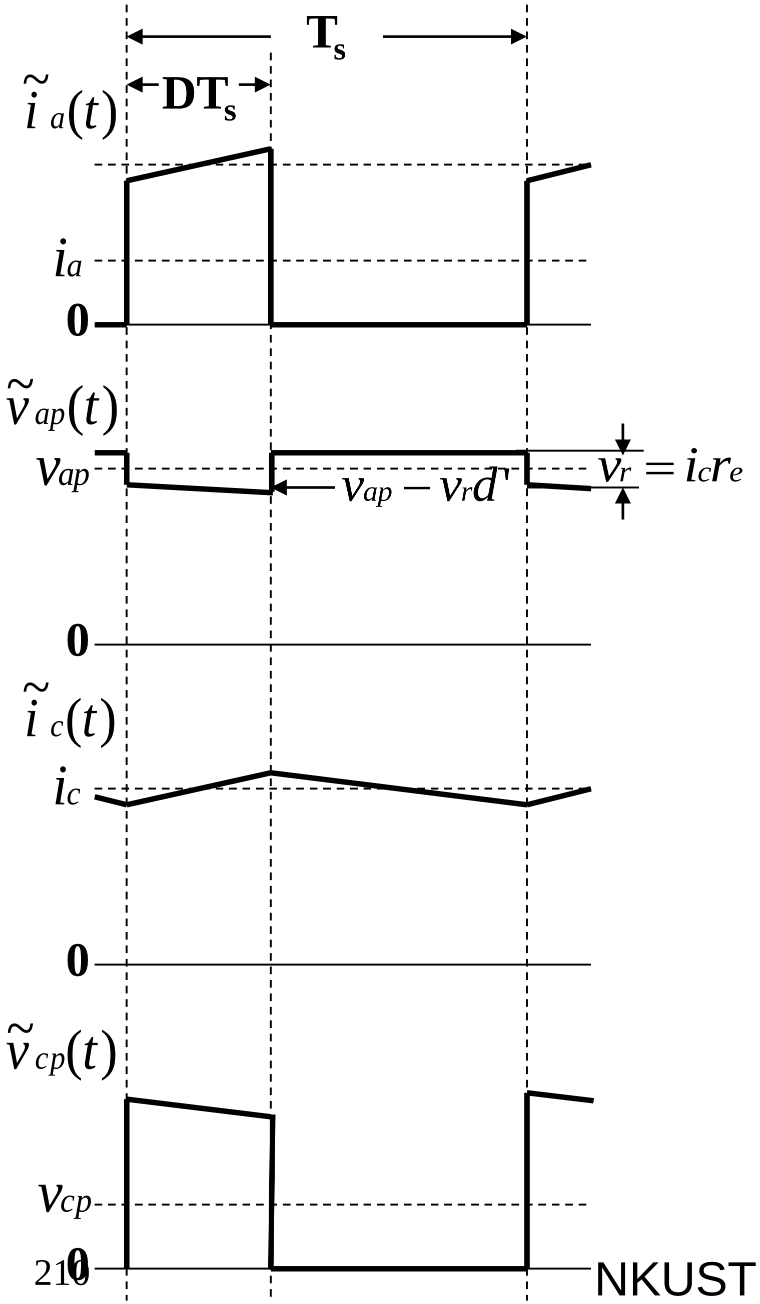
Cuk



PWM switch

$$\tilde{i}_a = \begin{cases} \tilde{i}_c(t), & 0 \le t \le DTs \\ 0, & Ts \le t \le DTs \end{cases}$$

$$\tilde{v}_{cp} = \begin{cases} \tilde{v}_{ap}(t), & 0 \le t \le DTs \\ 0, & Ts \le t \le DTs \end{cases}$$



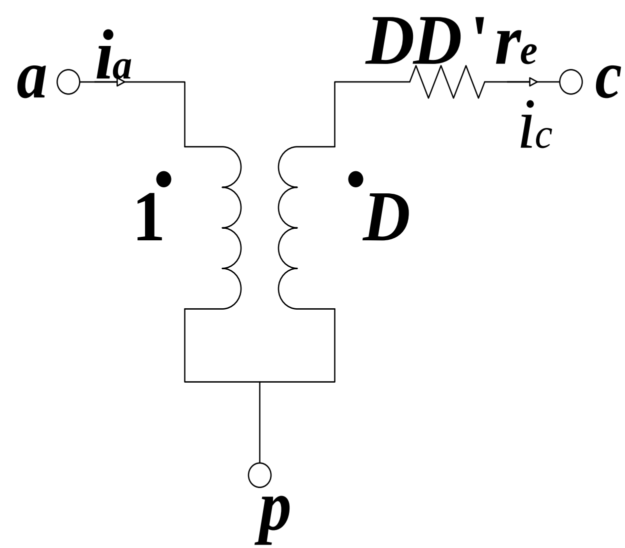
$$v_r = i_c r_e$$

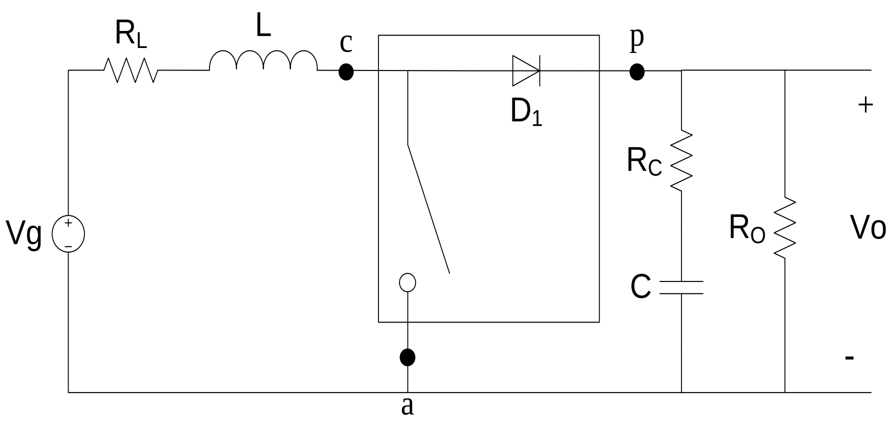
$$r_e = Rc \parallel Ro \qquad (\text{boost, buck - boost})$$

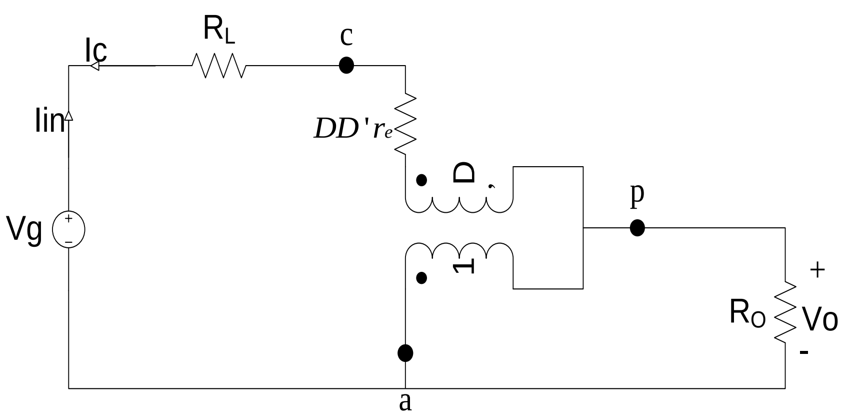
$$r_e = Rc \qquad (\text{Cuk})$$

$$i_a = d i_c$$

$$v_{cp} = d(v_{ap} - i_c r_e d')$$







$$V_{ap} = -Vo$$

$$Ic = -Iin = -\frac{Io}{D'}$$

$$M = \frac{Vo}{Vg}$$

$$= \frac{-Vap}{-\frac{Io}{D'}\left(R_L + r_e DD' + (D')^2 Ro\right)}$$

$$= \frac{IoRo}{\frac{Io}{D'}\left(R_L + r_e DD' + (D')^2 Ro\right)}$$

$$= \frac{D'Ro}{\left(R_L + r_e DD' + (D')^2 Ro\right)}$$

$$= \frac{1}{D'}\frac{1}{1 + \frac{R_L}{(D')^2 R} + \frac{r_e D}{RD'}}$$

$$r_e = Rc \parallel Ro$$

$$i_a = d i_c$$

$$v_{cp} = d(v_{ap} - i_c r_e d')$$

Perturbation and linearization

$$\hat{i}_a = D\hat{i}_c + I_c\hat{d}$$

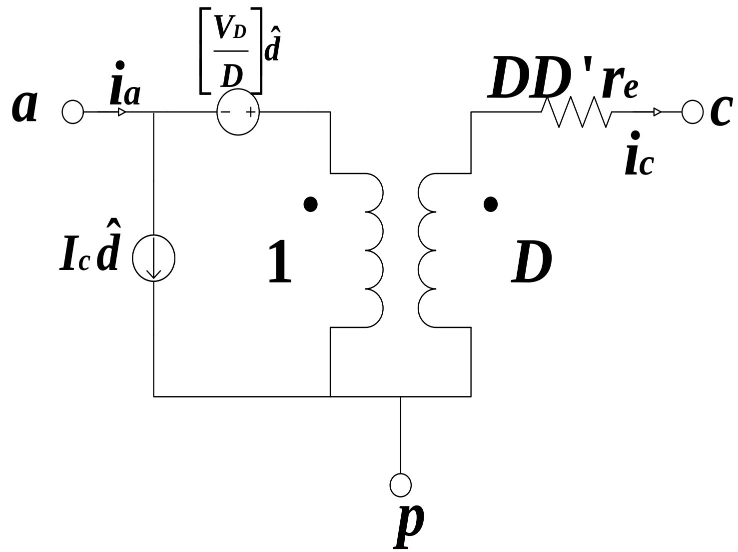
$$\hat{v}_{cp} = D(\hat{v}_{ap} + I_c r_e d' - \hat{i}_c r_e D') + \hat{d}(V_{ap} - I_c r_e D')$$

which can be rearranged as

$$\hat{v}_{ap} = \frac{\hat{v}_{cp}}{D} + \hat{i}_c r_e D' - \left[V_{ap} + I_c(D - D')r_e\right]\frac{\hat{d}}{D}$$

let $V_D = V_{ap} + I_c(D - D')r_e$

$$\hat{v}_{ap} = \frac{\hat{v}_{cp}}{D} + \hat{i}_c r_e D' - V_D\frac{\hat{d}}{D}$$



Line Regulation

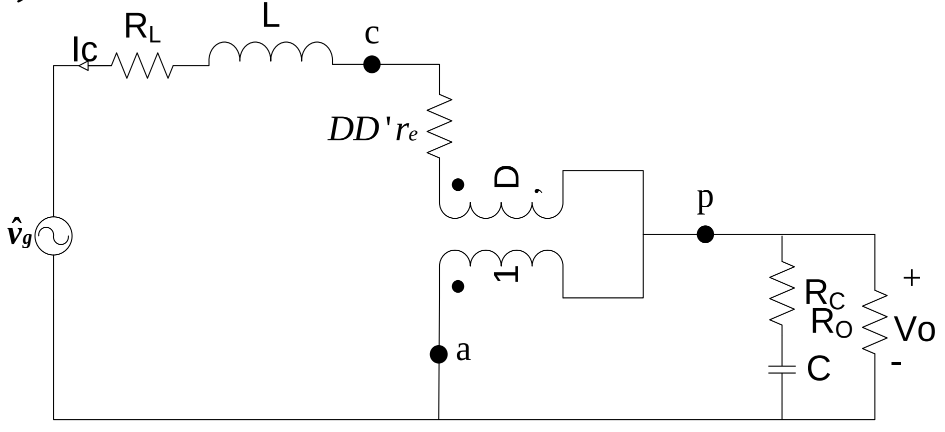
$$\frac{v_o(s)}{v_g(s)} = \frac{1}{D'}\frac{Ro \,\|\, \left(Rc + \frac{1}{sC}\right)}{\frac{R_L + r_e DD'}{(D')^2} + sL + Ro \,\|\, \left(Rc + \frac{1}{sC}\right)}$$

$$= M\,\frac{\left(1 + s / s_{z1}\right)}{1 + s/\omega_0 Q + s^2/\omega_0{}^2}$$

$$s_{z1} = \frac{1}{RcC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{R_L + r_e DD' + (D')^2 Ro}{Rc + Ro}}$$

$$Q = \frac{\omega_0}{\frac{R_L + r_e D'}{L} + \frac{1}{C(Rc + Ro)}}$$



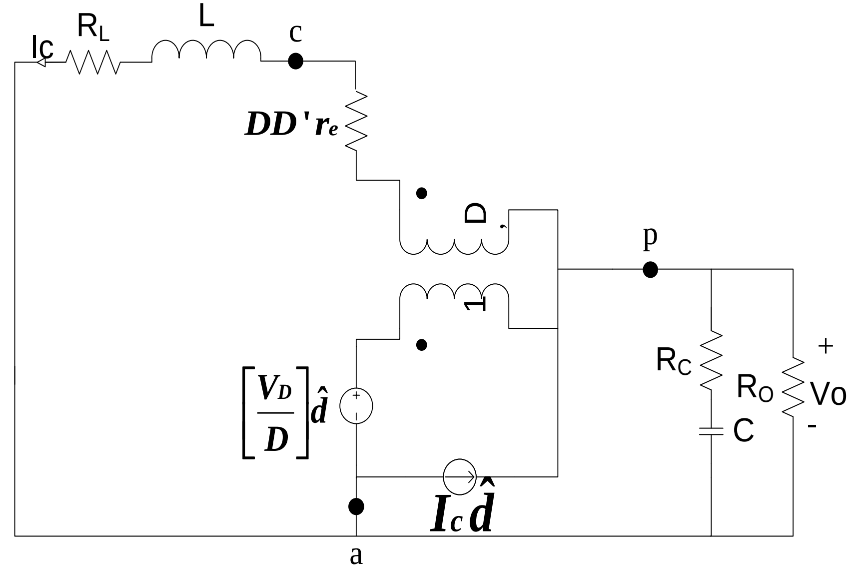
$$\frac{\hat{v}_o(s)}{\hat{d}(s)} = \frac{V_D}{D}\frac{Ro \parallel \left(Rc + \frac{1}{sC}\right)}{\frac{R_L + r_e DD'}{(D')^2} + sL + Ro \parallel \left(Rc + \frac{1}{sC}\right)} + Ic\left[\left(\frac{R_L + r_e DD'}{(D')^2} + sL\right) \parallel Ro \parallel \left(Rc + \frac{1}{sC}\right)\right]$$

$$= K_d \frac{(1 + s/s_{z1})(1 + s/s_{z2})}{1 + s/\omega_0 Q + s^2/\omega_0^2}$$

$$K_d = \frac{dVo}{dD} = Vi\frac{dM}{dD} \cong \frac{Vi}{(D')^2}$$

$$s_{z2} = \frac{1}{L}\left(D'\frac{V_D}{I_c} - R_L - r_e DD'\right)$$

$$= \frac{(D')^2}{L}\left(Ro - Ro \parallel Rc\right) - \frac{R_L}{L}$$



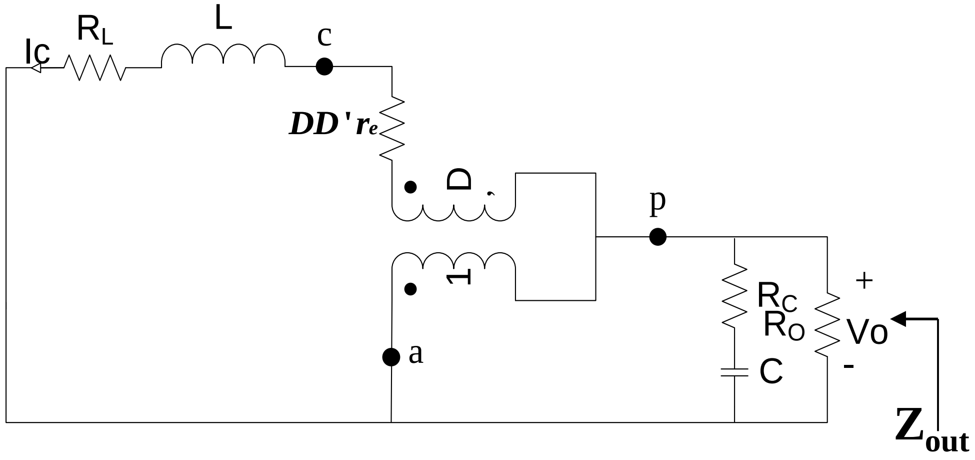
Load Regulation

$$Z_{out} = \left(\frac{R_L + r_e DD'}{(D')^2} + \frac{sL}{D'} \right) \| \left(Rc + \frac{1}{sC} \right) \| Ro$$

$$= R' \frac{(1 + s/s_{z1})(1 + s/s_{z0})}{1 + s/\omega_0 Q + s^2/\omega_0^2}$$

$$R' = Ro \| \frac{R_L + r_e DD'}{(D')^2}$$

$$s_{z0} = \frac{R_L + r_e DD'}{L}$$



Model parameters

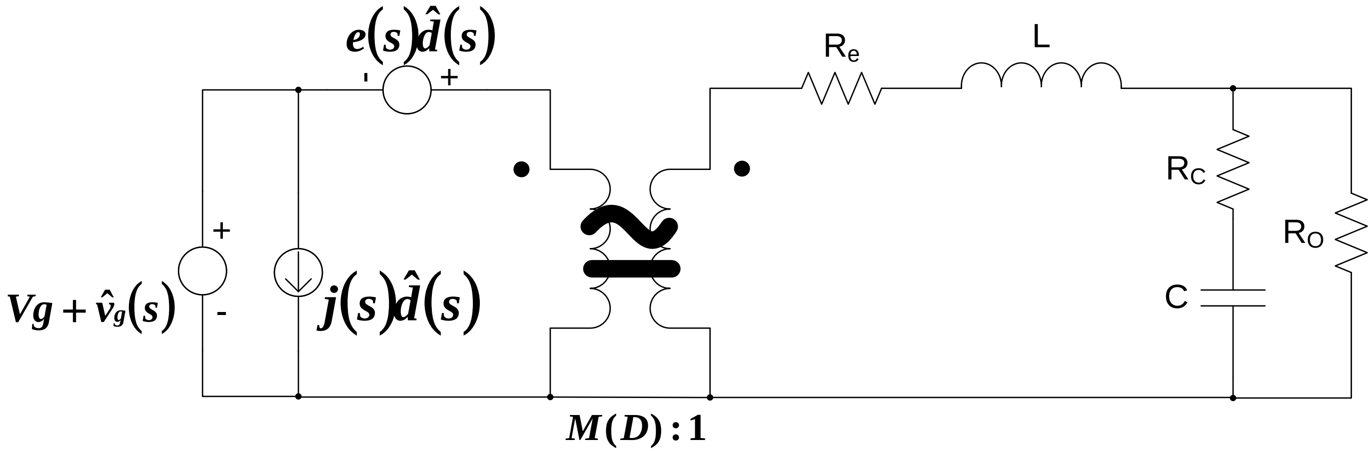
$$e(s)=V_O\left[\frac{R_O}{R_O+R_C}-\frac{R_L}{(D')^2 R_O}\right]\left[1-\left(\frac{sL}{\frac{(D' R_O)^2}{R_O+R_C}-R_L}\right)\right]$$

$$j(s)=\frac{V_O}{(D')^2 R_O}$$

$$Le=\frac{L}{(D')^2}$$

$$R=\frac{R_L+(R_O // R_C)DD'}{(D')^2}$$

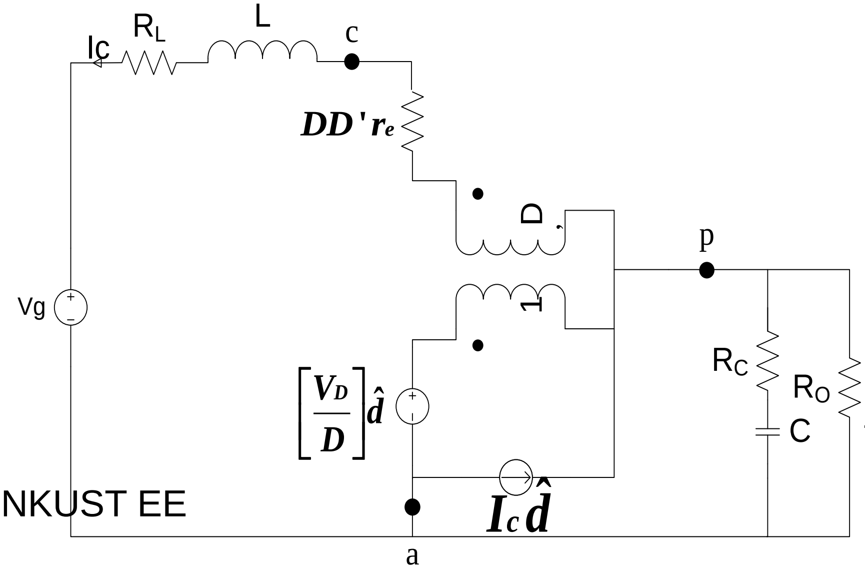
$$M(D)=D'$$



$$\hat{v}_{ap}=\frac{\hat{v}_{cp}}{D}+\hat{i}_c r_e D'-\left[V_{ap}+I_c(D-D')r_e\right]\frac{\hat{d}}{D}$$

$$let \quad V_D=V_{ap}+I_c(D-D')r_e$$

$$\hat{v}_{ap}=\frac{\hat{v}_{cp}}{D}+\hat{i}_c r_e D'-V_D\frac{\hat{d}}{D}$$



Perturbation and Linearization

1. The averaged converter equations
2. perturbation and linearization
3. Small-signal ac equations
4. Small-signal ac equivalent circuits
5. Calculate converter transfer function

Complex calculation

Easy understand

State-Space Averaging

1. The averaged converter equations
2. Change equation into state space functions and set up varied
3. Evaluate averaged matrices
4. Calculate converter transfer function

Matrices calculate easily

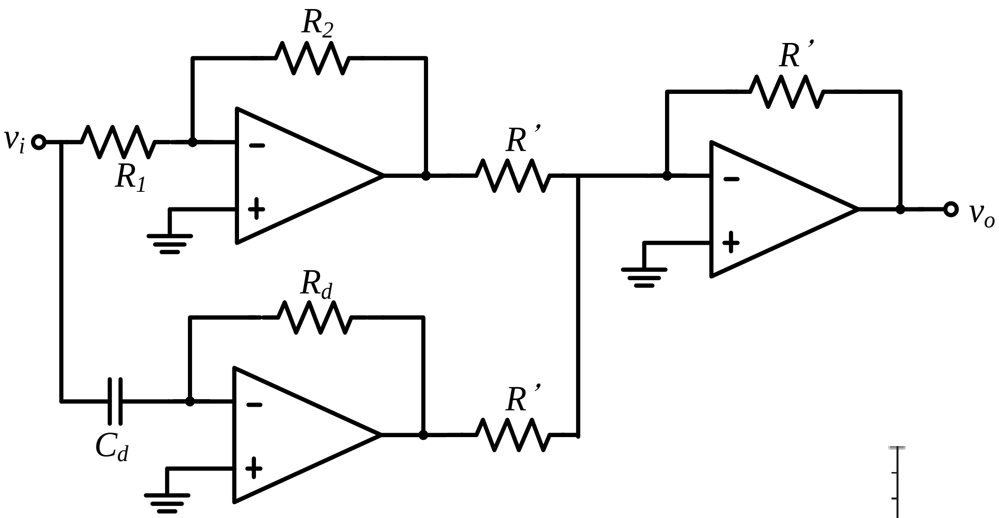
It is not necessary to build equivalent circuits

Simplified Analysis of PWM Converters Using Model of PWM Switch

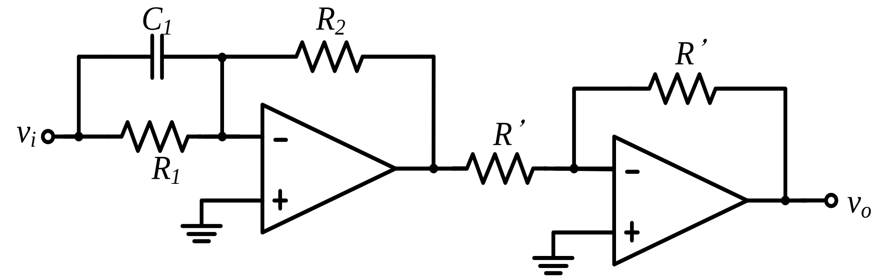
1. Model of PWM switch
2. Substitution converter
3. Calculate converter transfer function

Fast modeling

Lead (PD) Compensator



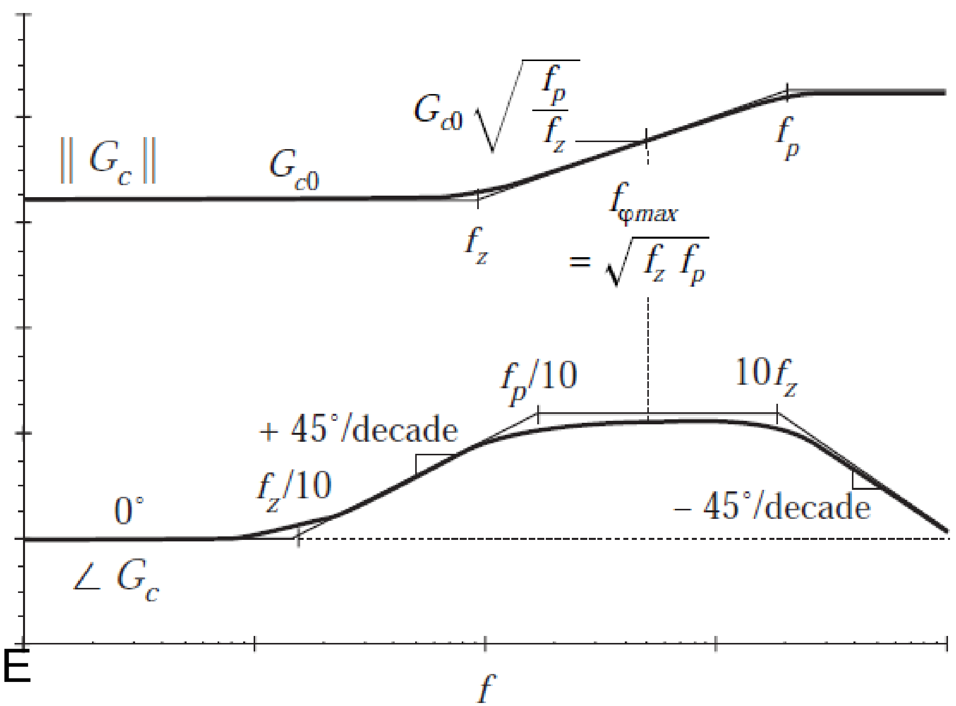
$K_p = \frac{R_2}{R_1}$, $K_d = R_d C_d$ (a)



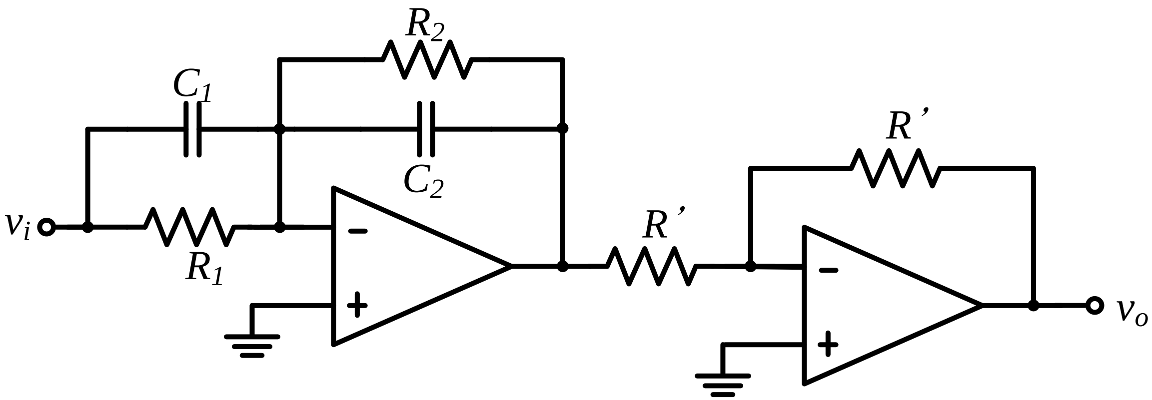
$K_p = \frac{R_2}{R_1}$, $K_d = R_2 C_1$ (b)

$$G_c = \frac{v_o(s)}{v_i(s)} = K_p + K_d s$$

$$= G_{c0} \frac{(1 + s/\omega_z)}{(1 + s/\omega_p)}$$

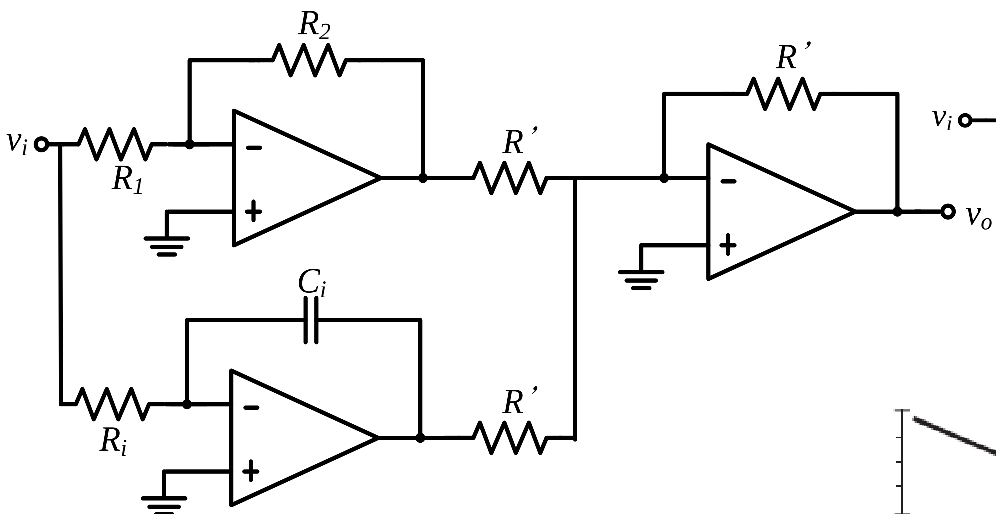


Lead-Lag Compensator

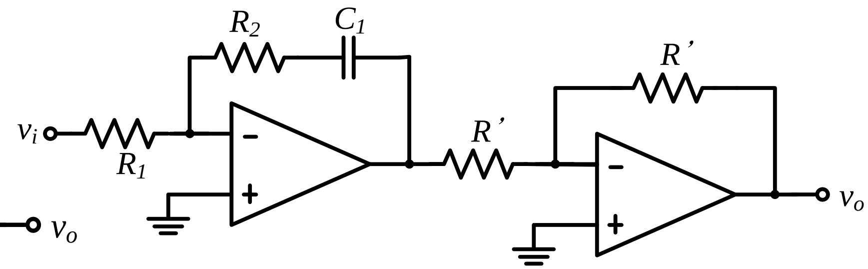


$$G_c = \frac{v_o(s)}{v_i(s)} = K_c \frac{(s+z_1)}{(s+p_1)}$$

Lag (PI) Compensator



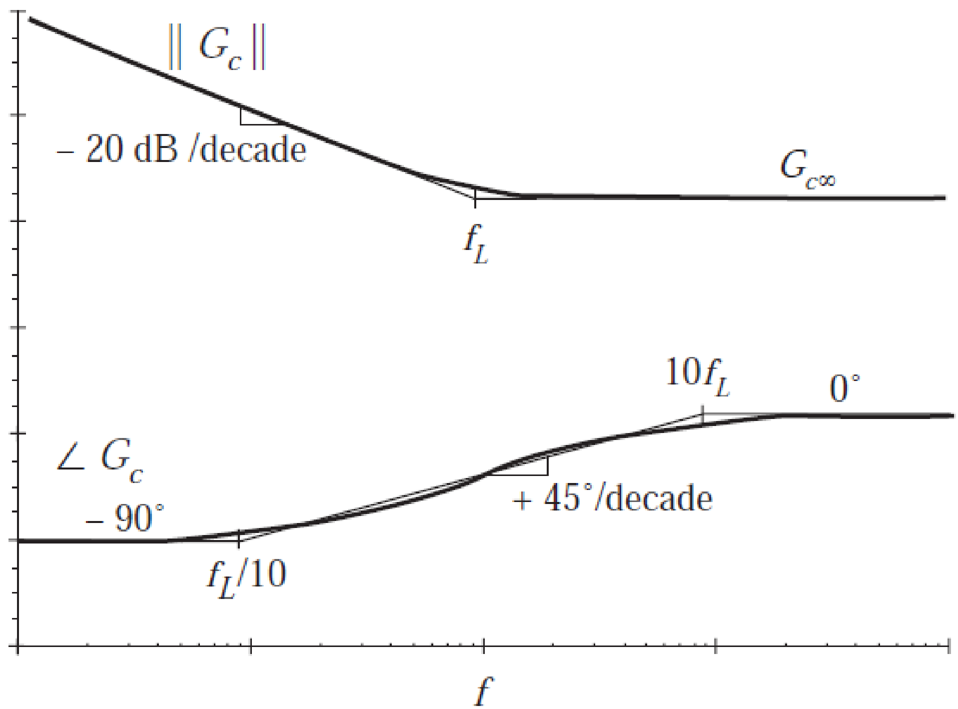
$$K_p = \frac{R_2}{R_1}, \; K_i = \frac{1}{R_i C_i} \quad (a)$$



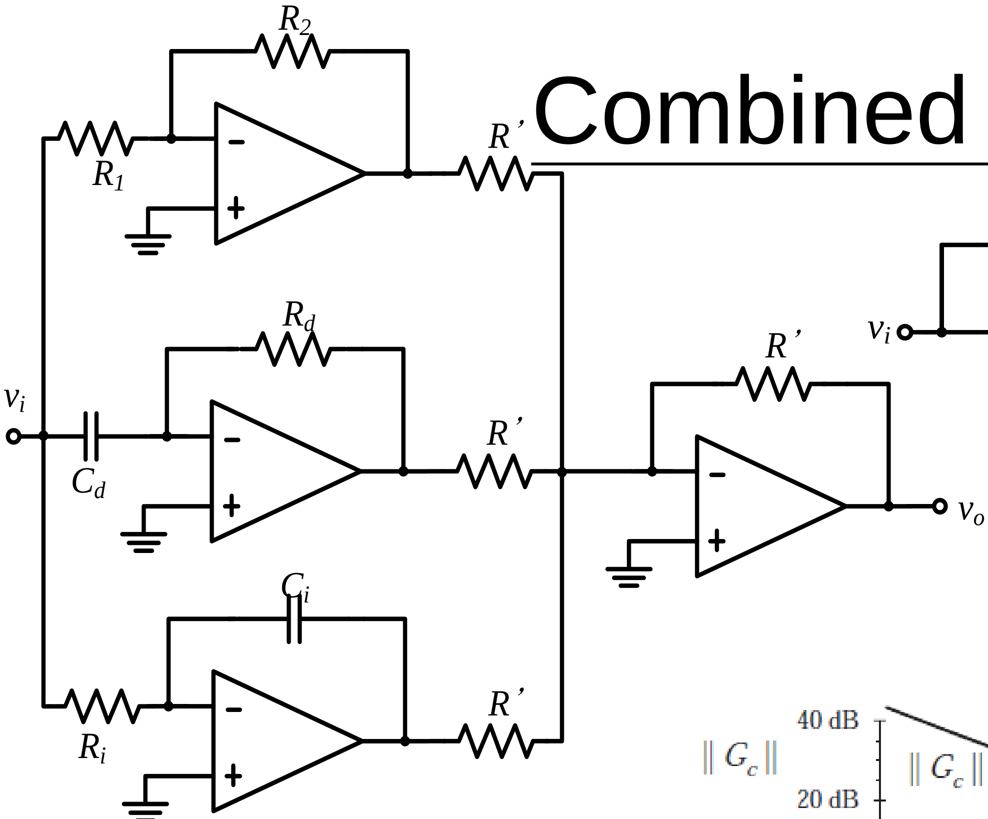
$$K_p = \frac{R_2}{R_1}, \; K_i = \frac{1}{R_1 C_1} \quad (b)$$

$$G_c = \frac{v_o(s)}{v_i(s)} = K_p + K_i / s$$

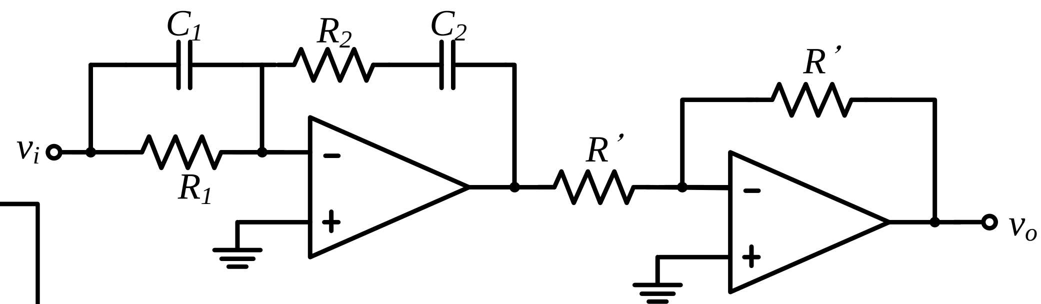
$$= G_{c\infty}(1 + \omega_l / s)$$



Combined (PID) Compensator



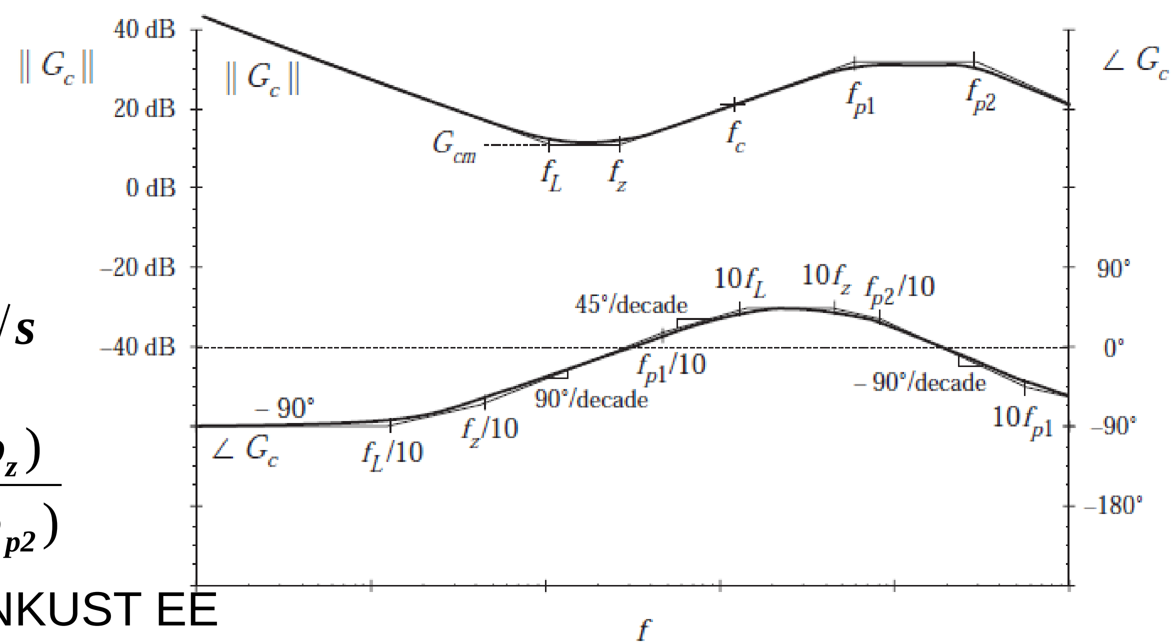
$$K_p = \frac{R_2}{R_1}, \; K_i = \frac{1}{R_i C_i} \quad \text{(a)}$$



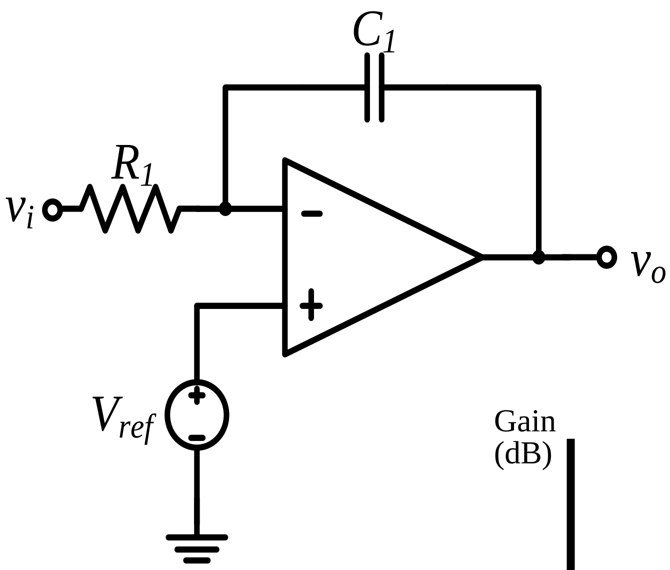
$$K_p = \frac{R_2}{R_1}, \; K_i = \frac{1}{R_1 C_1} \quad \text{(b)}$$

$$G_c = \frac{v_o(s)}{v_i(s)} = K_p + K_d s + K_i / s$$

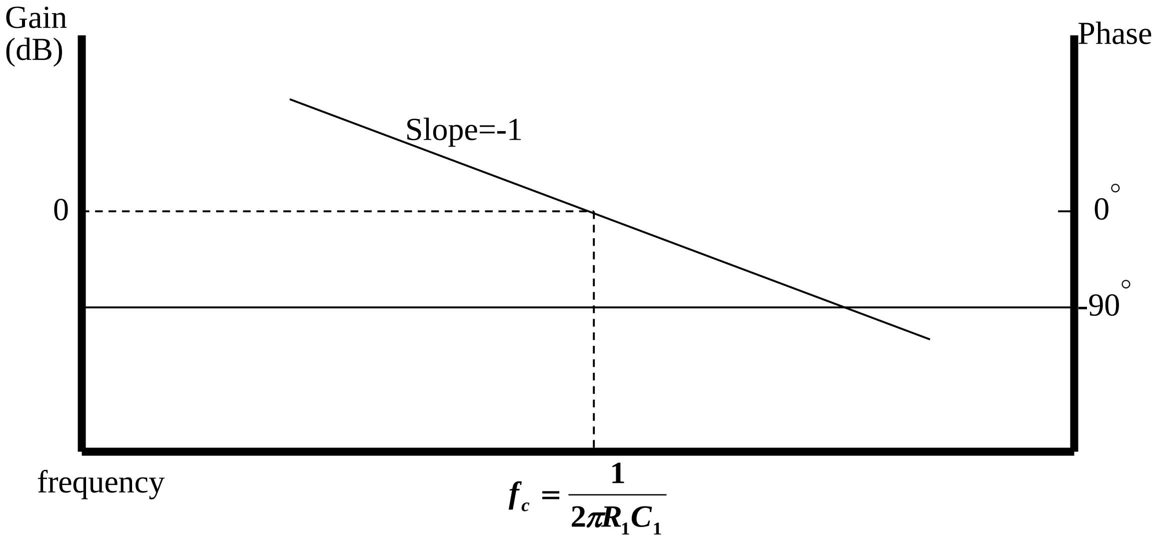
$$= G_{cm} \frac{(1 + s/\omega_L)(1 + s/\omega_z)}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$



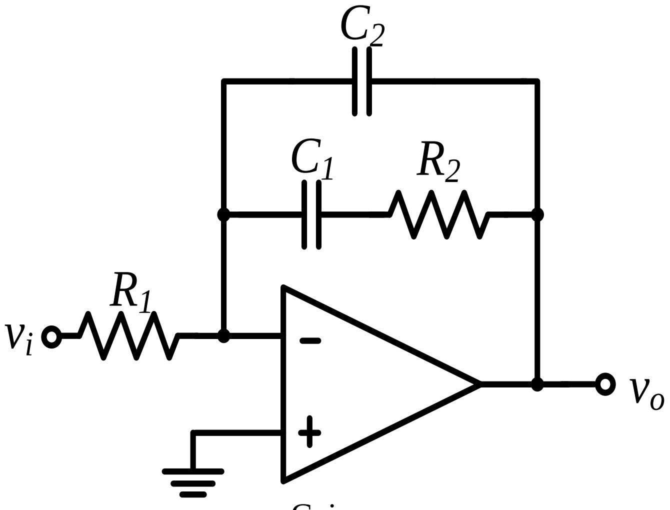
Type I



$$\frac{v_o(s)}{v_i(s)} = \frac{1}{R_1 C_1 s}$$

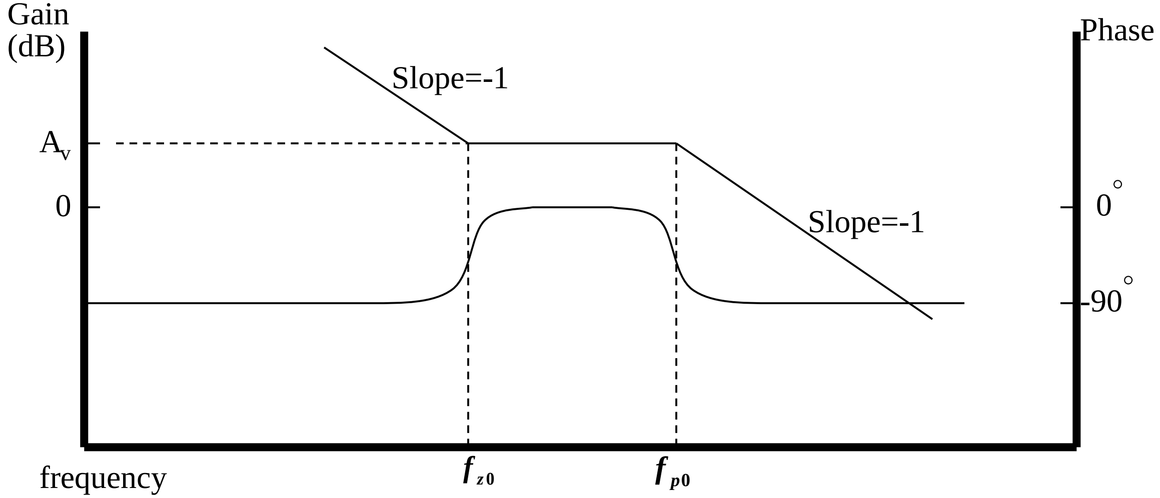


Type II

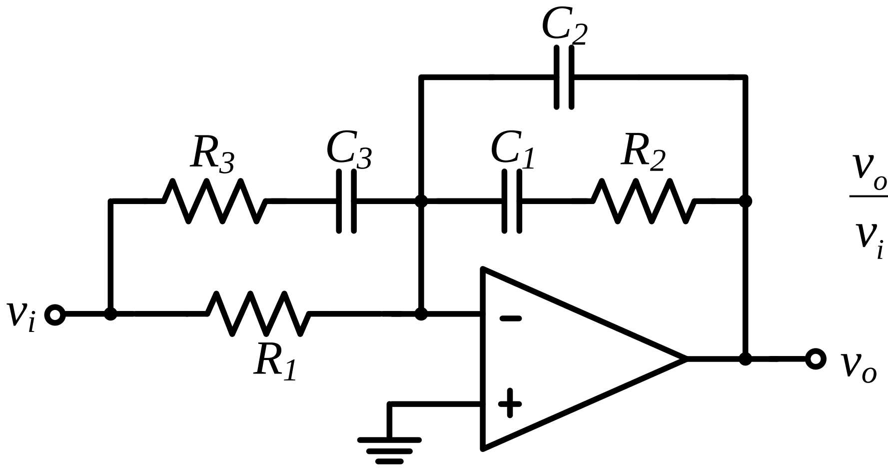


$$\frac{v_o(s)}{v_i(s)} = \frac{1 + sR_2C_1}{sR_1(C_1 + C_2)\left[1 + \frac{R_2C_1C_2}{(C_1 + C_2)}s\right]}$$

$$\approx \frac{1 + sR_2C_1}{sR_1C_1(1 + sR_2C_2)}$$

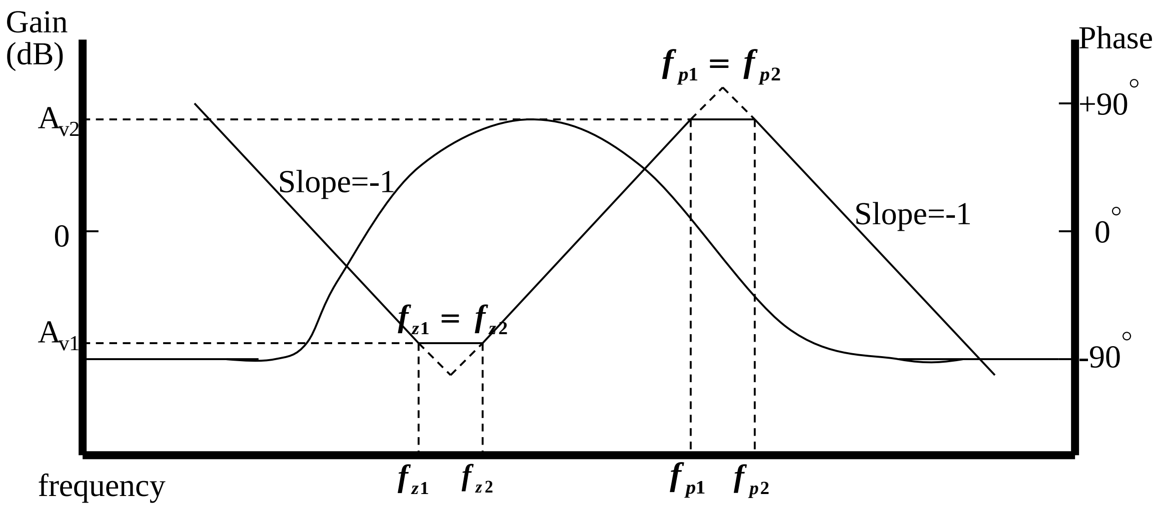


Type III



$$\frac{v_o(s)}{v_i(s)} = \frac{(1+sR_2C_1)[1+s(R_1+R_3)C_3]}{sR_1(C_1+C_2)(1+sR_3C_3)\left[1+\frac{R_2C_1C_2}{(C_1+C_2)}s\right]}$$

$$\approx \frac{(1+sR_2C_1)(1+sR_1C_3)}{sR_1C_1(1+sR_2C_2)(1+sR_3C_3)}$$



K-factor

	Type I	Type II	Type III
Enhance phase	0°	$0^\circ < P < 90^\circ$	$0^\circ < P < 180^\circ$
$v_o(s)/v_i(s)$	$1/sR_1C_1$	$\frac{1+sR_2C_1}{sR_1C_1(1+sR_2C_2)}$	$\frac{(1+sR_2C_1)(1+sR_1C_3)}{sR_1C_1(1+sR_2C_2)(1+sR_3C_3)}$
$K=f_c/f_z=f_p/f_c$	1	$Tan[(P/2)+45]$	$Tan[(P/4)+45]$
Specifications	$C=1/2\pi f_c G$	$R_2=K/2\pi f_c C_1$ $C_1= C_2(K^2-1)$ $C_2=1/2\pi f_c GKR_1$	$R_2=K/2\pi f_c C_1$ $C_1= C_2(K^2-1)$ $C_2=1/2\pi f_c GR_1$ $R_3=R_1/(K^2-1)$ $C_3=1/2\pi f_c GKR_3$
	R_1 is based on actual need to select		

Simulation of Boost

Result:

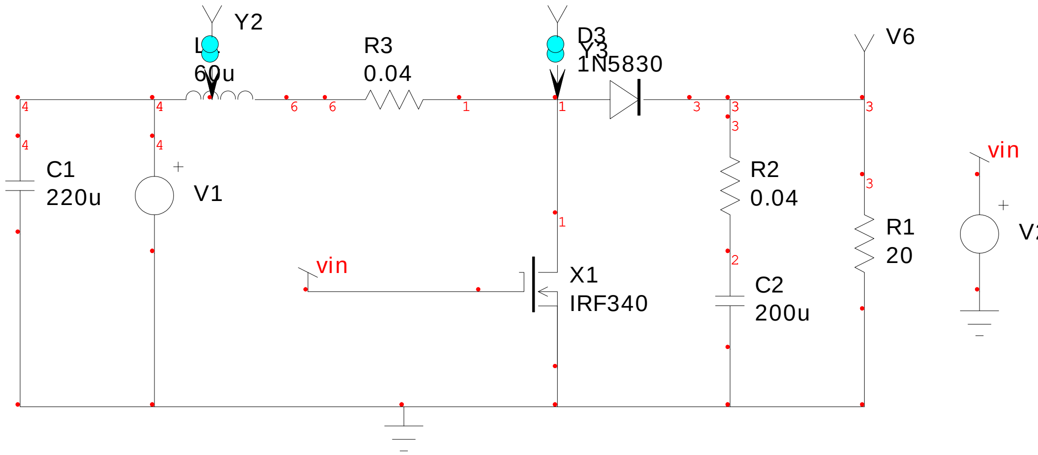
Av=-0.8dB

Q=8.05dB

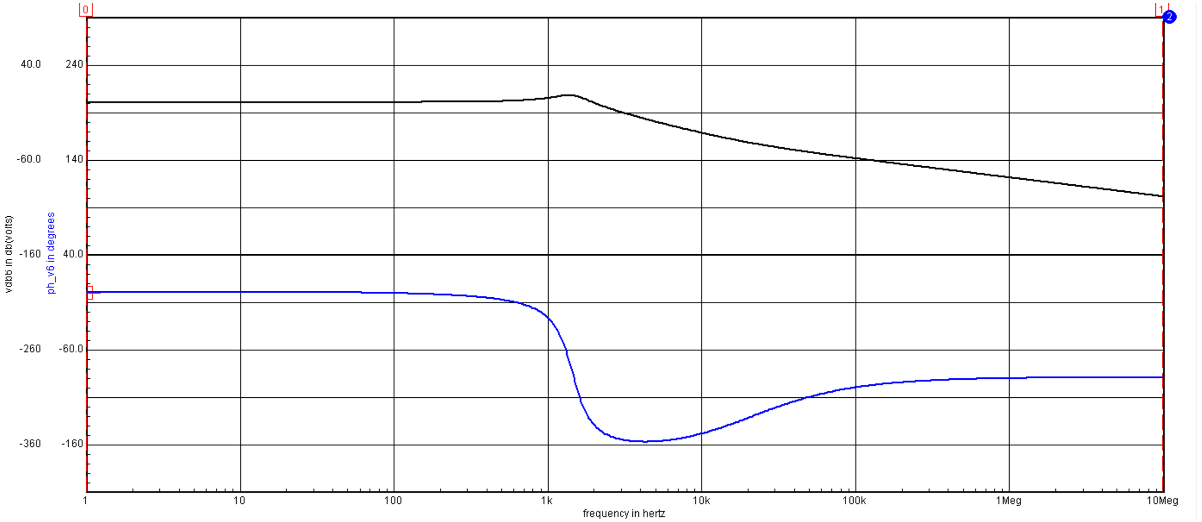
f_{p0}=1.35kHz

PM=42°

f_{z0}=4kHz



D=0.5



Design Example

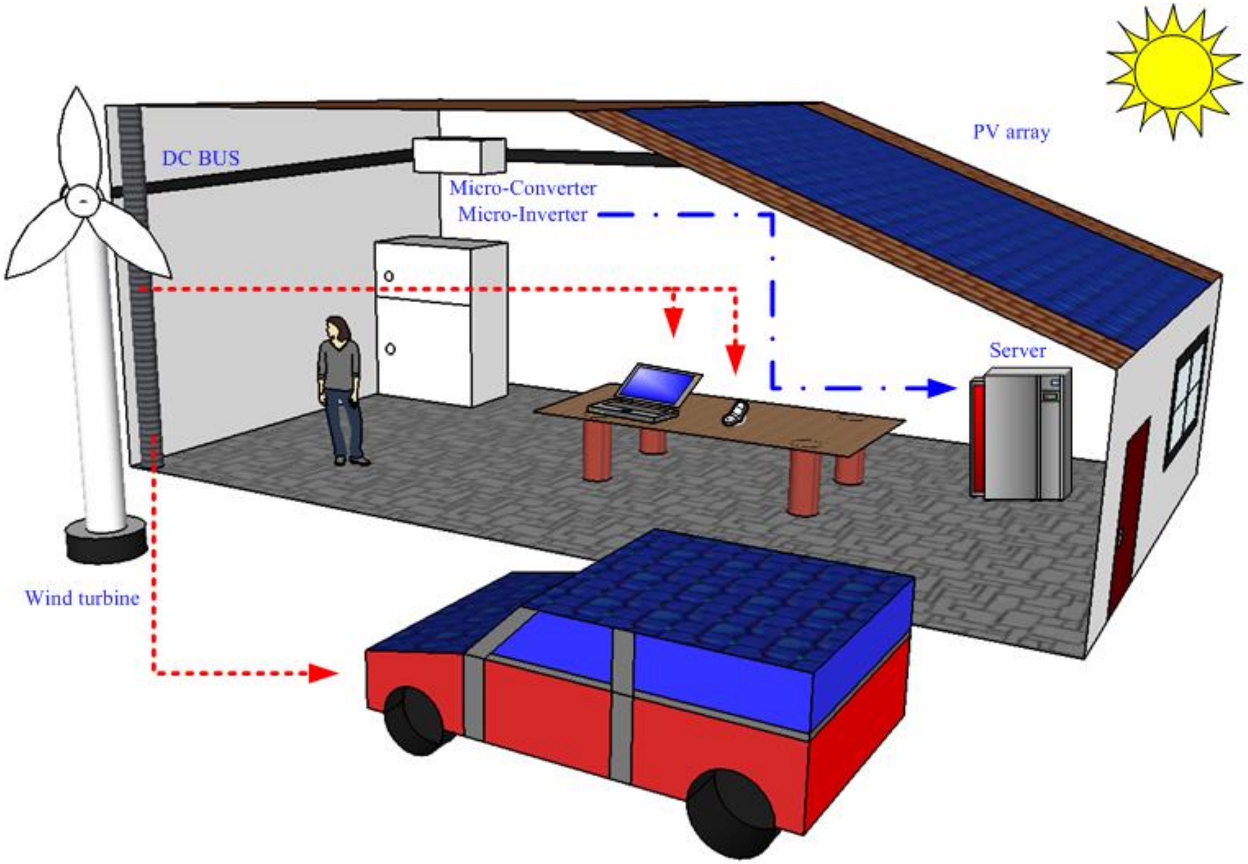
Wireless Energy Harvesting IC and System Design for Smart Home

Advisor：Yeong-Chau Kuo (郭永超 副教授)
Graduate Student： Yi-Ming Huang (黃羿銘)

Outline

- Smart Home
- Battery Model and Charging Method
- Micro-grid
- Wireless Charger
- Controller Architecture
- Synthesis Software
- Measurement
- Conclusion

Smart Home


DC BUS
Micro-Converter
Micro-Inverter
PV array
Server
Wind turbine

Outline

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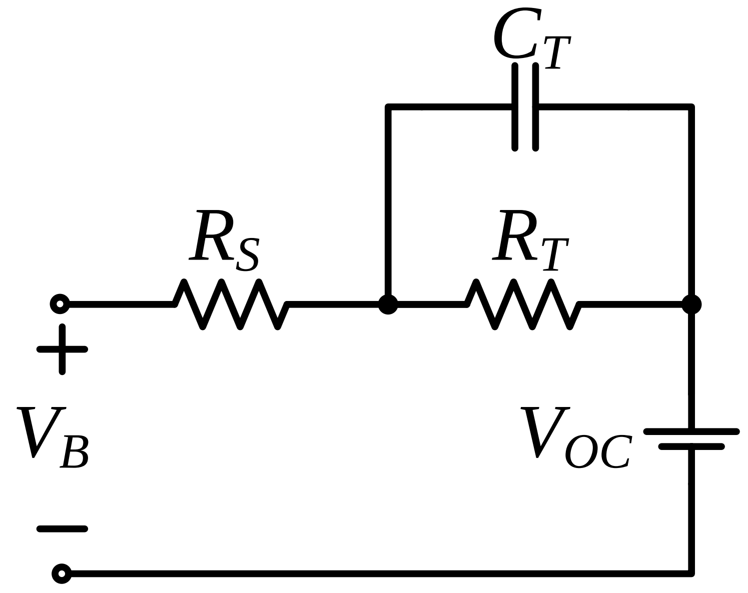
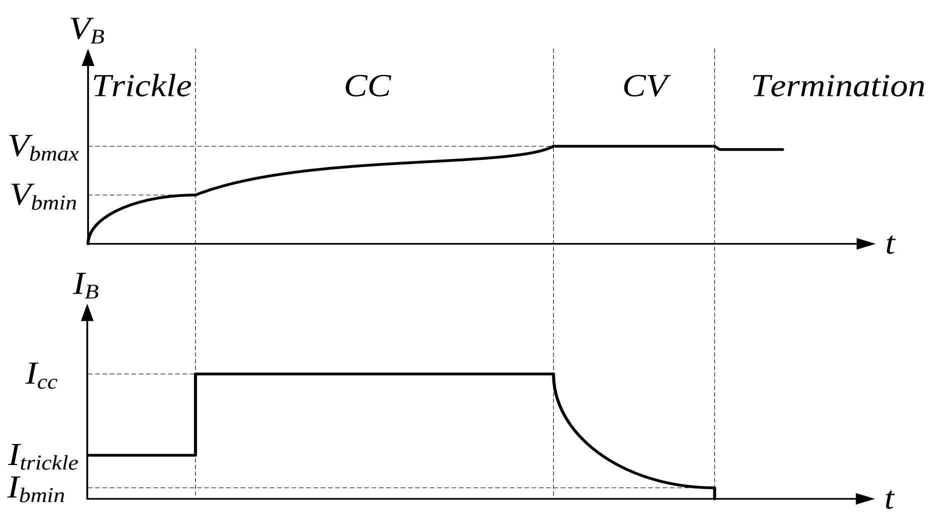
Battery Model and Charging Method (1/2)

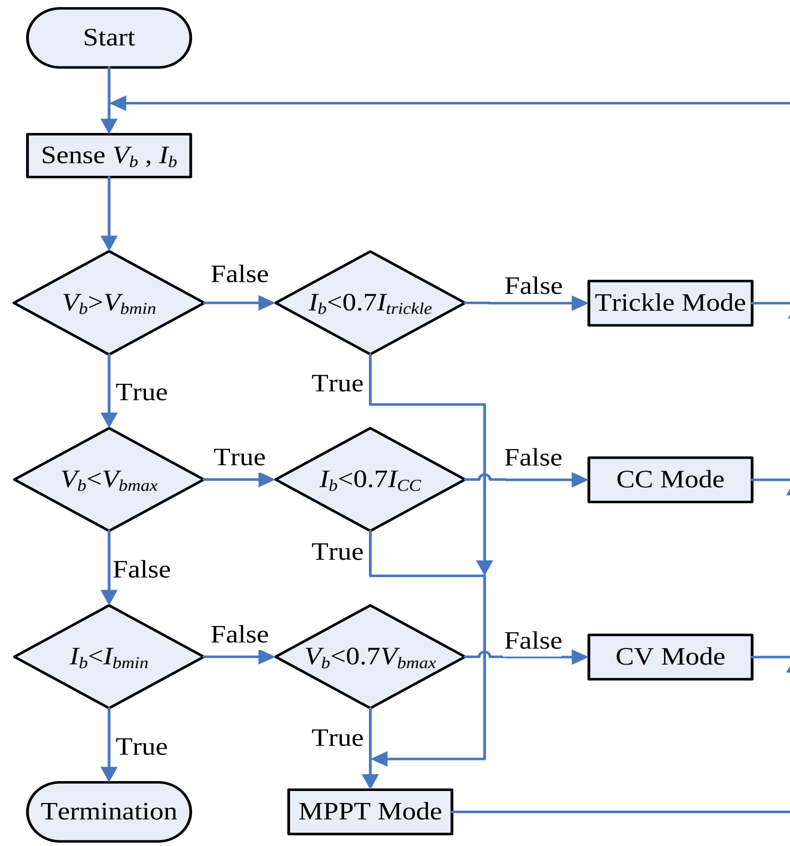
- 戴維寧模型
  - 串聯電阻(R_S)
  - 自放電電阻($R_{Self\text{-}discharge}$)
  - 暫態響應的並聯電阻(R_T)與電容(C_T)
  - 開路電壓(V_{OC})



Battery Model and Charging Method (2/2)

- 充電策略影響了電池的充電效率與壽命。
- 改良型三階段充電法
  - 最大功率追蹤(MPPT)演算法
  - 涓流充電(Trickle Mode)
  - 定電流(CC Mode)
  - 定電壓(CV Mode)





Outline

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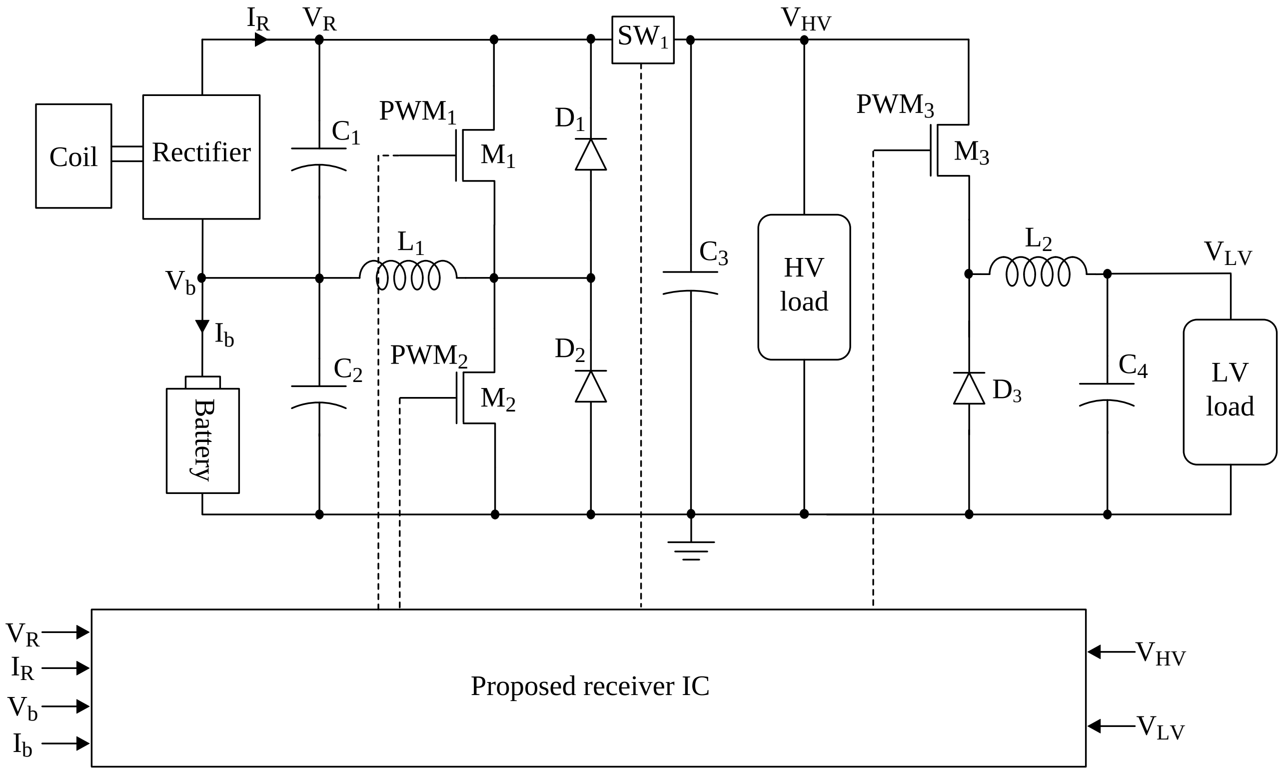
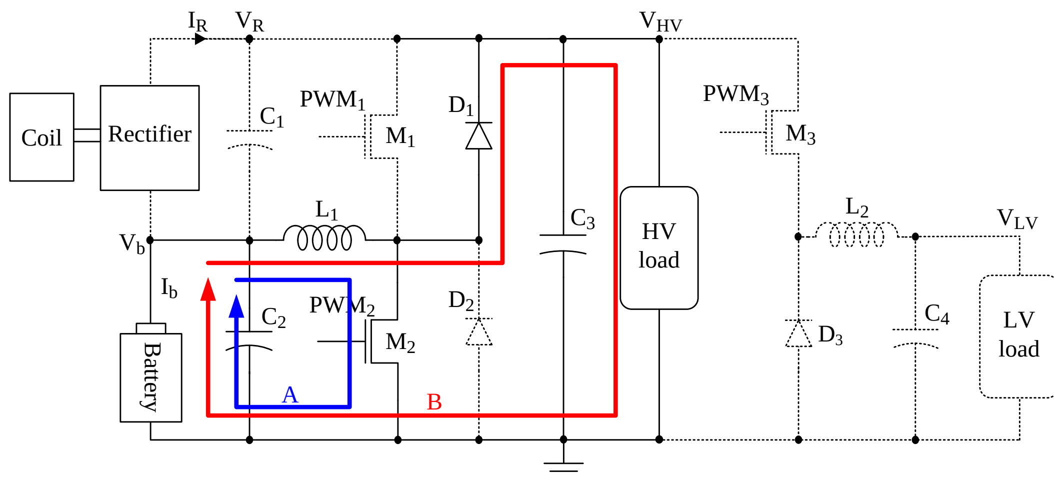
Micro-grid (1/5)

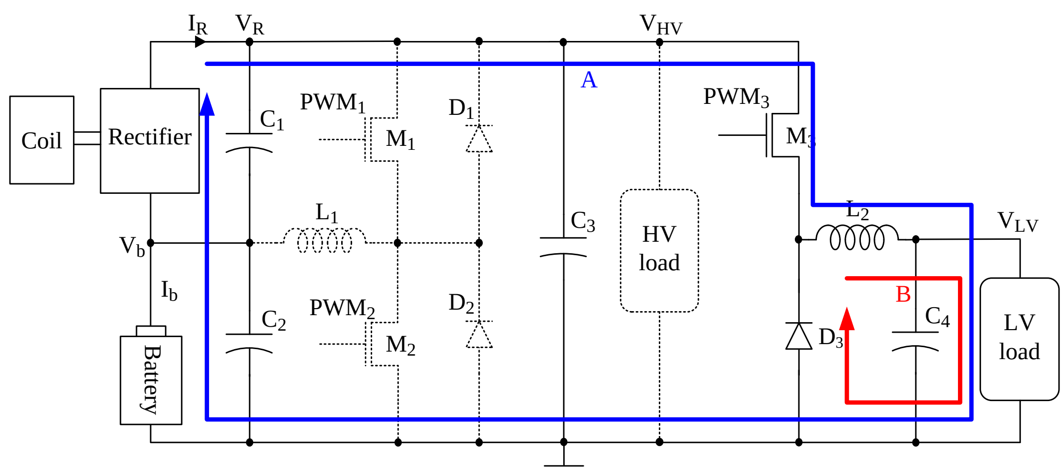
- **Micro-Converter**



Micro-grid (2/5)

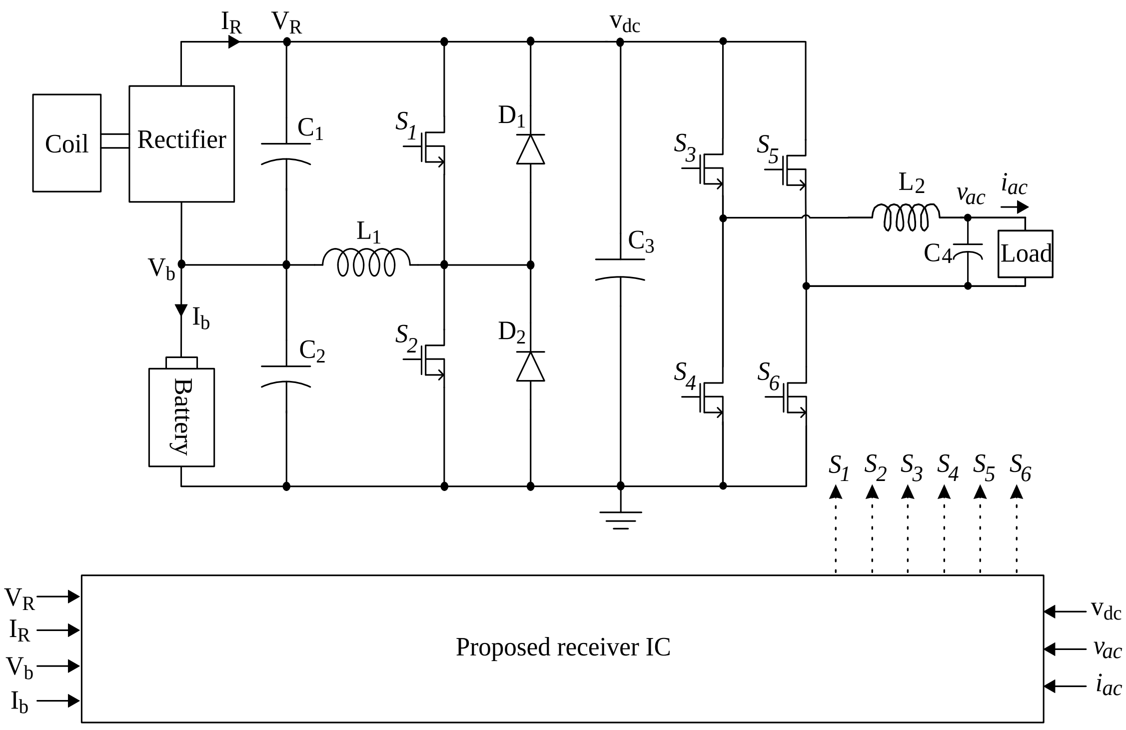
- 動作原理





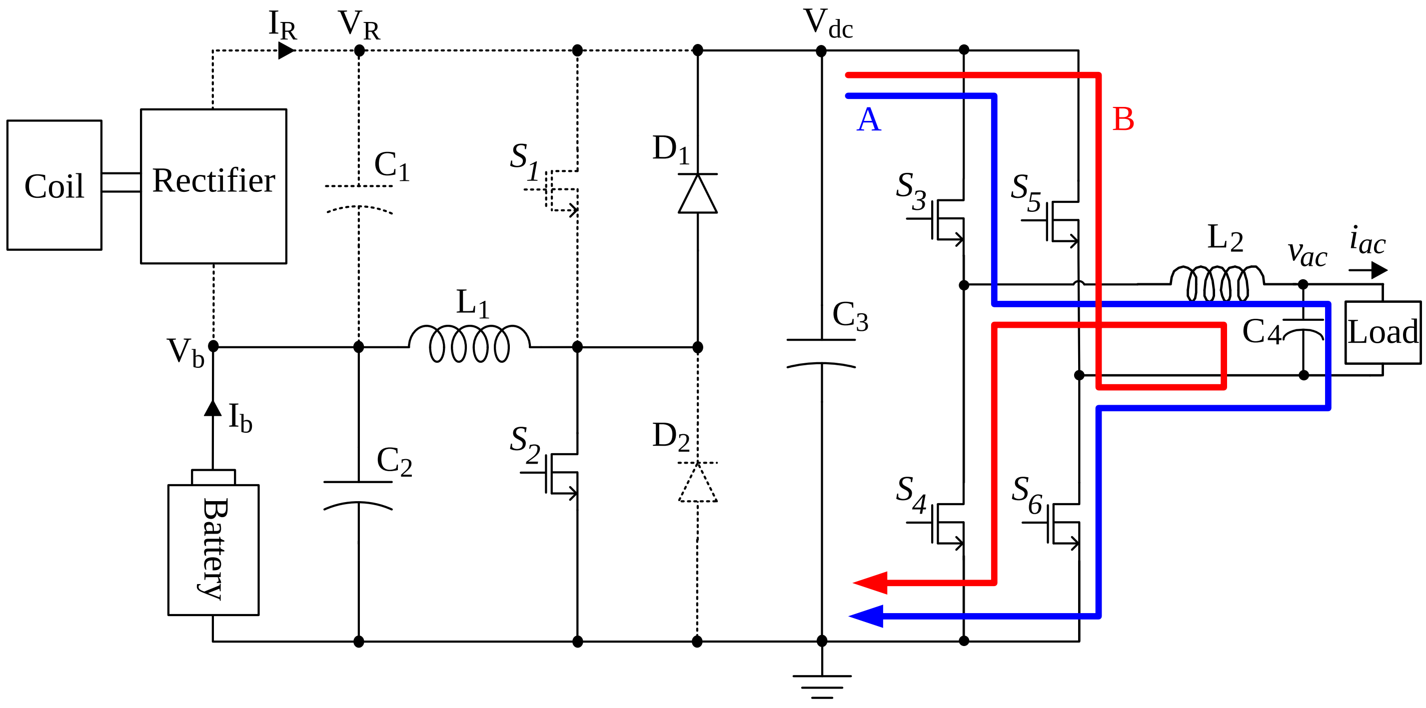
Micro-grid (3/5)

- **Micro-Inverter**



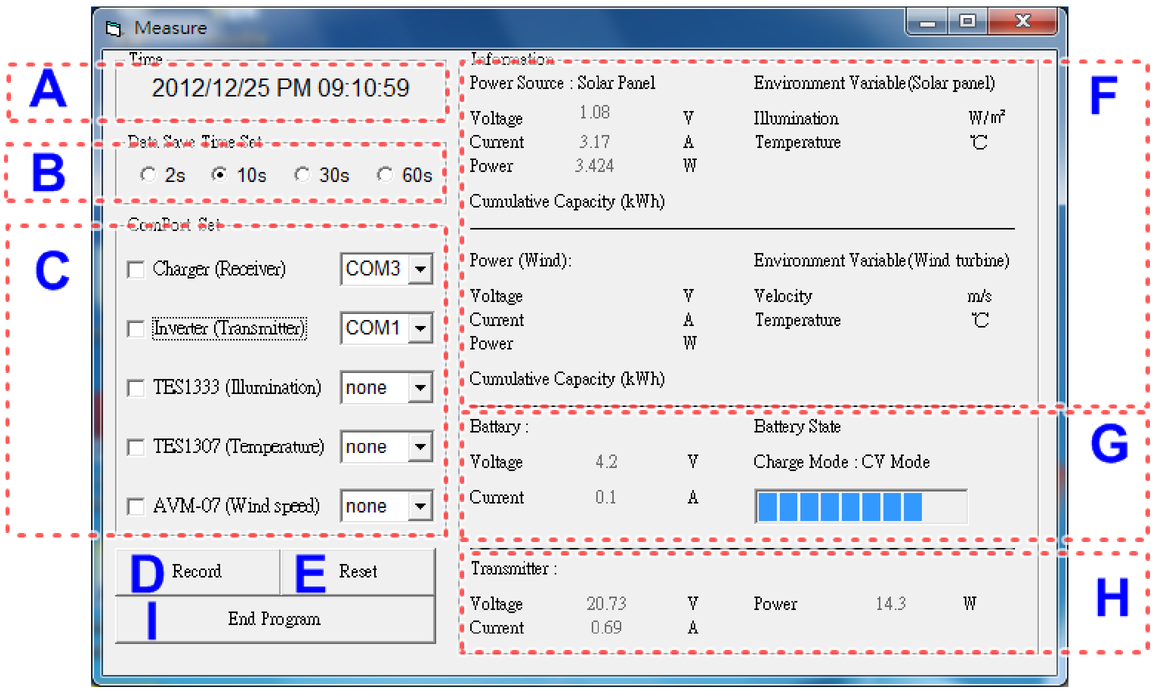
Micro-grid (4/5)

- 動作原理



Micro-grid (5/5)

- Smart meter



介面說明	
A	目前時間
B	資料記錄時間間隔
C	充電器的COMPORT選擇
D	開啟紀錄
E	重新設定
F	再生能源資訊端
G	無線充電接收端充電資訊
H	無線充電傳送端發電資訊
I	離開

Outline

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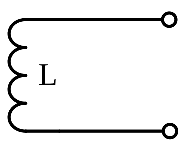
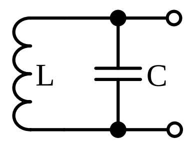
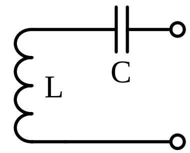
Wireless Charger (1/10)

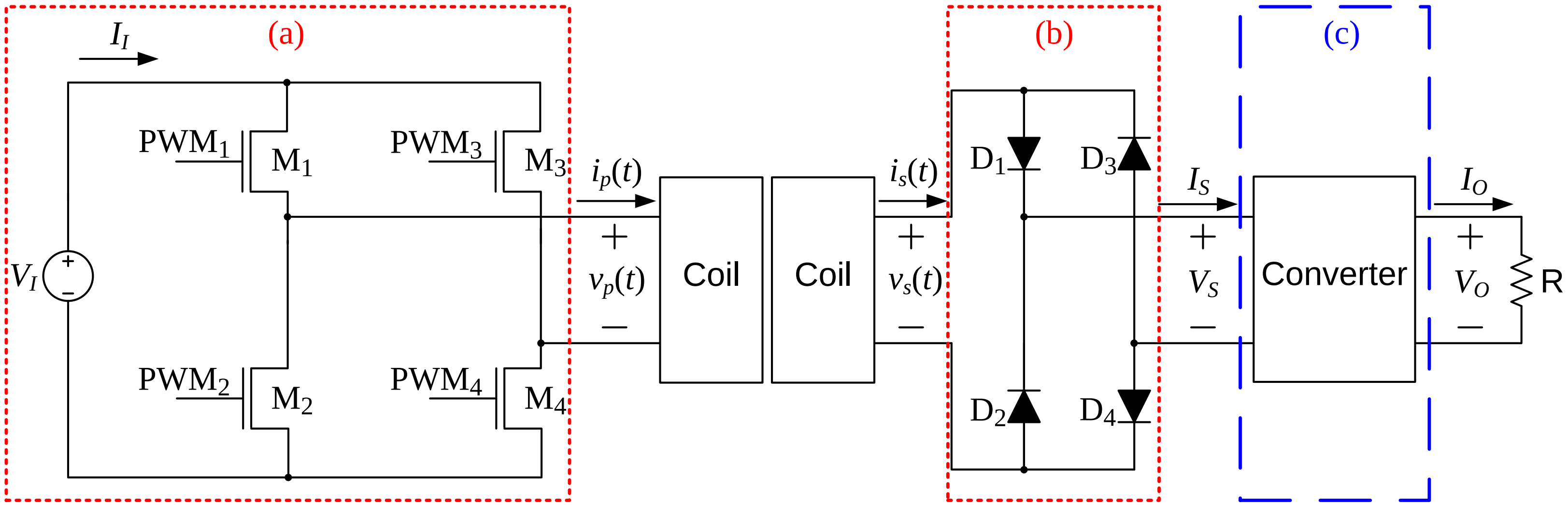
架構			
	磁感應	磁共振(並聯型)	磁共振(串聯型)
有效距離	短	長	中
效率	差	高	高
調整匹配	不用	容易	精密
充電系統	不適用	低功率	高功率

Wireless Charger (2/10)

頻率	125K Hz	13M Hz	1G Hz
產品	有 (Palm，Sanyo)	無，發表 (MIT，Qualcomm)	有 (Powercast)
功率	5W	60W	0.1W
距離	5公厘	1公尺	數公尺
EMC	通過	未通過	未知
成本	低	高	高

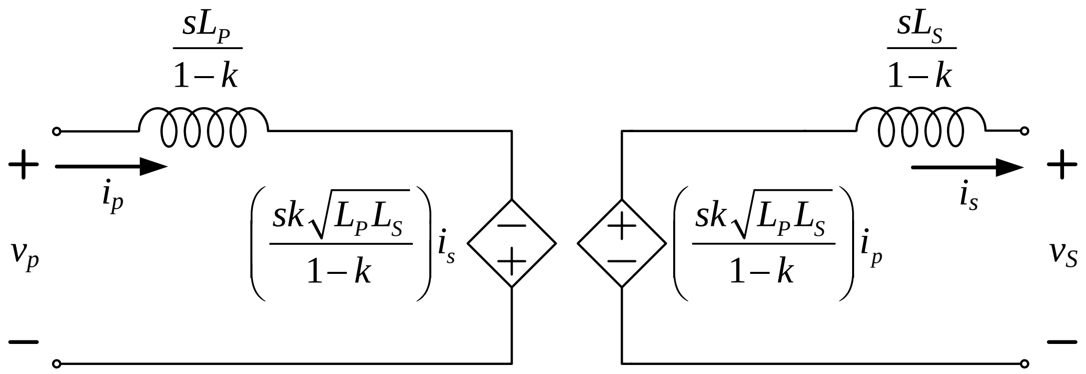
Wireless Charger (3/10)

- 系統架構
  a. DC/AC
  b. AC/DC
  c. DC/DC



Wireless Charger (4/10)

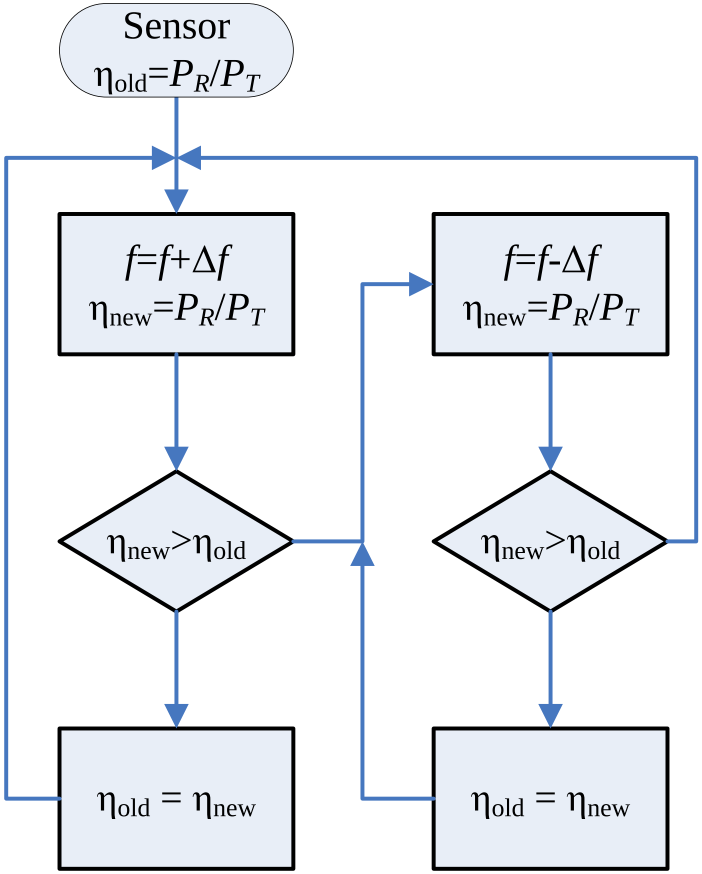
- 線圈模型



- 線圈補償架構
  - ZPA (Zero Phase Angle)
  - SS，SP，PS，PP

Wireless Charger (5/10)

- 變頻追蹤演算法
  - P_T:傳送端功率
  - P_R:接收端功率
  - η:傳送效率



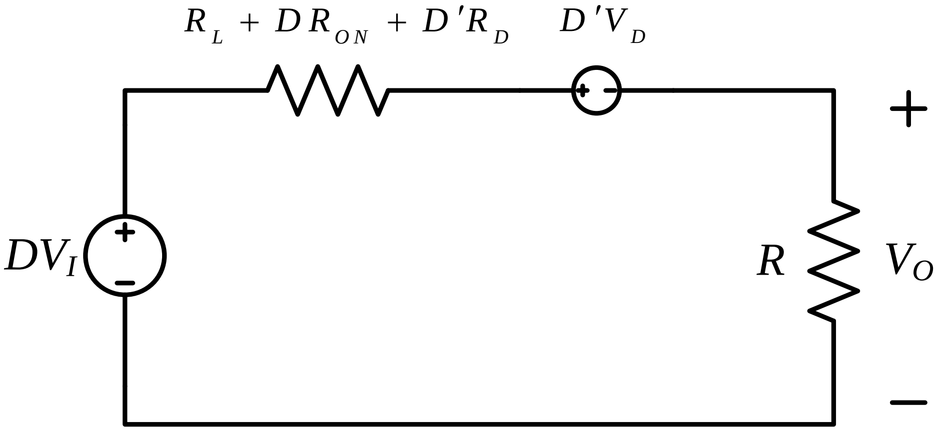
Wireless Charger (6/10)

- 轉換器設計

	直流轉換率	電感值	電容值
Buck	$\frac{V_O}{V_I}=D$	$L \ge \frac{V_O T_S(1-D)}{2\Delta i_L}$	$C \ge \frac{V_O T_S^2(1-D)}{16L\Delta v_o}$
Boost	$\frac{V_O}{V_I}=\frac{1}{D'}$	$L \ge \frac{D V_I T_S}{2\Delta i_L}$	$C \ge \frac{D I_O T_S}{2\Delta v_o}$
Buck-Boost	$\frac{V_O}{V_I}=-\frac{D}{D'}$	$L \ge \frac{V_O T_S(1-D)^2}{2\Delta i_L}$	$C \ge \frac{D I_O T_S}{2\Delta v_o}$

Wireless Charger (7/10)

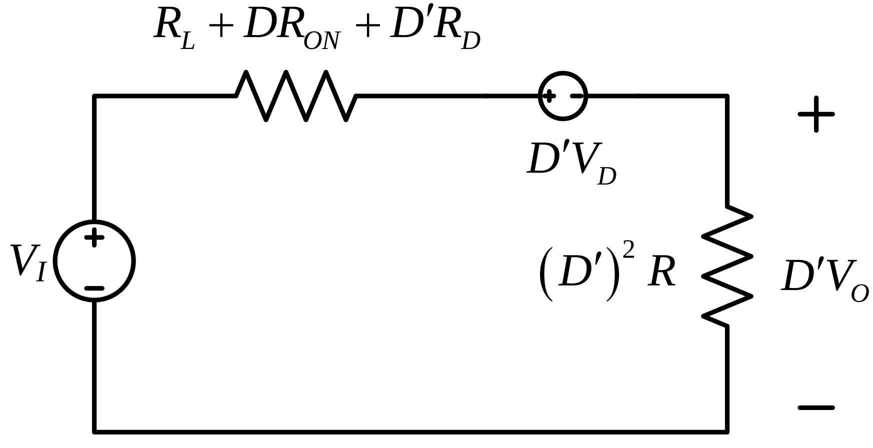
- Buck



$$\eta = \frac{V_O I_O}{DV_I I_I} = \frac{\left(1 - \frac{D'V_D}{DV_S}\right)R}{R_L + DR_{ON} + D'R_D + R}$$

Wireless Charger(8/10)

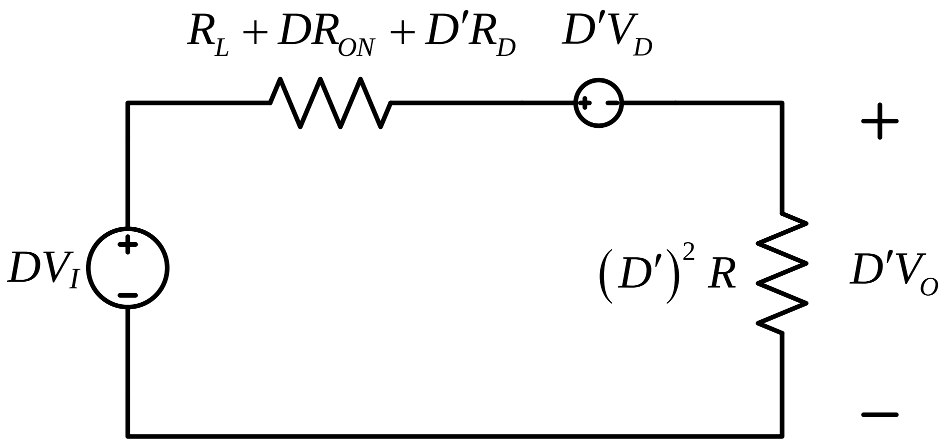
- Boost



$$\eta = \frac{D'V_O I_O}{V_I I_I} = \frac{\left(1 - D'\frac{V_D}{V_I}\right)(D')^2 R}{R_L + DR_{ON} + D'R_D + (D')^2 R}$$

Wireless Charger (9/10)

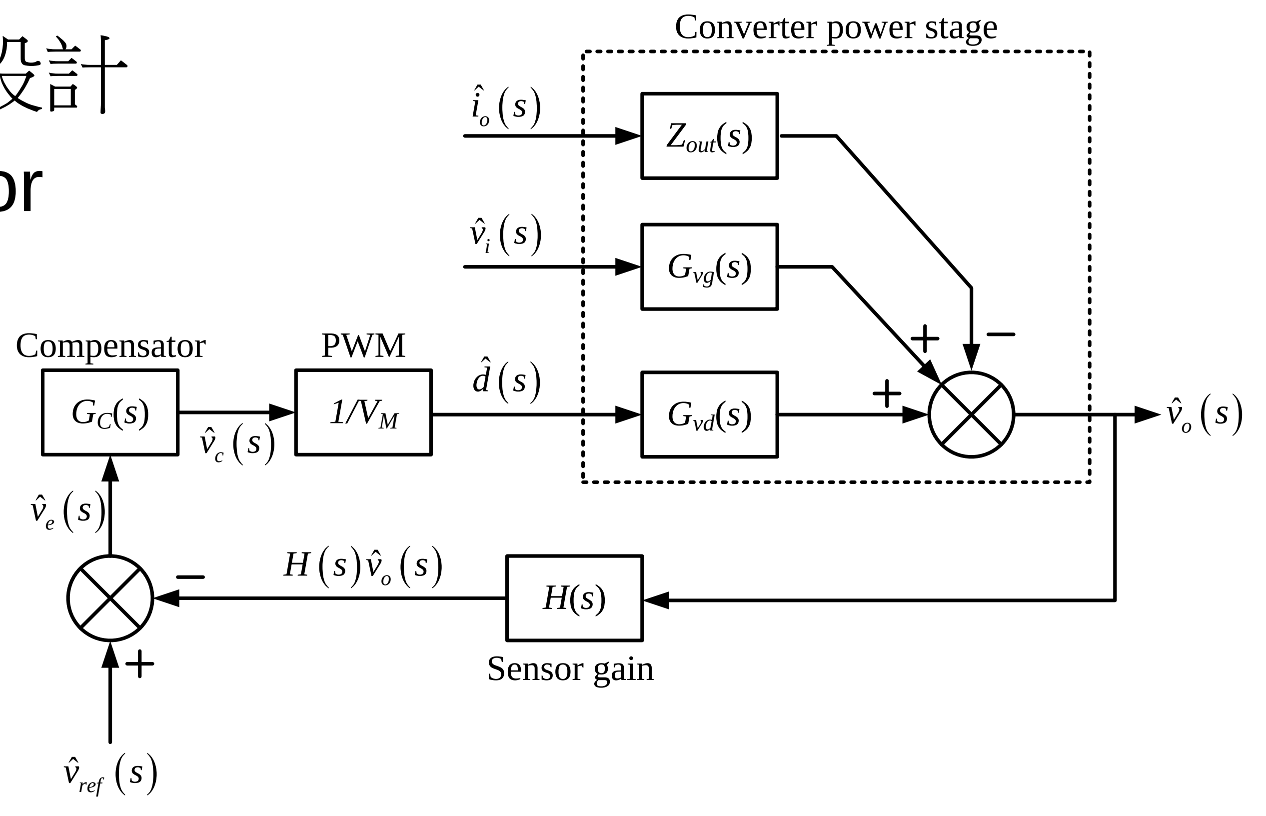
- Buck-Boost



$$\eta = \frac{D'V_O I_O}{DV_I I_I} = \frac{\left(1 - \frac{D'V_D}{DV_I}\right)(D')^2 R}{R_L + DR_{ON} + D'R_D + (D')^2 R}$$

Wireless Charger (10/10)

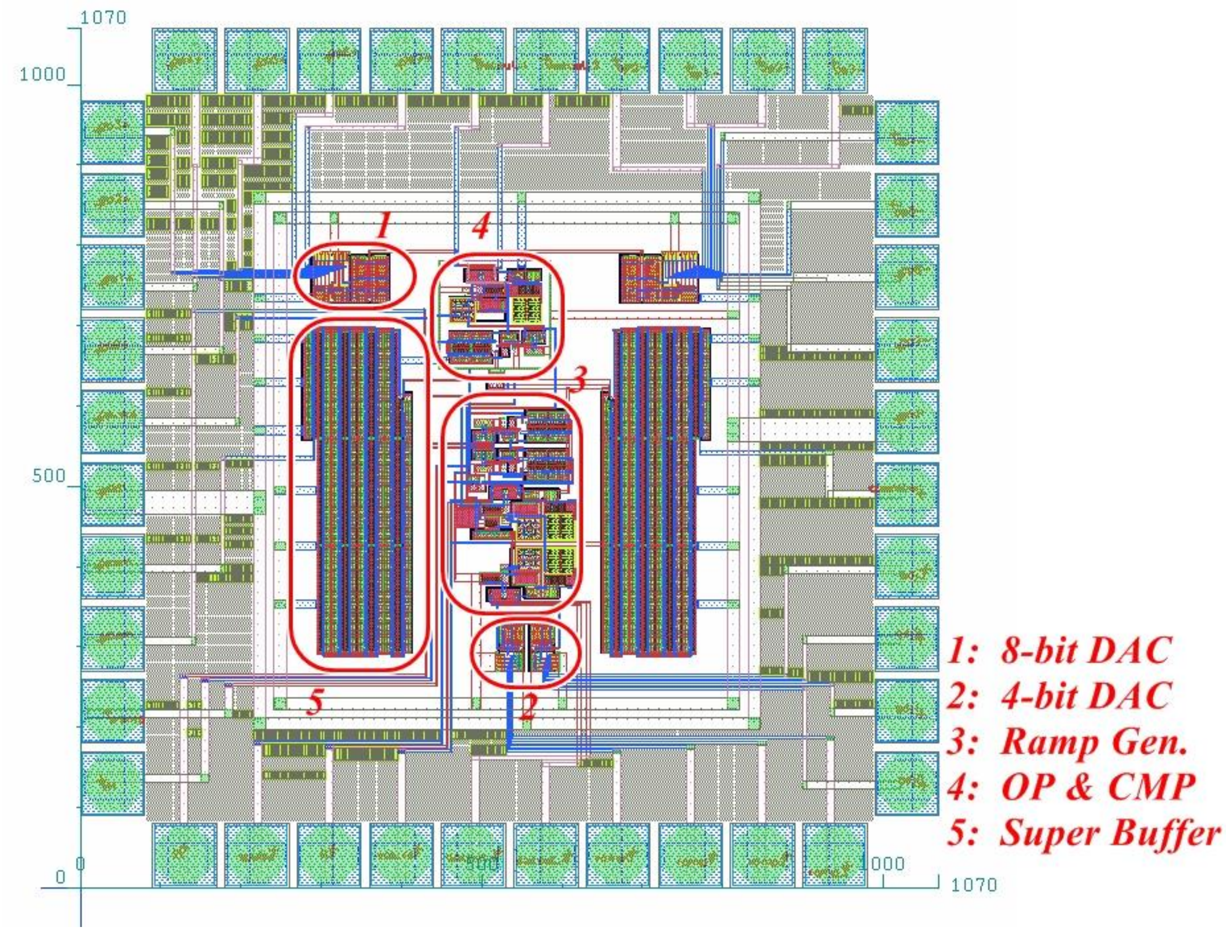
- 暫態分析
- 補償器設計
  - K factor



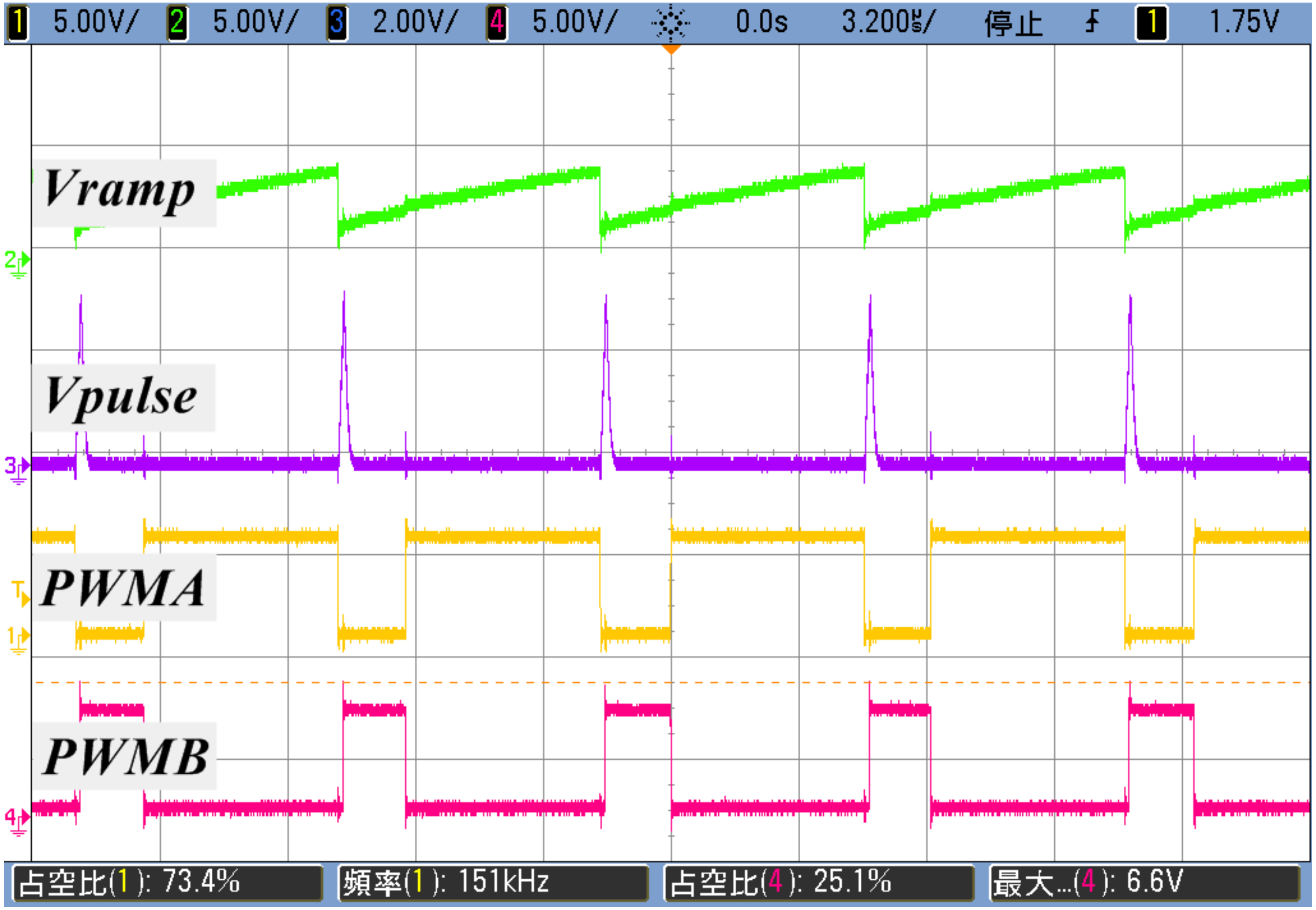
Outline

- Smart Home
- Battery Model and Charging Method
- Micro-grid
- Wireless Charger
- Controller Architecture
- Synthesis Software
- Measurement
- Conclusion

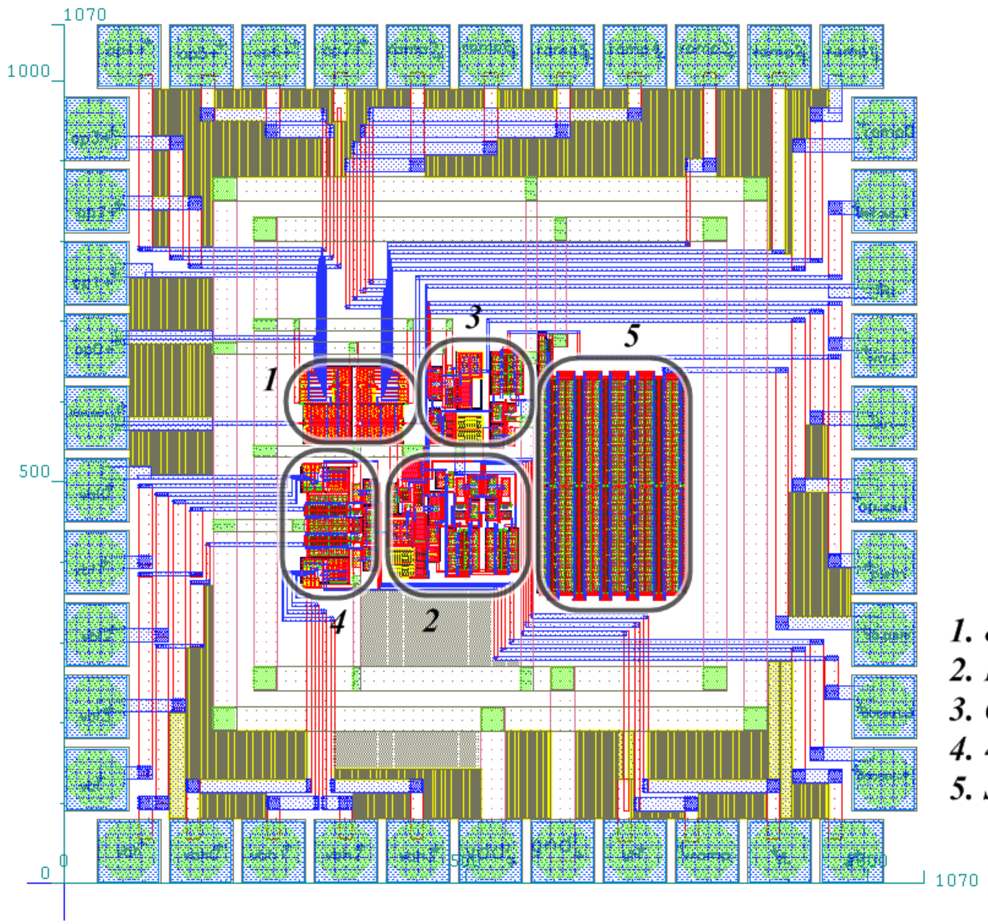
Controller Architecture (1/3)



Controller Architecture (2/3)



Controller Architecture(3/3)



1. 8-bit DAC
2. Ramp Gen.
3. OP & CMP
4. 4-bit DAC & CMP
5. Super Buffer

Outline

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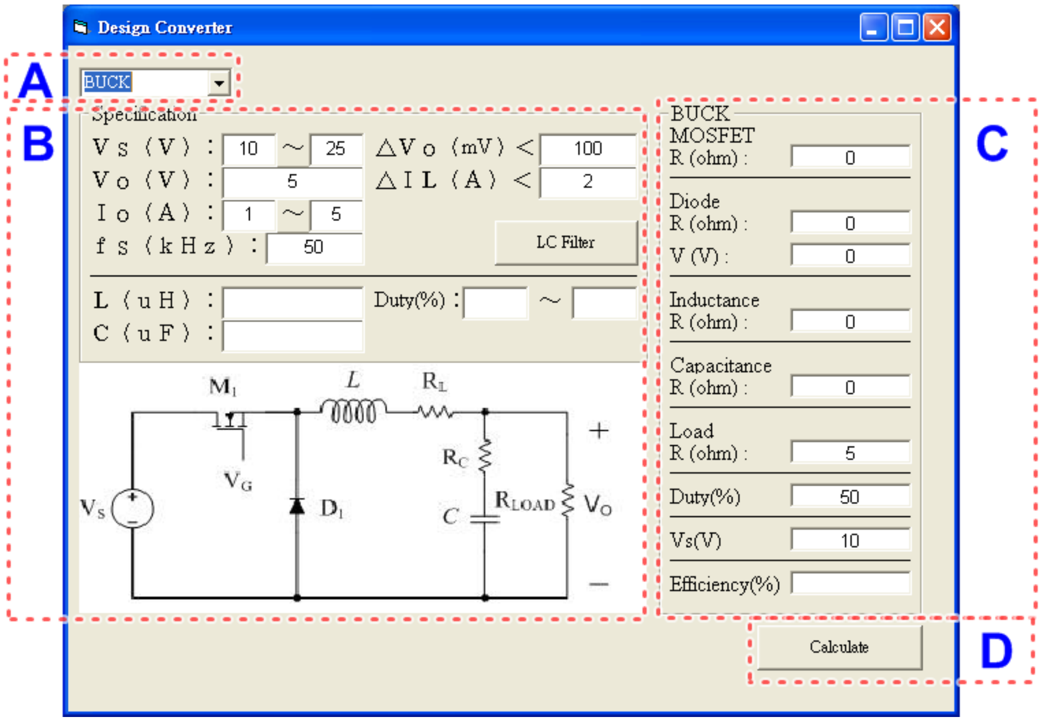
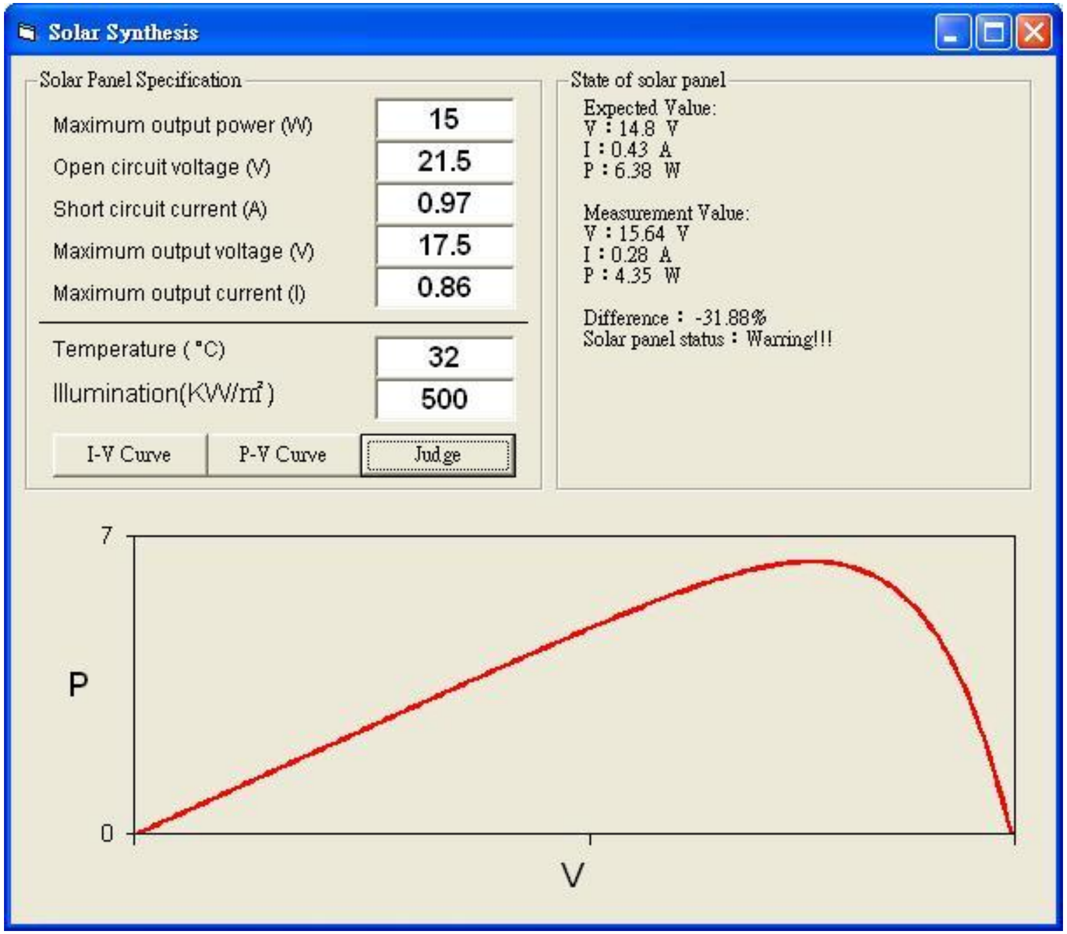
Synthesis Software (1/3)



轉換器合成系統	
A	選擇轉換器 (BUCK，BOOST，BUCK-BOOST)
B	規格設定
C	效率估算
D	計算各參數值

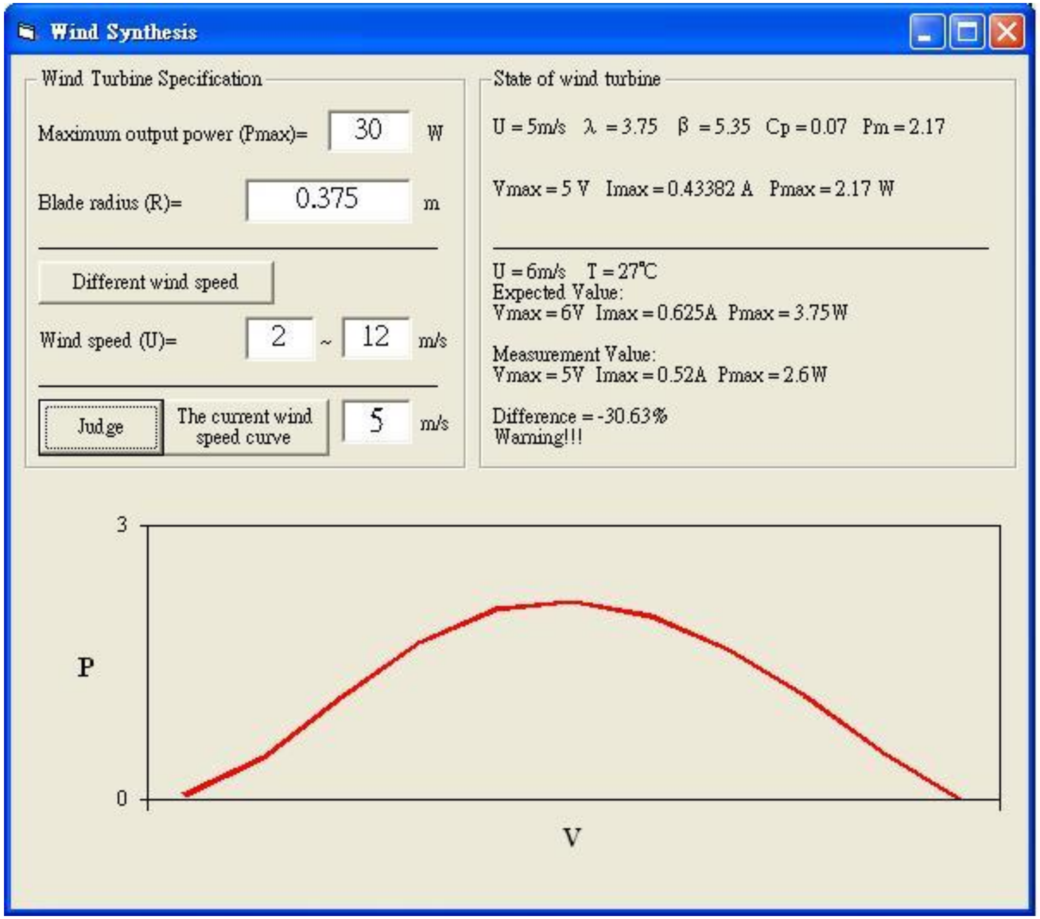
Synthesis Software (2/3)

- 太陽能診斷系統



Synthesis Software (3/3)

- 風能診斷系統



Outline

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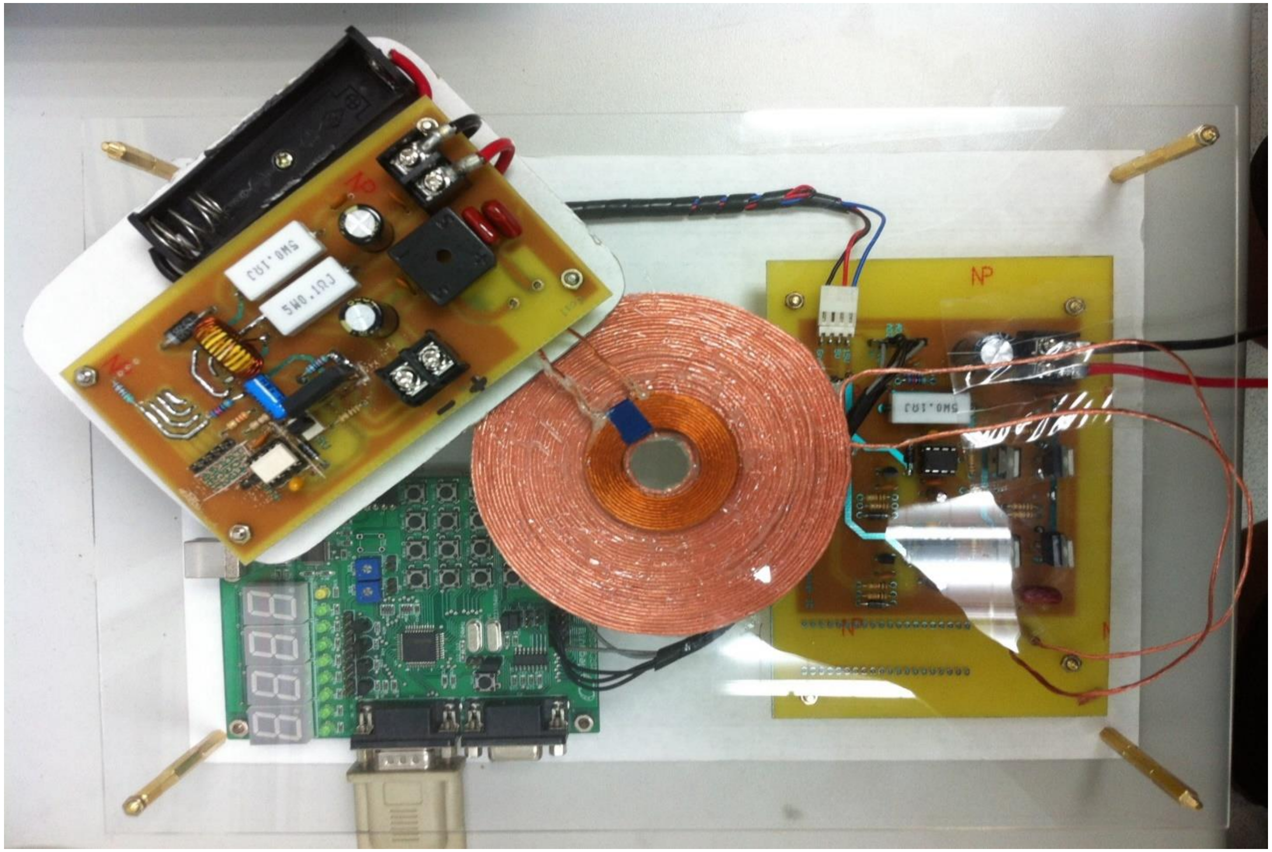
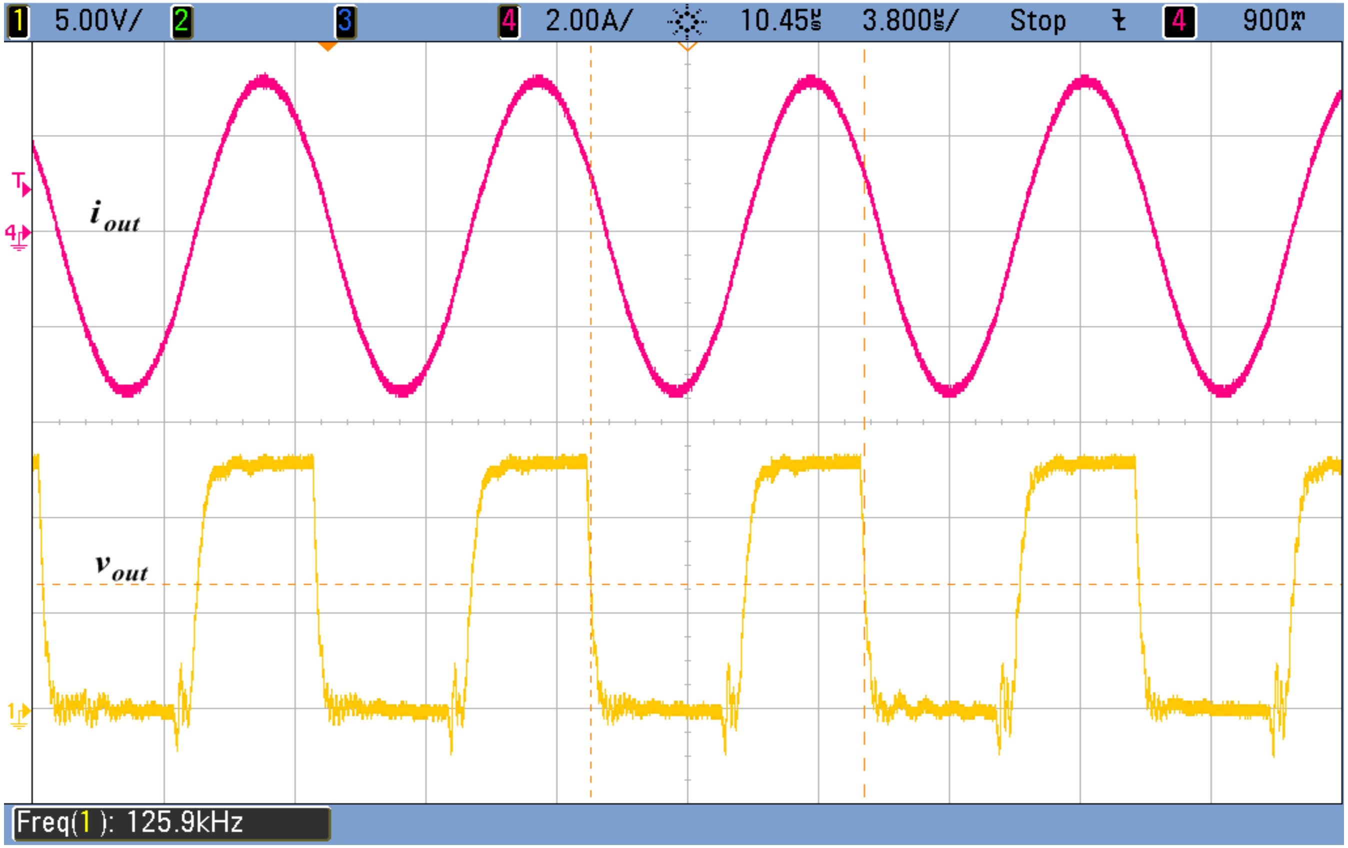
Measurement (1/4)

- 無線充電成品



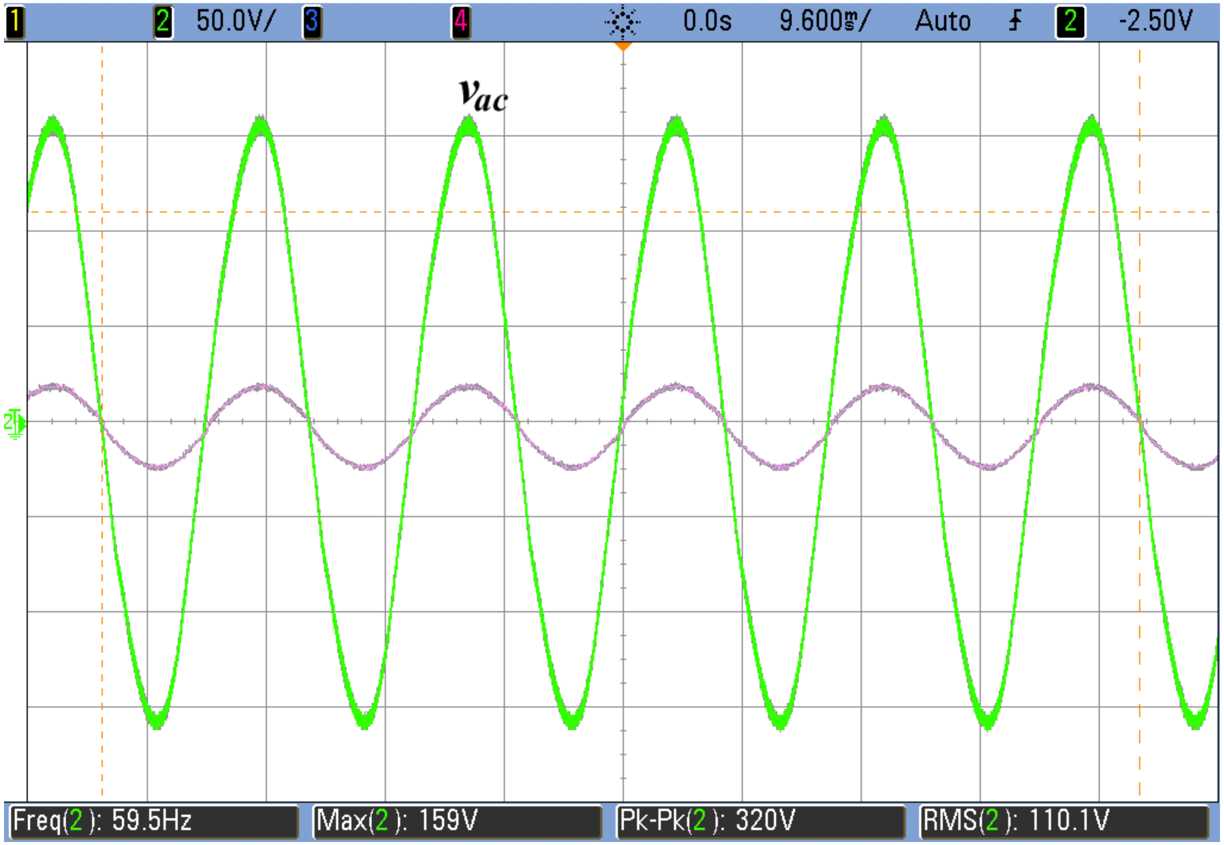
Measurement (2/4)

- 無線充電系統



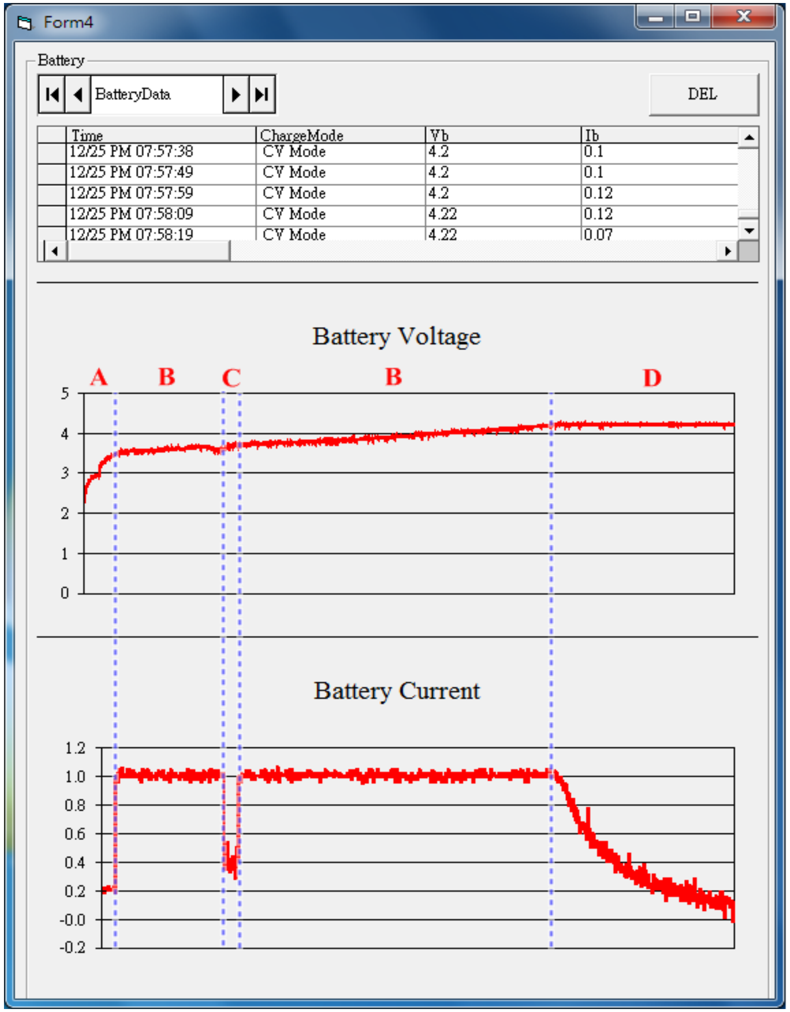
Measurement (3/4)

- Micro-Inverter



Measurement (4/4)

- 充電結果
  - A. Trickle Mode
  - B. CC Mode
  - C. MPPT Mode
  - D. CV Mode



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Conclusion

- 無線充電系統藉由各種模型的分析設計，提高其穩定度與可靠度，利用自行撰寫的電表系統，可管理各再生能源的用電情況與診斷分析系統運作情況，建立了未來智慧型家庭的雛型系統。

- 未來採用32位元的微控制器系統，將會使系統有更精確的控制，達到更佳的效率與穩定度，結合所開發的傳送器與接收器積體電路，可更縮小系統的體積與成本。

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[4] V. Vorperian, “Simplified analysis of PWM converters using model of PWM switch” *IEEE Trans. Aerospace Electron. Syst.,* vol. 26, pp. 490-496, May 1990.

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Thanks for your attention!!